

A METHOD OF THE OPTIMAL SCHEDULING FOR A PROJECT WITH RESOURCE RESTRICTIONS*

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Abstract

So far, several methods have been presented for the strictly optimal scheduling under limited resources. There seems, however, no one for practical and not small projects. Such a method is given and its computer program is applied to some practical examples of the regular overhaul of a steam-power station. The method is to examine all the possible cases efficiently by the principle of the branch-and-bound method. The concept of subnetwork scheduling is also used for the treatment of not small projects. A network with 53 arcs and one with 92 arcs under the crane limit are successfully calculated by this idea.

1. Introduction

The application of PERT systems to practical scheduling problems are widely performed and the calculation method of PERT/TIME [3] and PERT/COST are well developed [1] [4]. However, most practical schedulings concern with resource restrictions and such problems or PERT/LOAD problems are hardly solved except simple models or heu-

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Arc	i	j	Day	Crane
a	1	2	1	0
b	2	3	2	1
c	3	4	1	1
d	1	4	2	1
e	4	5	2	0

Table 1. Data of a Network

ristic approximate treatments [5]. In this paper, we shall consider the strictly optimal scheduling with resource restrictions and apply its computer program to some practical problems.

2. An Illustrative Example

We shall consider the scheduling of the project given by Table 1. For every work or every arc, the preceding and the succeeding node numbers, the work time for completion and the number of necessary cranes are assigned. It is assumed that only one crane is available in this work field. The problem is to make a minimum completion time schedule under the crane limitation.

The loading for earliest starting times is over the crane limit as seen in Fig. 1. Then, we shall examine whether it is possible or not to schedule the crane limited project for a given completion time, say $T=8$ days. The earliest starting and the latest finishing times of all the arcs for $T=8$ days are calculated as in Fig. 2. For example, they are the 1st day and 3rd day for the arc a. The arcs a, b and c are

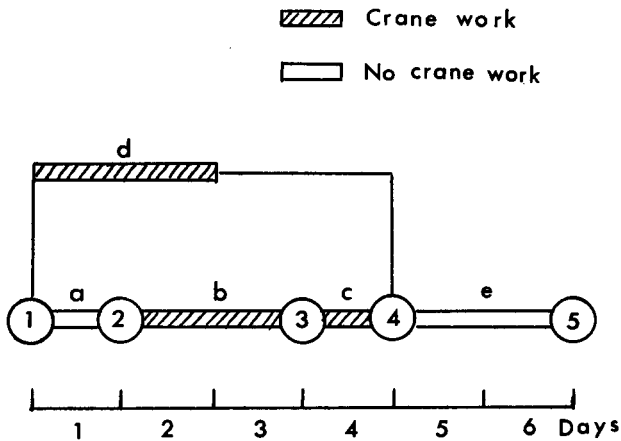


Fig. 1. Earliest Starting Schedule

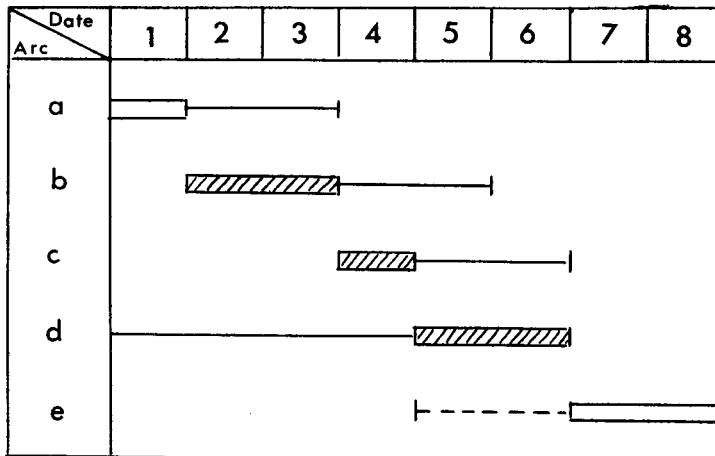


Fig. 2. Schedule for 8 Days

able to be scheduled at their earliest starting times respectively. The arc d is possible to be scheduled only at the 5th and 6th days under the crane limit. Then, the earliest starting time of the arc e is changed

into 7 from 5. The schedule of the arc e is also possible. Therefore, we have seen that the scheduling under the crane restriction is possible for completion time $T=8$ days.

Next, let us examine the schedule for completion time $T=7$ days. The earliest starting time and the latest finishing time are given as in Fig. 3a. The arcs a, b and c are scheduled at the earliest starting times. The arc d must be scheduled between the 1st day and the 5th day, but it is impossible when the crane limit is considered. So, the preceding arc c goes ahead by one day as in Fig. 3b. Then, the earliest time of the node 4 becomes 6, *i.e.*, $ES(4)=6$ and the old time 5 is memorized as $JE(c)=5$. Since the scheduling of the next arc d is also impossible under the crane limit, the schedule of the arc c must further ahead. But the schedule would be over the latest finishing time, then we must treat the arc b. Before we go to the arc b, we recover the earliest starting time $ES(4)$ of the node 4 as $ES(4)=5$ making use of the memory $JE(c)=5$ mentioned above. Next, we make the arc b go ahead by one day and put $ES(3)=5$ and $JE(b)=4$. The arc c is scheduled at the 5th day and $ES(4)=6$, $JE(c)=5$. The arcs d and e are possible to be scheduled as Fig. 3c. We have seen that the schedule of completion time $T=7$ days is possible. We can see easily that the schedule of $T=6$ is impossible and finally conclude that $T=7$ is the minimum completion time.

Now, we can show the above procedure from the stand point of the branch-and-bound method as in Fig. 4. In each circle, the starting date of the related arc is indicated. For example, the chain 1-3-5-1-6 means the optimal schedule shown in Fig. 3c [2].

3. Flow Chart

First, data of each arc such as the i -node number, the j -node number, the work duration and the necessary number of cranes are read in (1). Using these data, PERT/TIME is calculated and total floats obtained (2). To shorten calculation time, the arcs are topologically ordered

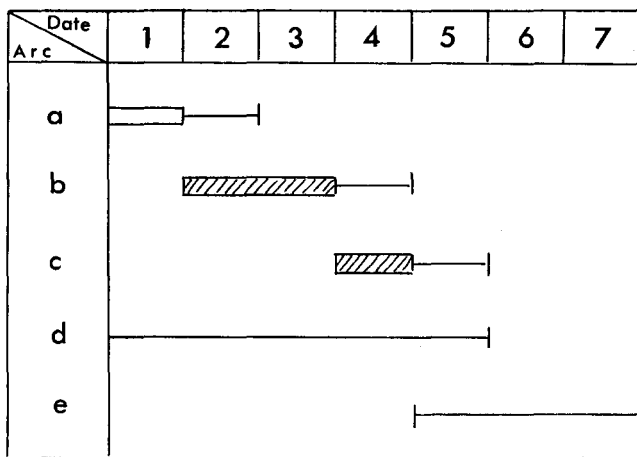


Fig. 3a. A Trial for 7 Days

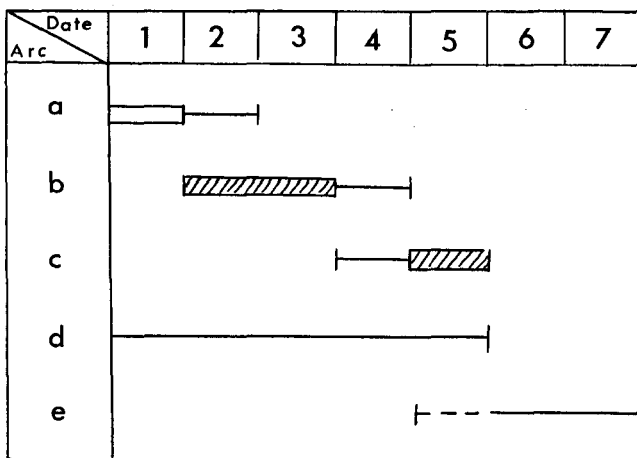


Fig. 3b. Another Trial for 7 Days

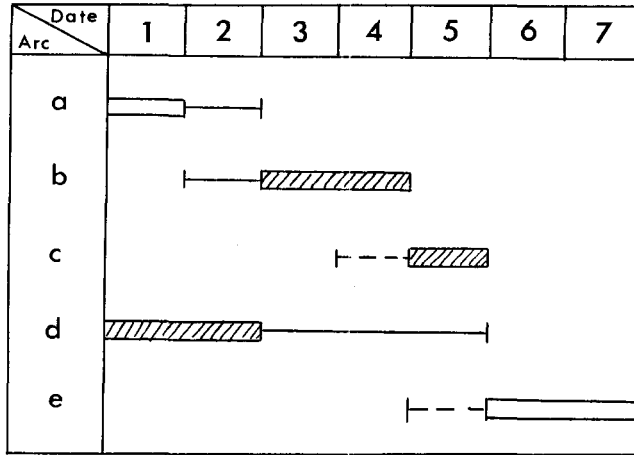


Fig. 3c. Schedule for 7 Days

such that arcs with less total floats precede (3). From the programming point of view, it is also an idea that arcs with less latest starting times precede.

If it is found that the scheduling under crane limit is possible within overall project duration T , such examination is made for $T-1$ and so on (5).

To determine the feasible interval for the schedule of each arc, the earliest starting time and the latest starting time are calculated for given T (4). Then, the following calculation is made by turns for the topologically ordered arcs.

Assuming that the arcs to the p -th one from 1st are scheduled, we shall determine the schedule of the $(p+1)$ -th arc. A schedule within its float is examined (8). If the schedule satisfies the resource condition, the loading is made and the next arc is treated (9). If the schedule is over the resource limit, it goes ahead and the earliest starting time $ES(j)$ of the j -node of that arc is changed for the calculation of the total float of the arcs emanating for the node (10). When the schedule for the p -th arc is over its latest finishing time, the earliest starting time

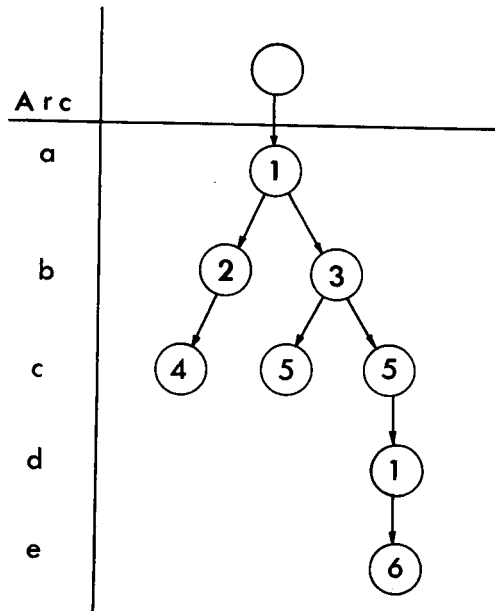


Fig. 4. Correspondence to Branch-and-Bound Method

of the j -node is recovered as before the p -th arc is treated (7) (11) and then $(p-1)$ -th one is treated (12).

4. Application to Practical Examples

We have applied the method to the periodical overhaul of a steam-power station. Among its works, the turbine part is most critical, because it requires precision and heavy works under the condition that there is only one crane in this work field. We shall cite two examples A and B: A is concerned with the part before the inspection by authority and B is [with that after the inspection. The part A has 92 arcs and B has 53 arcs as shown in Fig. 7 and Fig. 8.

We cannot apply our method directly to practical problems, because we would take too long calculation time. Then, we use the concept of

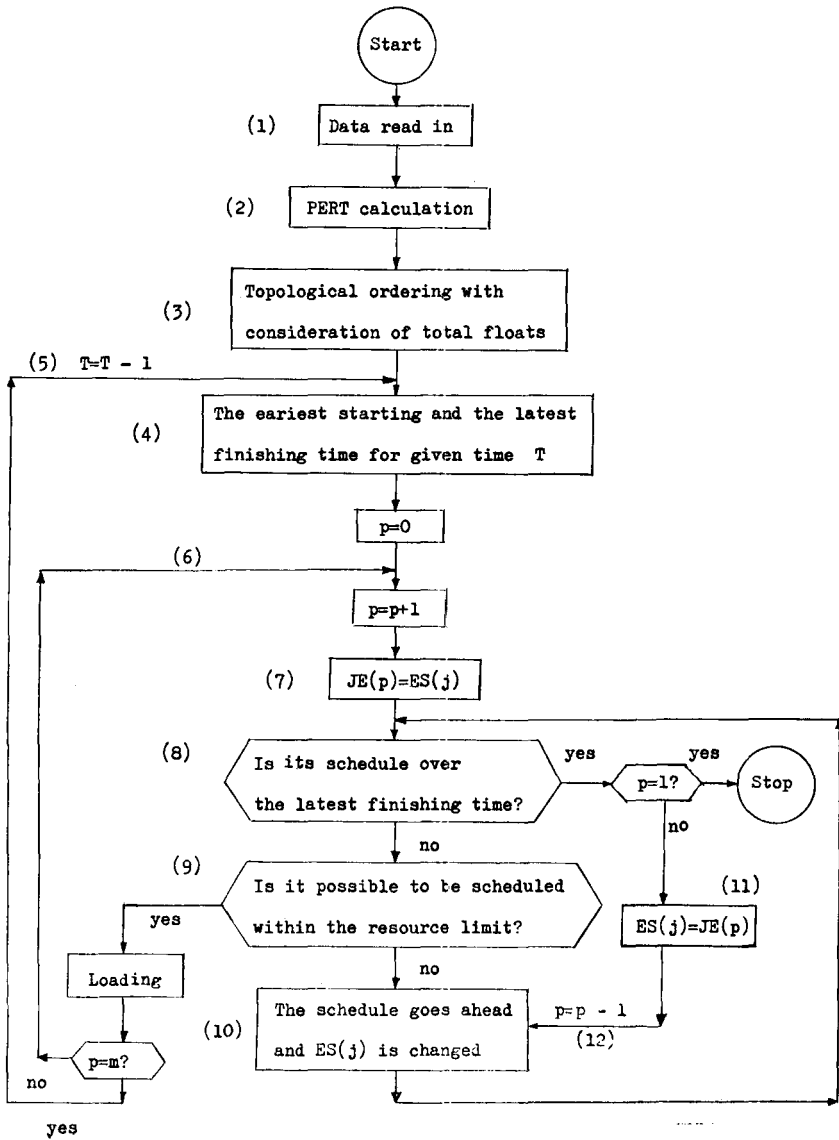


Fig. 5. Flow Chart

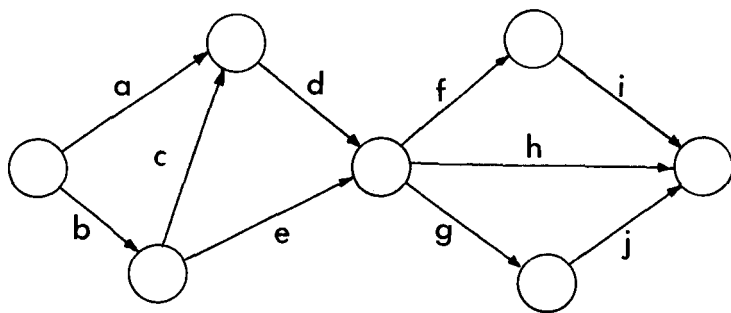


Fig. 6. Subnetworks

subnetworks in order to shorten the calculation time. The network in Fig. 6 is able to be considered as the tandem connection of the subnetwork (a, b, c, d, e) and the subnetwork (f, g, h, i, j). In this case, the schedules of the two parts can be independently calculated. Now, we can expand such an idea to general networks.

Namely, in the network B of Fig. 8, we first calculate the optimal schedule of the subnetwork S1. We can see that the minimum completion time of S1 is 78 hours. Then, using this result that the completion time from node 30 to node 40 is 78 hours at least, we go to the subnetwork S2 including S1 and we can also see that the completion time of S2 is 160 hours at least. Finally, we go to the total network. For a given time T , the latest finishing time of node 30 can be considered as $T-78$ and that of node 10 can as $T-160$. Using such informations, we can get the result that the minimum completion of the total network B is 196 hours and we show an optimal schedule in Fig. 10. The total calculation time of this network is 30 seconds by IBM 7040.

For the network A of Fig. 7, we treat three subnetworks. The minimum completion time is 147 hours and an optimal schedule is shown in Fig. 9. The calculation time is 4 minutes.

We have showed examples for the case of one crane limitation. As seen from the flow chart in Fig. 5, however, we can easily expand the

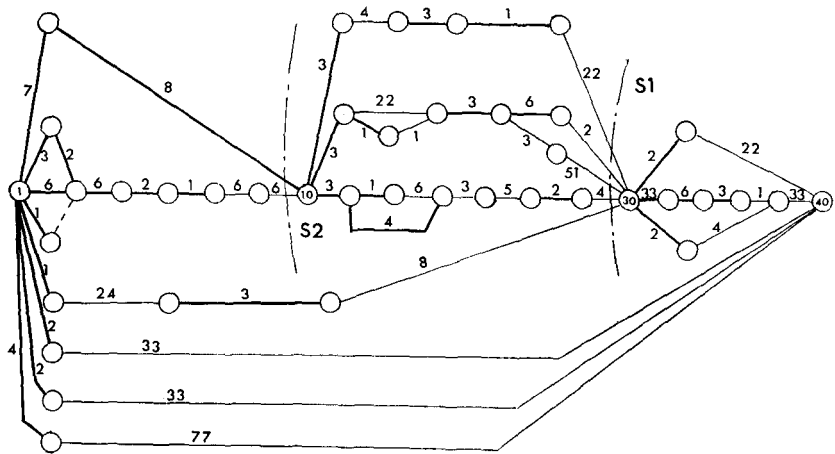


Fig. 8. Network B

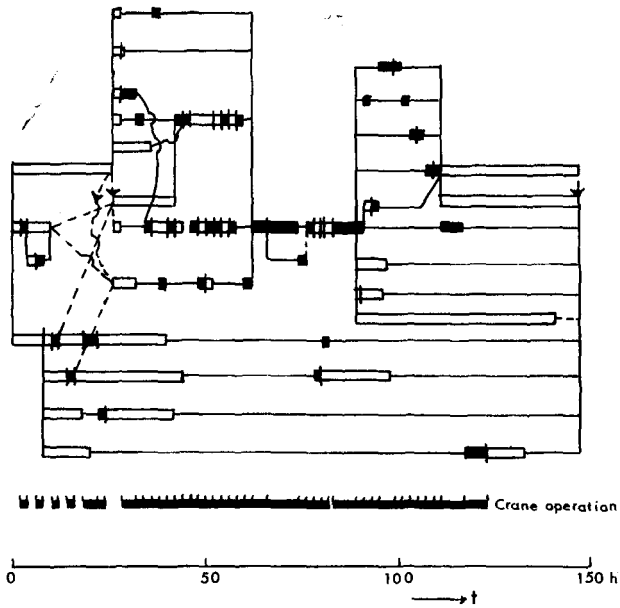


Fig. 9. An Optimal Schedule for Network A

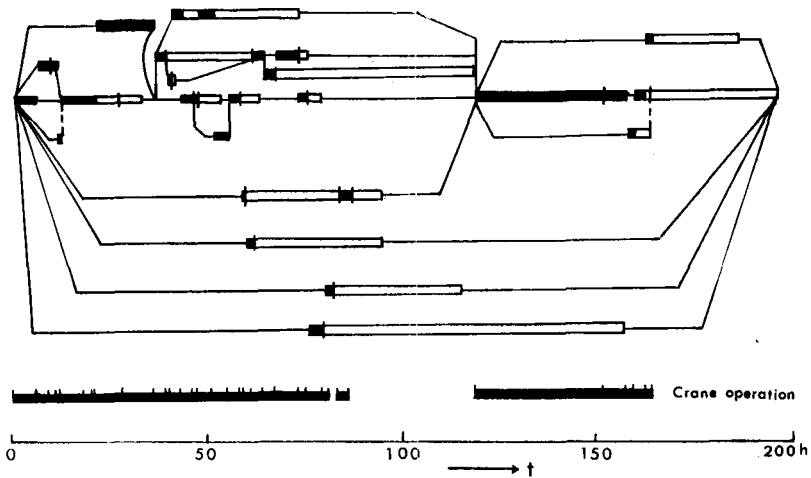


Fig. 10. An Optimal Schedule for Network B

method to the case that two or more cranes are usable or that several kinds of resource are necessary for each activity and the total amount usable per unit time for each resource is limited within a given value.

5. Conclusion

The main results are as follows.

i) The calculation method to obtain the strictly optimal schedule under several resource restrictions is developed. The computer program of this method is not difficult and the computer memory for the calculation is about the same amount as in PERT/TIME or PERT/COST calculations.

ii) If the concept of subnetworks is used, the method is not confined within small projects. Really, we have succeeded in the calculation of practical and not small problems.

iii) So far, however, not so many examples have been solved that it is necessary to apply this method to various practical examples in order to know its applicable region.

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