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OUTPUT OF M/M/1/N QUEUE

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Abstract

Th output distribution of a single server Markovian queuing system with finite waiting space is obtained, and explicit expressions for the moments are given.

Introduction

Makino [1] has studied the output distribution of the M/G/1 system and several other related queuing systems. He has also obtained the output distribution from a tandem-type system with two and three stages. He has assumed throughout an ifinite queue in front of the first server. The purpose of this note is to obtain the output distribution of the M/M/1 system with the restriction that there can not be more than N custmers in the system at any time. This output distribution is. obtained in the steady state, and its moments are given explicitly.

Output Distribution from M/N/1/N Queue

Customers arrive at a single-service facility, following an orderly stationary Poisson stream with parameter λ. Whenever there are N customers in front of the server, the new arriving customers are lost to the system. The service time distribution is negatively exponential, with parameter μ.

Let P_0 be the steady state probability that there are no customers in the system, and let $L_A(\theta)$, $L_S(\theta)$, $L_U(\theta)$ be the Laplace-Stieltjes transform of the inter-arrival distribution, service time distribution, and the output distribution, respectively.

Then, following the arguments in Makino [1], $L_U(\theta)$ can be written as

$$L_U(\theta)=P_0\, L_A(\theta)\, L_S(\theta)+(1-P_0)\, L_S(\theta) \tag{1}$$

It is well known that

$$L_A(\theta)=\frac{\lambda}{\lambda+\theta}$$

$$L_S(\theta)=\frac{\mu}{\mu+\theta}$$

and

$$P_0=\frac{1-\rho}{1-\rho^{N+1}}$$

for the M/M/1/N system

where

$$\rho=\frac{\lambda}{\mu}$$

Substituting these values in (1), we get, after simplification,

$$L_U(\theta)=\frac{(1-\rho^{N+1})\,(\lambda+\theta)\,\mu-(1-\rho)\,\mu\theta}{(1-\rho^{N+1})\,(\lambda+\theta)\,(\mu+\theta)} \tag{2}$$

$$=\frac{1}{1-\rho^{N+1}}\left[\frac{\lambda}{\mu+\theta}-\rho^{N+1}\frac{\mu}{\mu+\theta}\right]$$

By inverting (2), we can write the density function of the interdeparture times as follows:

$$g(t)=\frac{1}{1-\rho^{N+1}}\left[\lambda e^{-\lambda t}-\rho^{N+1}\mu e^{-\mu t}\right] \tag{3}$$

$$=\frac{\lambda}{1-\rho^{N+1}}\left[e^{-\lambda t}-\rho^{N}e^{-\mu t}\right] \qquad t\geq 0$$

Moments of the output distribution are given by

$$E(t^r)=\int_0^\infty t^r g(t)\,dt=\frac{r!\,\lambda}{1-\rho^{N+1}}\left[\frac{1}{\lambda^{r+1}}-\frac{\rho^N}{\mu^{r+1}}\right] \tag{4}$$

$$=\frac{r!}{\lambda^r}\left[\frac{1-\rho^{N+1+r}}{1-\rho^{N+1}}\right]$$

From (4), the mean and the variance can easily seen to be:

$$E(t)=\frac{1}{\lambda}\,\frac{1-\rho^{N+2}}{1-\rho^{N+1}}$$

$$Var(t)=\frac{(1-\rho^{N+1})\,(1-\sigma^{N+3})-\rho^{N+1}\,(1-\rho)^2}{\lambda^2\,(1-\rho^{N+1})^2}$$

REFERENCE

[1] Makino, T. "On a Study of Output Distribution." *Journal of the Operations Research Society of Japan*, 8, 3 (1966), 109–133.