Journal of the
Operations Research
Society of Japan

VOLUME 13 *January 1971* NUMBER 3

ON THE INTERFERENCE TIME DURATION AT THE RAILWAY INTERSECTION

—— A Mathematical Model ——

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(Received November 2, 1970)

1. Introduction

In the present paper we consider three fundamental types of the intersections in the railway traffic, and calculate the interference time duration in each type. The first type is the simple cross of two one-way lines. In this type, the interference duration is obtained directly by considering that there can not be two or more trains simultaneously in the interference intervals at the intersection. The second type is the merging, and we consider an one-way line with another one-way line merging to the former. The interference duration in this case is obtained by considering the congestion after merging in addition to the above circumstance. In the third type, the mixed type of the simple cross and the merging, the interference duration is calculated by using the distributions of the next arrival intervals under the assumption of independent interarrival intervals of trains.

As to the stationary case of the third type, a simplified formula

$$T\left\{1-\lambda_a\lambda_c\int_{\varepsilon}^{\infty}(1-F(x))\,dx\int_{\zeta}^{\infty}(1-H(x))\,dx\right\}\qquad \text{(in the case that } t_a=t_c>\varepsilon\text{)}$$

is given in [1] b) and [2], while in this paper we deal with the non-stationary case and more precise formulas are given.

2. Notation, Terminology, and Assumptions

1) 3 types of the intersections in the railway traffic illustlated by Fig. 1 are considered.

2) In Fig. 1, PQ, RS, RQ′, and P′Q′ are called the interference intervals at the intersection. There can not be two or more trains simultaneously in the interference intervals at the intersection.

3) The trains on the A-route, B-route, and C-route are called A-trains, B-trains, and C-trains, respectively.

4) A-trains and C-trains have priority over B-trains.

Then, the interference time duration I_B of B-trains by the A-trains or C-trains is in question.

[Assumption I] The time interval X_i of A-trains at the place P is a random variable whose minimum value is $\varepsilon_i>0$. The time interval Z_i of C-trains at the place P′ is a random variable whose minimum value is $\delta_i>0$. Let θ_i be the minimum time interval between train C_i (or train C_{i+1}) and train B which merges in the interval Z_i of C-train, C_i and C_{i+1}. (For $R>0$ which is defined in the sequel, we assume that $\theta_i\geqq R$.)

N.B. 1. We may consider the limit cases $\varepsilon_i=0$ or $\delta_i=0$.

N.B. 2. In [1] and [2], we have assumed $\varepsilon_i=\delta_i=\theta_i=\varepsilon$.

[Assumption II] Let t_{a_i}, t_b, and t_{c_j} be the time durations for train A_i, train B, and train C_j to pass the interference interval respectively, and suppose $t_b\geqq t_{c_i}$, $t_{c_{i+1}}$ in the type (II) and the type (III). Whenever $t_b\leqq\max(t_{c_i}, t_{c_{i+1}})$, we modify t_b to such t_b^* that $\min(t_{c_i}, t_{c_{i+1}})\leqq t_b^*\leqq\max$

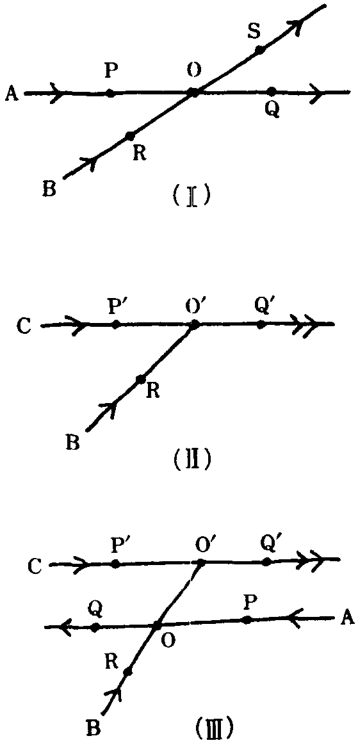

Fig.1. 3types of the intersections

$(t_{c_i}, t_{c_{i+1}})$ by decreasing the speed of train B in the interference interval.

N.B. 3. We may practically include the redundancy into t_a, t_b, and t_c. It implies that the theoretical interference intervals are extended. The quantities t_a, t_b, and t_c are influenced by the length of the train.

[Assumption III] In the type (II), (III), we consider the interference on the C-route from the intersection to the next shunting station of the intersection, but not the railway capacity of C-route after the station.

[Assumption IV] In the type (II), (III), the higher speed the train has, the shorter time duration it takes to pass through the interference interval. (This assumption is always adequate so far as the lengths of the interference intervals do not differ too much in the same intersection and the lengths of the trains also do not differ too much each other.)

Notation and terminology

T : The length of the period in question. (T=const.) E.g. $T=24$ hours.

I_B : The total of the interference time durations. (A random variable.) ($I_B = T-$[The total of the periods through which B-trains are able to pass.], $I_B \leqq T$.)

ξ_i^{I}, ξ_i^{II}, etc.: The interference constants. (The constants determined in the following.)

F_i : The distribution of the interval X_i of A-trains at P.

H_i : The distribution of the interval Z_i of C-trains at P'.

N_a : The number of A-trains in the period T.

N_c : The number of C-trains in the period T.
(N_a and N_c are constants in the type (I), (II), while they are random variables in the type (III).)

R : The minimum of the safety time interval from the arrival of B-trains to the arrival of the train C_{i+1} at the shunting station. ($\theta_i \geqq R > 0$.)

3. Interference Time Durations

Type (I) In the distance-time diagram, B-trains can be inserted between A-trains A and A^* if any only if the time interval between A and A^* is larger than

$$(1) \qquad \xi_i^{\mathrm{I}} = \max(t_a + t_b,\ \varepsilon_i) \ .$$

In the period T, the total length of the time through which B-trains are able to pass is

(2) $$T_a=\sum_{i=0}^{Na}(X_i-\xi_i^{\mathrm{I}})^+ .^{*)}$$

Then,

(3) $$I_B=T-T_a,$$

and its mathematical expectation

(4) $$\mathrm{E}I_B=T-\mathrm{E}T_a=T-\sum_{i=0}^{Na}\int_{\xi_i^{\mathrm{I}}}^{\infty}(1-F_i(x))\,dx ,$$

where $\xi_i^{\mathrm{I}}=\max(t_{a_i}+t_b,\ \varepsilon_i)$.

N.B. 4. In the case $\xi_i^{\mathrm{I}}=\varepsilon_i$ (i.e. $t_a+t_b<\varepsilon_i$),

$$\int_{\xi_i^{\mathrm{I}}}^{\infty}(1-F_i(x))\,dx=\mathrm{E}X_i-\varepsilon_i, \text{ and } \mathrm{E}I_B=\sum_{i=1}^{Na}\varepsilon_i .$$

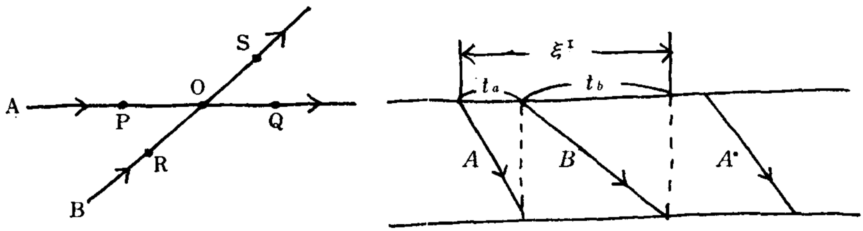

Fig.2. The interfernce constant ξ^{I} in Type(I).

Type (II) Let U_c and U_b be the driving time (the time needed to go) from Q′ to the next shunting station of C-trains and B-trains respectively. Then, from Fig. 3,

(6) $$\xi_i^{\mathrm{II}}=\max(t_{c_i},\ \theta_i)+t_b+\max[(\theta_i-t_{c_{i+1}})^+,\ (U_b-U_{c_{i+1}}+R-t_{c_{i+1}})^+].$$

The total of the time durations through which B-trains are able to pass is

*) $(x)^+\equiv\max(0,x)$.

(7) $$T_c=\sum_{i=0}^{Nc}(Z_i-\xi_i^{II})^+ ,$$

and then the total of the interference time durations is

(8) $$I_B=T-T_c=T-\sum_{i=0}^{Nc}(Z_i-\xi^{II})^+ ,$$

and its mathematical expectation

(9) $$\mathrm{E}I_B=T-\sum_{i=0}^{Nc}\int_{\xi_i^{II}}^{\infty}(1-H_i(x))dx .$$

Especially, if $\xi_i^{II}\leqq\delta_i$ for every i, $\mathrm{E}I_B=\sum_{i=1}^{Nc}\delta_i$.

In the case that both C-trains and B-trains have the same spead, say the case of ***no interfesence from the speed difference,*** the 3rd term of the right hand side of the formula (6) of ξ_i^{II} reduces to

(10) $$\max[(\theta_i-t_{c_{i+1}})^+,\ (R-t_{c_{i+1}})^+]=(\theta_i-t_{c_{i+1}})^+.$$

N.B. 5. From the assumption IV,

(11) $$U_b\gtreqless U_c \Longleftrightarrow t_b\gtreqless t_c .$$

Then,

(12) $$U_b\geqq U_c ,$$

since by the assumption II, $t_b\geqq t_c$.

N.B. 6. In the case $t_{c_i}=t_{c_{i+1}}=t_c$ in the formula (6) for ξ^{II},

(13) $$\xi_i^{II}=\begin{cases} t_c+t_b+(U_b-U_c+R-t_c)^+, & (t_c\geqq\theta_i) \\ \theta_i+t_b-t_c+\max(\theta_i,\ U_b-U_c+R). & (t_c\leqq\theta_i) \end{cases}$$

Furthermore, if $U_b=U_c$, then under the assumption $\theta_i\geqq R$

(14) $$\xi_i^{II}=\begin{cases} t_b+t_c , & (t_c\geqq\theta_i) \\ 2\theta_i+t_b-t_c . & (t_c\leqq\theta_i) \end{cases}$$

Type (III) Let T_i be the time when the train A_i passes the place P, and S_j be the time when the train C_j passes the place P′, then

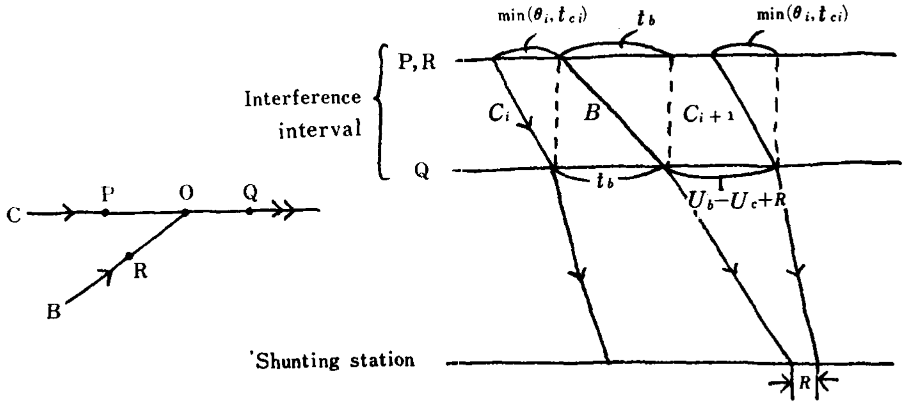

Fig. 3. The interference constant ξ in Type (II).

$$(15)\qquad \begin{aligned} X_i &= T_{i+1} - T_i\ , && (i=0,1,2,\cdots) \\ Z_j &= S_{j+1} - S_j\ , && (j=0,1,2,\cdots) \end{aligned}$$

where $T_0=S_0=0$. The distributions of T_i and S_j are denoted by G_i and K_j respectively, i.e.

$$(16)\qquad G_i(x)=\mathrm{P}\{T_i\leqq x\} \quad \text{and} \quad K_j(x)=\mathrm{P}\{S_j\leqq x\}$$

[Assumption V] A-trains and C-trains satisfy the following conditions:

(i) X_i $(i=0,1,2,\cdots)$: independent,
Z_j $(j=0,1,2,\cdots)$: independent,

(ii) $\{T_i\}$ and $\{S_j\}$: independent.

[Assumption VI] Classifying the gap of A-trains and C-trains through which B-trains are able to pass into two classes, (i) those which begin after the passages of an A-train (after the times T_i's) and (ii) those of C-train (after the times S_j's); and we call them the first class of gaps and the second class of gaps, denoted by V_i^a and V_j^c. (An exception of V_0 after the time $T_0=S_0=0$.)

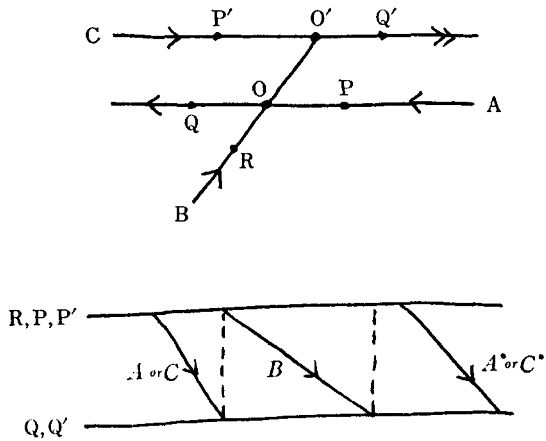

Fig.4. The interference constants ξ, ζ in Type(III).

In the type (III), the numbers N_a and N_c of A-trains and C-trains which are able to pass through the period T are considered as random variables, i.e.

$$\{N_a=m\}=\{T_m\leqq T<T_{m+1}\} \text{ and } \{N_c=m\}=\{S_m\leqq T<S_{m+1}\} \tag{17}$$

The mean interference time duration is

$$\begin{aligned} \mathrm{E}I_B &= T-\mathrm{E}\left(V_0+\sum_{i=1}^{Na} V_i^a+\sum_{j=1}^{Nc} V_j^c\right) \\ &= T-\mathrm{E}\left(V_0+\sum_{i=1}^{Na} \mathrm{E}^{Na} V_i^a+\sum_{j=1}^{Nc} \mathrm{E}^{Nc} V_j^c\right) .^{*)} \end{aligned} \tag{18}$$

We have only to calculate the right hand side of the above formula.

*) $\mathrm{E}^Y X$ denotes the conditional expectation of X given Y.

(19)
$$V_0=\begin{cases} \min[(X_0-\xi_0)^+,\ (Z_0-\zeta_0)^+] & \text{for } X_0,\ Z_0\leqq T\ , \\ (X_0-\xi_0)^+ & \text{for } X_0\leqq T<Z_0\ , \\ (Z_0-\zeta_0)^+ & \text{for } Z_0\leqq T<X_0\ , \\ T & \text{for } X_0,\ Z_0>T\ . \end{cases}$$

For $x>0$,

(20)
$$\begin{aligned} P\{V_0>x\} &= P\{X_0-\xi_0>x,\ Z_0-\zeta_0>x,\ X_0\leqq T,\ Z_0\leqq T\}+ \\ &\quad +P\{X_0-\xi_0>x,\ X_0\leqq T<Z_0\}+ \\ &\quad +P\{Z_0-\zeta_0>x,\ Z_0\leqq T<X_0\}+ \\ &\quad +P\{T>x,\ X_0>T,\ Z_0>T\} \\ &= P\{T\geqq X_0>x+\xi_0\}P\{T\geqq Z_0>x+\zeta_0\}+ \\ &\quad +P\{T\geqq X_0>x+\xi_0\}P\{Z_0>T\}+ \\ &\quad +P\{T\geqq Z_0>x+\zeta_0\}P\{X_0>T\}+ \\ &\quad +P\{T>x\}P\{X_0>T\}P\{Z_0>T\}\ . \end{aligned}$$

(21)
$$\begin{aligned} EV_0 &= \int_0^\infty P\{V_0>x\}dx=\int_0^T P\{V_0>x\}dx= \\ &= \int_0^{T-\max(\xi_0,\zeta_0)} P\{T\geqq X_0>x+\xi_0\}P\{T\geqq Z_0>x+\zeta_0\}dx+ \\ &\quad +\int_0^{T-\xi_0} P\{T\geqq X_0>x+\xi_0\}P\{Z_0>T\}dx+ \\ &\quad +\int_0^{T-\zeta_0} P\{T\geqq Z_0>x+\zeta_0\}P\{X_0>T\}dx+ \\ &\quad +\int_0^T P\{X_0>T\}P\{Z_0>T\}dx \\ &= \int_0^T [1-F_0(x+\xi_0)]\ [1-H_0(x+\zeta_0)]dx- \\ &\quad -\int_{T-\max(\xi_0,\zeta_0)}^T [F_0(x+\xi_0)-F_0(T)]\ [H_0(x+\zeta_0)-H_0(T)]dx+ \\ &\quad +[1-H_0(T)]\int_{T-\xi_0}^T [F_0(x+\xi_0)-F_0(T)]dx+ \\ &\quad +[1-F_0(T)]\int_{T-\zeta_0}^T [H_0(x+\zeta_0)-H_0(T)]dx\ . \end{aligned}$$

Since

(22)
$$F_0(x)=\sum_{i=1}^{\infty}\int_0^x [1-F_i(x-u)]dG_i(u)\ , \qquad \text{and}$$
$$H_0(x)=\sum_{j=1}^{\infty}\int_0^x [1-H_j(x-u)]dK_j(u)\ , \qquad \text{(cf. [4])},$$

$\mathrm{E}V_0$ is expressed by F_i, H_j, G_i, and K_j.

(23) $$\xi_0=\max[t_b,\ \varepsilon_0], \qquad \text{(usually, } \varepsilon_0=0.)$$

(24) $$\zeta_0=t_b+\max[(\theta_0-t_{c_1})^+,\ (U_b-U_{c_1}+R-t_{c_1})^+]$$
$$=t_b+(\max[\theta_0,\ U_b-U_{c_1}+R]-t_{c_1})^+.$$

We have approximately

(25) $$\mathrm{E}V_0=\int_0^T\left[1-\sum_{i=1}^{\infty}\int_0^{x+\xi_0}(1-F_i(x+\xi_0-u))dG_i(u)\right]\left[1-\sum_{j=1}^{\infty}\int_0^{x+\zeta_0}(1-H_j(x+\zeta_0-u))dK_j(u)\right]dx\ .$$

Under the condition $C_j{}^i=\{S_j\leqq T_i<S_{j+1}\}$,

(26) $$V_i{}^a=\begin{cases}\min[(S_{j+1}-T_i-\eta_{ij})^+,\ (X_i-\xi_i{}^1)^+] & \text{for } T_{i+1}\leqq T,\ S_{j+1}\leqq T,\\ (S_{j+1}-T_i-\eta_{ij})^+ & \text{for } T_{i+1}>T,\ S_{j+1}\leqq T,\\ (X_i-\xi_i{}^1)^+ & \text{for } T_{i+1}\leqq T,\ S_{j+1}>T,\\ (T-T_i)^+ & \text{for } T_{i+1}>T,\ S_{j+1}>T.\end{cases}$$

(27) $$\mathrm{E}\sum_{i=1}^{Na}V_i{}^a=\mathrm{E}(\sum_{i=1}^{Na}\mathrm{E}^{Na}V_i{}^a)=\sum_{m=1}^{\infty}\sum_{i=1}^{m}\mathrm{E}(V_i{}^a|N_a=m)\mathrm{P}\{N_a=m\}$$
$$=\sum_{m=1}^{\infty}\sum_{i=1}^{m}\int_0^T\mathrm{P}\{V_i{}^a>x|N_a=m\}dx\ \mathrm{P}\{N_a=m\}$$
$$=\sum_{i=1}^{\infty}\sum_{m=i}^{\infty}\int_0^T\mathrm{P}\{V_i{}^a>x,\ N_a=m\}dx$$
$$=\sum_{i=1}^{\infty}\int_0^T\mathrm{P}\{V_i{}^a>x,\ T_i\leqq T\}dx\ .$$

(28) $$\mathrm{P}\{V_i{}^a>x,\ T_i\leqq T,\ C_j{}^i\}=$$

$$
\begin{aligned}
&= \mathrm{P}\{S_{j+1} - T_i - \eta_{ij} > x,\ X_i - \xi_i^{\mathrm{I}} > x,\ T_{i+1} \leqq T,\ S_{j+1} \leqq T,\ T_i \leqq T,\ C_j^i\} + \\
&\quad + \mathrm{P}\{S_{j+1} - T_i - \eta_{ij} > x,\ T_{i+1} > T,\ S_{j+1} \leqq T,\ T_i \leqq T,\ C_j^i\} + \\
&\quad\quad + \mathrm{P}\{X_i - \xi_i^{\mathrm{I}} > x,\ T_{i+1} \leqq T,\ S_{j+1} > T,\ T_i \leqq T,\ C_j^i\} + \\
&\quad\quad\quad + \mathrm{P}\{T - T_i > x,\ T_{i+1} > T,\ S_{j+1} > T,\ T_i \leqq T,\ C_j^i\}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathrm{P}\{T - S_j \geqq Z_j > x + \eta_{ij} + T_i - S_j,\ T - T_i \geqq X_i > x + \xi_i^{\mathrm{I}},\ S_j \leqq T_i\} + \\
&\quad + \mathrm{P}\{T - S_j \geqq Z_j > x + \eta_{ij} + T_i - S_j,\ X_i > T - T_i,\ S_j \leqq T_i\} + \\
&\quad\quad + \mathrm{P}\{Z_j > T - S_j,\ T - T_i \geqq X_i > x + \xi_i^{\mathrm{I}},\ S_j \leqq T_i\} + \\
&\quad\quad\quad + \mathrm{P}\{Z_j > T - S_j,\ X_i > T - T_i,\ S_j \leqq T_i < T - x\}
\end{aligned}
$$

$$
\begin{aligned}
&= \int_0^{T - x - \max(\xi_i^{\mathrm{I}}, \eta_{ij})} \int_0^t [H_j(T - S) - H_j(x + \eta_{ij} + t - s)] \\
&\qquad\qquad [F_i(T - t) - F_i(x + \xi_i^{\mathrm{I}})] dK_j(s) dG_i(t) + \\
&\quad + \int_0^{T - x - \eta_{ij}} \int_0^t [H_j(T - S) - H_j(x + \eta_{ij} + t - s)] \\
&\qquad\qquad (1 - F_i(T - t)) dK_j(s) dG_i(t) + \\
&\quad + \int_0^{T - x - \xi_i^{\mathrm{I}}} \int_0^t (1 - H_j(T - s)) [F_i(T - t) - F_i(x + \xi_i^{\mathrm{I}})] dK_j(s) dG_i(t) + \\
&\quad + \int_0^{T - x} \int_0^t (1 - H_j(T - s)) (1 - F_i(T - t)) dK_j(s) dG_i(t)
\end{aligned}
$$

$$
= \int_0^T \int_0^t [1 - H_j(x + \eta_{ij} + t - s)] dK_j(s) dG_i(t) (1 - F_i(x + \xi_i^{\mathrm{I}})) - \sum_{K=1}^{4} \varphi_K(x, i, j),
$$

where
$$
\varphi_1(x, i, j) = \int_{T - x - \max(\xi_i^{\mathrm{I}}, \eta_{ij})}^{T} \int_0^t [H_j(x + \eta_{ij} + t - s) - H_j(T - s)] [F_i(x + \xi_i^{\mathrm{I}}) - F_i(T - t)] dK_j(s) dG_j(t)\ ,
$$

$$
\varphi_2(x, i, j) = -\int_{T - x - \eta_{ij}}^{T} \int_0^t [H_j(x + \eta_{ij} + t - s) - H_j(T - s)] [1 - F_i(T - t)] dK_j(s) dG_i(t)\ ,
$$

$$
\varphi_3(x, i, j) = -\int_{T - x - \xi_i^{\mathrm{I}}}^{T} \int_0^t [1 - H_j(T - s)] [F_i(x + \xi_i^{\mathrm{I}}) - F_i(T - t)] dK_j(s) dG_i(t)\ ,
$$

$$
\varphi_4(x, i, j) = \int_{T - x}^{T} \int_0^t (1 - H_j(T - s))(1 - F_i(T - t)) dK_j(s) dG_i(t)\ .
$$

Then,

$$\text{(29)}\qquad \mathrm{E}\sum_{i=1}^{Na} V_i^a = \sum_{i=1}^{\infty}\sum_{j=0}^{\infty}\int_0^T\Big[\int_0^T\int_0^t (1-H_j(x+\eta_{ij}+t-s))dK_j(s)dG_i(t)$$
$$\times(1-F_i(x+\xi_i^{\mathrm{I}}))\Big]dx-\sum_{i=1}^{\infty}\sum_{j=0}^{\infty}\Big[\int_{T-\max(\xi_i^{\mathrm{I}},\eta_{ij})}^T \varphi_1(x,i,j)dx+$$
$$+\int_{T-\eta_{ij}}^T \varphi_2(x,i,j)dx+\int_{T-\xi_i^{\mathrm{I}}}^T \varphi_3(x,i,j)dx+$$
$$+\int_{T-x}^T \varphi_4(x,i,j)dx\Big],$$

where $\xi_i{}^{\mathrm{I}}=\max(t_{a_i}+b,\ \varepsilon_i)$,

$\eta_{ij}=t_{a_i}+t_b+\max[(\theta_j+t_{c_{j+1}})^+,\ (U_b-U_{c_{j+1}}+R-t_{c_{j+1}})^+]$.

By the symmetry,

$$\text{(30)}\qquad \mathrm{E}\sum_{j=1}^{Nc} V_j^c = \sum_{j=1}^{\infty}\sum_{i=0}^{\infty}\int_0^T\Big[\int_0^T\int_0^t (1-F_i(x+\eta_j+t-s))dG_i(s)dK_j(t)$$
$$\times(1-H_j(x+\xi_j^{\mathrm{II}}))\Big]dx-\sum_{j=1}^{\infty}\sum_{i=0}^{\infty}\Big[\int_{T-\max(\eta_j,\xi_j^{\mathrm{II}})}^T \phi_1(x,i,j)dx+$$
$$+\int_{T-\eta_j}^T \phi_2(x,i,j)dx+\int_{T-\xi_j^{\mathrm{II}}}^T \phi_3(x,i,j)dx+$$
$$+\int_{T-x}^T \phi_4(x,i\ j)dx\Big],$$

where
$$\phi_1(x,i,j)=\int_{T-x-\max(\eta_j,\xi_j^{\mathrm{II}})}^T\int_0^t [F_i(x+\eta_j+t-s)-F_i(T-s)]$$
$$\times[H_j(x+\xi_j^{\mathrm{II}})-H_j(T-t)]dG_i(s)dK_j(t),$$

$$\phi_2(x,i,j)=-\int_{T-x-\eta_j}^T\int_0^t [F_i(x+\eta_j+t-s)-F_i(T-s)]$$
$$\times[1-H_j(T-t)]dG_i(s)dK_j(t),$$

$$\phi_3(x,i,j)=-\int_{T-x-\xi_j^{\mathrm{II}}}^T\int_0^t [1-F_i(T-s)]$$
$$\times[H_j(x+\xi_j^{\mathrm{II}})-H_j(T-t)]dG_i(s)dK_j(t),$$

$$\phi_4(x,i,j)=\int_{T-x}^T\int_0^t (1-H_j(T-t))(1-F_i(T-s))dG_i(s)dK_j(t).$$

We have approximately

$$\text{(31)} \qquad EI_B = T - \int_0^T \left[1 - \sum_{i=1}^{\infty} \int_0^{x+\xi_0} (1 - F_i(x+\xi_0-u))dG_i(u)\right]$$
$$\times\left[1 - \sum_{j=1}^{\infty} \int_0^{x+\zeta_0} (1 - H_j(x+\zeta_0-u))dK_j(u)\right]dx$$
$$-\sum_{i=1}^{\infty}\sum_{j=0}^{\infty}\int_0^T\left[\int_0^T\int_0^t (1-H_j(x+\eta_{ij}+t-s))dK_j(s)dG_i(t)\right.$$
$$\left.\times(1-F_i(x+\xi_i{}^{\mathrm{I}}))\right]dx$$
$$-\sum_{j=1}^{\infty}\sum_{i=0}^{\infty}\int_0^T\left[\int_0^T\int_0^t (1-F_i(x+\eta_j+t-s))dG_i(s)dK_j(t)\right.$$
$$\left.\times(1-H_j(x+\xi_j{}^{\mathrm{II}}))\right]dx \; .$$

In the special case that $F_1=F_2=\cdots=F$, $H_1=H_2=\cdots=H$, $\xi_1{}^{\mathrm{I}}=\xi_2{}^{\mathrm{I}}=\cdots=\xi_1$, $\eta_{11}=\eta_{12}=\cdots=\zeta_1$, $\eta_1=\eta_2=\cdots=\xi_2$ and $\xi_1{}^{\mathrm{II}}=\xi_2{}^{\mathrm{II}}=\cdots=\zeta_2$.

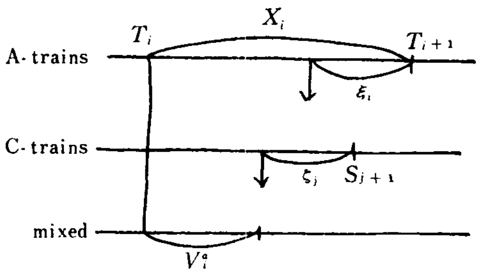

Fig.5. Gap V_i

$$\text{(32)} \qquad EI_B = T - \lambda_a\lambda_c\int_0^T\left[\int_x^{\infty}(1-F(u+\xi))du\int_x^{\infty}(1-H(u+\zeta))du\right]dx -$$

$$-\lambda_a\lambda_c\int_0^T\left[\int_0^T\int_x^{x+t}(1-H(u+\zeta_1))dudt(1-F(x+\xi_1))+\right.$$
$$\left.+\int_0^T\int_x^{x+t}(1-F(u+\xi_2))dudt(1-H(x+\zeta_2))\right]dx$$

N.B. 7. In [1]b) and [2], under the assumptions of the stationarity and Palm's flow condition and the independency of N and V, we have already obtained the following result:

(33) $$\mathrm{E}(\sum_{i=1}^{Na}V_i{}^a)=\mathrm{E}\mathrm{E}^{Na}(\sum_{i=1}^{Na}V_i{}^a)=\mathrm{E}N_a\mathrm{E}V^a=$$
$$=(\sum_r ra_r(T)\mathrm{E}V^a \qquad (a_r(t):\ \text{the probability there are } r \text{ arrivals in time } t)$$
$$=\lambda_a T\mathrm{E}V^a$$
$$=\lambda_a T\int_0^\infty(1-F(x+\xi_1))\,(1-K_1(x+\zeta_1))\,dx$$
$$=\lambda_a\lambda_c T\int_0^\infty\int_x^\infty(1-H(y+\zeta_1))\,dy\,(1-F(x+\xi_1))\,dx\ .$$

(34) $$\mathrm{E}(\sum_{j=1}^{Na}V_j{}^c)=\lambda_a\lambda_c T\int_0^\infty\int_x^\infty(1-F(y+\xi_2))\,dy\,((1-H(x+\zeta_2))\,dx\ .$$

$\xi_1=t_a+t_b$, $\zeta_1=t_a+t_b+\max((\theta-t_c)^+,\ (U_b-U_c+R-t_c)^+)$,
$\xi_2=\max(t_c,\ \theta)+t_b$, $\zeta_2=\max(t_c,\theta)+t_b+\max((\theta-t_c)^+,\ (U_b-U_c+R-t_c)^+)$.
In the case that $t_a=t_c>\theta$, $\xi_1=\xi_2=\xi$, $\zeta_1=\zeta_2=\zeta$,

(35) $$\mathrm{E}I_B\doteqdot T-(\mathrm{E}N_a\mathrm{E}V^a+\mathrm{E}N_c\mathrm{E}V^c)$$
$$=T\left\{1-\lambda_a\lambda_c\int_\xi^\infty(1-F(x))\,dx\int_\zeta^\infty(1-H(x))\,dx\right\}\ .$$

4. Comments

The improvement in the accommodation at the station yard or the marshaling yard entails enormous cost. To estimate the investment effect in the improvement, we need the measure of effectiveness of the interference situation of the trains. The author has adopted the mean number of the interference trains at the intersection as the measure of the opportunity cost of the interference before. (1962 [5]). But, in its formulation, we met with a difficulty and left the results unsatisfactory.

Recently, [January, 1969], the author was suggested it by Mr. Naoichi Hashikura in Japanese National Railways that there they usually regard the time duration of the railway occupation by the trains as a matter of importance. This is, what matters much to them is the total time duration of the interference which occurs in the situation where the trains on one-way line (B-route) are interfered by the trains on another line (A-route or C-route) which crosses (or merges to) the former. And this quantity is independent of the extent of utilization of B-route. As we had been afraid of the very fact that the importance degree of B-route is rather not reflected in this quantity, we had hesitated to deal with the interference time duration. But, its formulation is far easier than that of the opportunity cost problem stated above, and it can be the measure enough to fit for use in the case where the variation of the importance of B-route may be eliminated by some means or other. (It is also the case where the variation of the importance of B-route is not in question by nature.)

It can be applied to the following problems (cf. [6]):

1) The measure to estimate the investment effect of the improvement in the accommodation at the yard.
2) The assessment of the priority order of the improvement of the yards.
3) A criterion for the adoption of the cubic acrossing.
4) The problem of the examination of the necessity of the crossover.
5) The interception rate at the crossing gate.

In reply to the Mr. Hashikura's question, we could immediately send to him a solution for the type (I). It affords the very simple calculation particularly under the assumption of the same exponential type distribution of the intervals of the trains. As for type (II), (III), we reported in [1] a) and b). (At that time we made a slight revision and a post entry on its abstract.) As for type (III) we were obliged to discuss on the independency assumption.

Our method of considering the distribution of the diagram seems to be scarcely understood even among specialists. Such an idea that the railway diagram, different from the road traffic, never fluctuates by chance is now prevalent. Such being the case, a few words may not be amiss in this connection as to our point of view which we have had for these ten years.

Given a fixed diagram, the distribution of the intervals of the trains of course degenerates into the biassed unit distribution and our formulas are reduced to the corresponding simple arithmetic calculations. It seems, however, that the diagram is by no means fixed and it is better to regard the present diagram as a realized value subjected to the various demands and chances than as being fixed. Until we adopt such a treatment, we can neither make a fair comparison between yards nor give the estimation of the measure of the interference for the unknown one such as the future diagrams.

In this sense, the present paper also challenges the absolutism of the each realized diagram past or present and we have intended to point out that the estimation problem of the measure of the interference can theoretically be reduced to and deduced from that of the random variable diagram.

Certainly the estimation of the random variable diagram is, however, not so easy practically. It must be performed by the positive method and must not result in a metaphysical evasive answer. We feel the need of studying the theoretical structure of the random variable diagram.

For example, suppose that many individuals who have their own different opininions determine their own diagrams respectively, it may be justified that we should adopt the same exponential distribution of the trains intervals on an average. Under stronger conditions, however, we may obtain other various distributions. The establishment of the theory to indicate what condition will yield what distribution is a pending question. At present, it is the only course open to us that we can devise the method to estimate the distribution as the prolongation of the past ones. Of course the prolongation method will be different, depending on the long-range plans or the short-range ones. Again, at the estimation we may accept the subjective probability of the specialists by the simulation experiments in this case as well as in PERT.

It is expected that the failure of the independency assumption in type (III) may be yielded by the so-called successive trains group which consists of the trains running in parallel and in a batch. In such a case we have to try to eliminate the fault by the devise of adopting the biassed degenerated distribution of the trains interval or treating the group as a single train.

With the social progress the kaleidoscopic revisions of the diagrams are being made. The cost of the improvement in accommodations have far higher order than the cost of the changes of the diagrams. Then, without finding the measure of the interference relative to each given special diagram, we have considered the general diagram fluctuating by chance in a certain range and have tried in this paper to formulate the methods of finding the measure of the interference, given the so-called random variable diagram.

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