J. Operations Research Soc. of Japan
Vol. 10, Nos. 3 & 4, June 1968

A NETWORK MODEL FOR COMMUTER TRANSPORTATION*

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(Received December 4, 1967)

Summary

Most of the commuters in a big metropolitan area such as Tokyo are transported by a complex system of railways and buses. A network flow model of this system was constructed and tested against actual observations. The model was applied to the prediction of the load distribution among alternative routes in the eastern section of Tokyo under various assumptions, including the building of a new subway line, an express train line, as well as the increase of population.

1. Introduction

A complex system of railways carries more than a million commuters to and from the business area of Tokyo. In Fig. 1, JNR (Japanese National Railways) lines, private railway lines, and subway lines are shown. Average number of passengers carried by trains on each line

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* This paper was presented at the 14th international meeting of the Institute for Management Sciences, held in Mexico City on August 22~26, 1967.

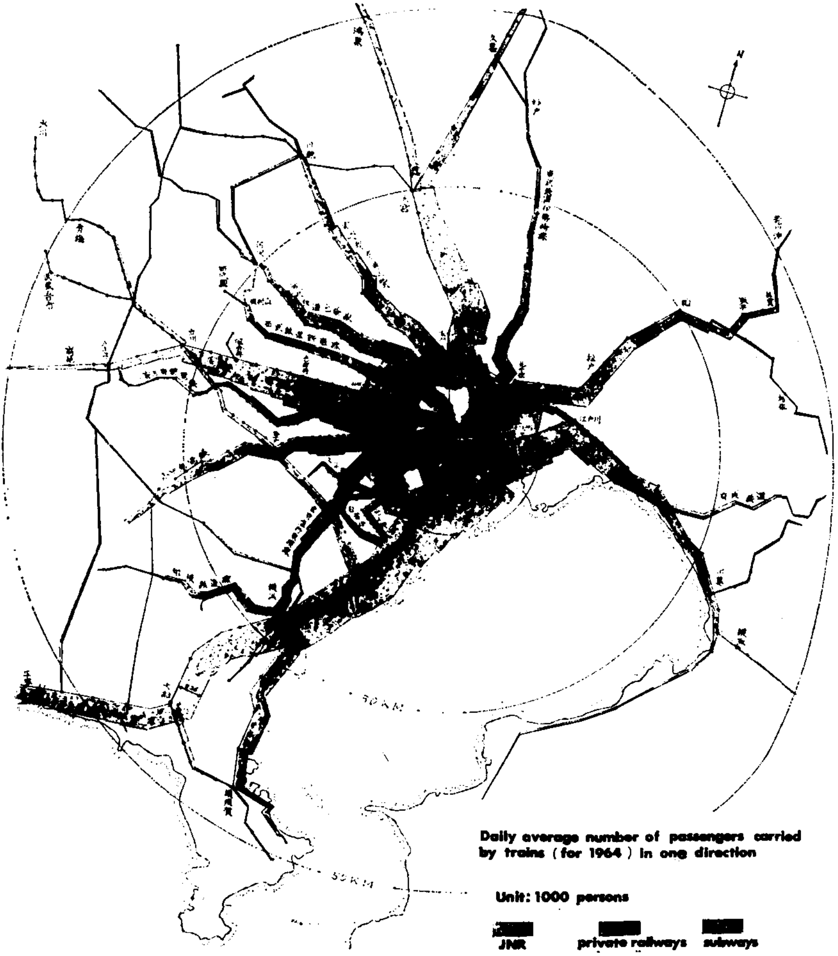

Fig. 1. Passenger traffic on railroads around Tokyo

is shown in units of 1,000 persons. Thus Tokaido-line of JNR carries 365,000 persons from the south, Chuo-line 344,000 from the west, Tohoku-line 302,000 from the north, and Sobu-line 169,000 from the east. These numbers are the numbers of passengers in one direction per day. Private railways such as Odakyu, Seibu, Tobu, etc. carry commuters living in the sectors left between the lines of JNR.

It used to be a policy of JNR that every private railway line shall have a terminal at one of the stations on Yamate-line, which is a circular line of JNR, so that JNR had a monopoly of transportation from that point on. But in recent years tremendous increase of the number of commuters from all directions made it necessary to allow those private railways to extend their lines into the area inside Yamate-line, and to carry the passengers directly to the downtown business districts. Under these circumstances, subway constructions are now going on quite extensively. In a few years from now, almost every main street in that area will come to have a subway line under it, which extends at both ends into some railway lines, either JNR or private.

Our problem is to analyze this system of railways by a model, which we call "network flow model."

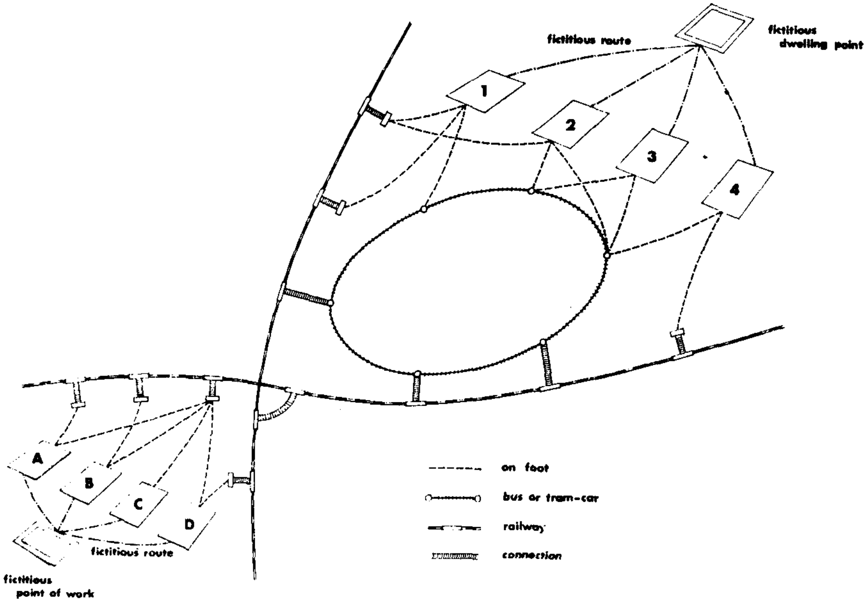

Fig. 2. The description of the model, I: the network

2. The description of the model

A typical commuter leaves his home in the morning and goes on foot to perhaps a bus stop. Then he takes a bus up to a railway station. There he goes through the wicket and stands on the platform until he catches a train. There may be a change or two on the way. At a station he leaves the train, gets out of the station, and, either on foot or by bus, goes to his office. This takes place for every commuter. His route consists of several different legs, such as a segment of a railway line, bus or tram-car, connection, and a street on which he walks (Fig. 2).

We divide the residential areas into blocks, and also the business areas into blocks. The number of commuters originating in, or flowing into, each block can be estimated.

In applying the theory to this system, it is convenient to assume that all commuters originate at a fictitious dwelling point and go from there to each residential area via a fictitious route, with a certain capacity limitation. And also at the end, we assume, people go from each business district to a certain fictitious point of work through a fictitious route.

As a typical leg of the route of a commuter, let us take a segment of railway which he travels in a car of a train. We assume the characteristic of this leg to be as shown in Fig. 3. " Cost " of taking this leg is measured in general by a mixture of the fare and the time. The relative weight (importance) of the time to the distance is given as parameter α. This cost is the cost per passenger. It is assumed to remain constant as long as the number of passengers in a car does not exceed a certain limit. At first, this limit was taken to be the " ultimate capacity " above which the cost goes up to infinity. By a very brave experiment, this ultimate capacity was found to be 300% of the " nominal capacity," a magic number which *was* a practical figure in good olden days. Later, after discussions with the managers of JNR, we refined the model a little bit, and introduced a multiplication factor β, and assumed that, between 260% and 300% of the nominal capacity, the mar-

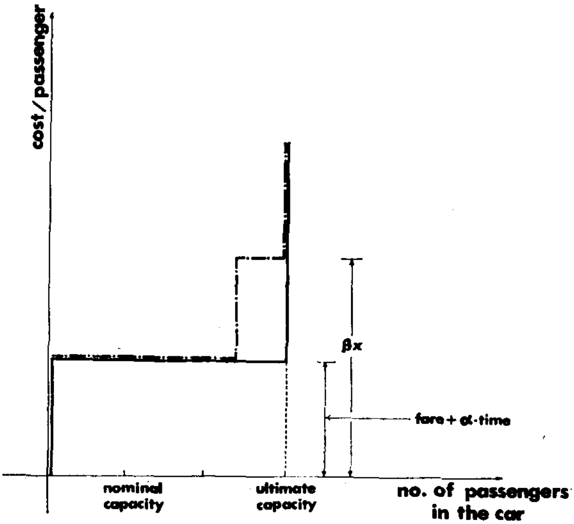

Fig. 3. The description of the model, II: the characteristic of a typical leg

ginal cost per passenger becomes β times the normal cost.

Similar and much simpler assumptions are also made to other kinds of legs of the route.

Now our fundamental assumption is as follows:—*The flow of commuters is such that it maximizes the flow from the fictitious dwelling point to the fictitious point of work subject to all the capacity limitations and in so doing minimizes the total "cost" in the system as a whole.* Here the cost of each leg is to be calculated by taking the area under the curve up to the number of passengers present in the car.

Any standard technique of network flow analysis could be applied here. What we actually used was one described in Iri [1]. The computer program itself was written in ALGOLIP—a dialect of ALGOL.

3. Test of the model

For the purpose of testing how good our model is and how well our fundamental assumption can describe the actual phenomenon, we took the example shown in Fig. 4. The network consists of Yamate-line — described here as a rectangle —, portions of Chuo-line, Sobu-line, and Tohoku-line.

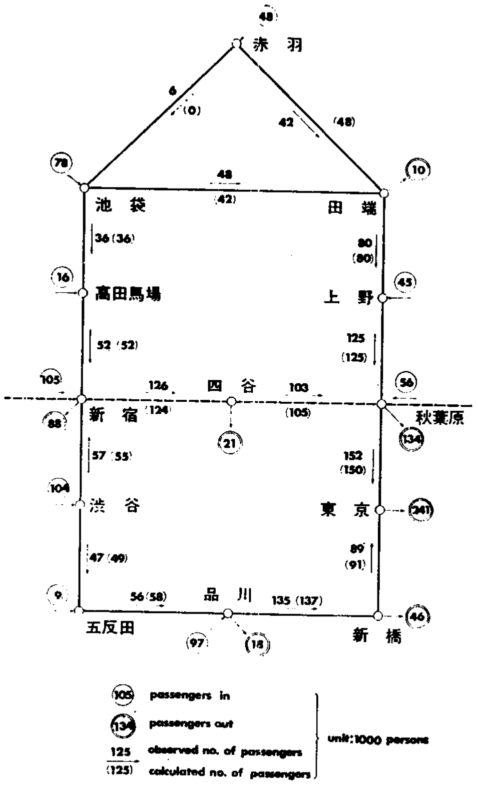

Fig. 4. Test of the model

Number of passengers either coming in or going out of this system at each major station was given as data. Calculated number of passengers on each segment is shown between parentheses in units of 1,000 persons per hour. It agrees very well with the corresponding observed number which is shown outside the parentheses.

4. The area and the railways investigated

Encouraged by that result, we carried out a series of computations under various conditions, applying our model to the area shown in Fig. 5.

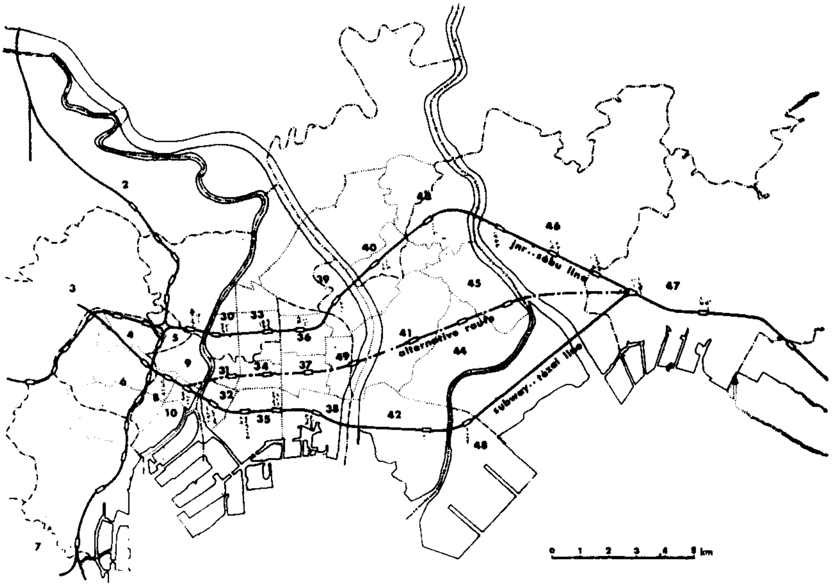

Fig. 5. The area and the railways investigated

The following story will, we hope, illustrate how and to what extent this kind of simple model can be utilized in the evaluation of alternative plans for the future.

Sobu-line of JNR is the only existing railway line in this area which carries commuters from the east. It is under very heavy load at present in rush hours. In order to help it, two projects are under way. One is the construction of an express line of JNR. Another is the construction of a subway line called Tozai-line. The original plan of its route is shown by a solid line. For comparison, we also investigated a possible alternative route which is shown by a dot-dash line.

Fig. 6 is a schematic representation of the network model which we

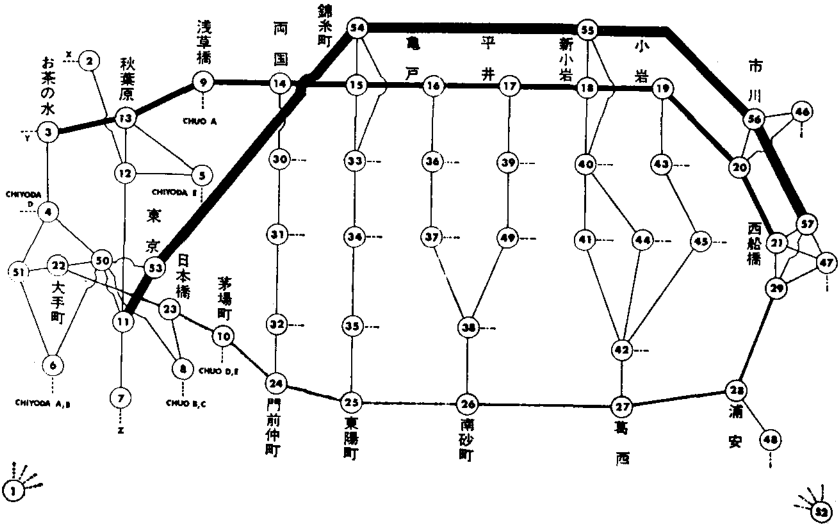

Fig. 6. Network model, the area investigated

used in this investigation. Circle 52 represents the " fictitious dwelling point." From there, "through fictitious routes" shown by dot-dash lines, passengers go to real residential areas like 30, 31, $\cdots$, 48. In particular, Circle 47 represents a point where the commuters from the east flow into this system. There are three heavy lines connecting Circle 47 to the business districts 2, 3, $\cdots$, 10. These are Sobu express, Sobu local, and Tozai-subway, respectively. From each business district a " fictitious route" represented by a dotted line leads to Circle 1, which is the " fictitious point of work."

5. Factors and their ranges of variation

Fig. 7 shows the factors which we varied in our series of computations. Express line of JNR is either absent or present. Number of commuters in the area is assumed to be either 90,000 persons per hour (which is approximately the present situation), or twice as many. In one case, we assumed it to be 190,000, which is the number we expect when the area gets saturated.

express line JNR : absent, present
no. of commuters in the area : 90,000 , 180,000 , 190,000 saturation
routes between the living quarters and the station : standard, alternative
parameter α in the cost function : 0 , 1 , ∞
JNR fare : 1, 2, 3, 4, 5 x current fare
multiplication factor β for 260 ~ 300% nominal capacity : 1, 2, 3, ∞
subway route : standard, alternative
superposition of local commuters : absent, present

Fig. 7. Factors and their ranges of variation

Standard routes between the living quarters and the stations are the shortest routes. Alternative routes are those which go oblique, and in fact actual bus routes in this area are somewhat of that nature.

Parameter α, that is the relative importance of time to fare, is taken to be 1 (yen per minute, that is) as standard. But sometimes it was taken to be either 0 or ∞.

As the investigation went on, it became clear that the load of Sobu-line would never be appreciably reduced by the construction of the subway line as is proposed now. So we investigated the possibility of changing the picture by increasing the fare of JNR considerably. We went up to 5 times the current fare.

Multiplication factor β, explained in Fig. 3, was varied in the range 1, 2, 5, ∞. Two subway routes are considered alternatively. "Superposition of local commuters" was provided as the means of checking its effect on the main flow of through passengers.

6. Results of computations

Fig. 8~16 illustrate the results of computations for various cases (reproduced from [2]). In all these cases, the parameters α and β were kept $=1$.

Fig. 8 is the picture we would expect if Tozai-line is built as planned and if there is no change in other aspects. Notice that 74 thousand out of 90 thousand commuters ride Sobu-line and only 16 out of 90 ride the new subway line.

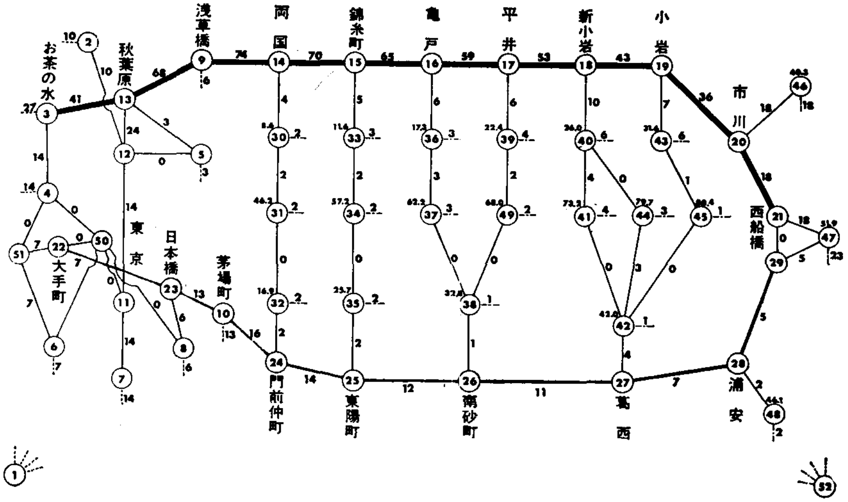

Fig. 8. Standard, no express, 90,000 commuters

Attached to each circle for a residential area, there is a number which we call "inconvenience index." It is by definition

$$\frac{\text{total cost of all commuters originating from that area}}{\text{number of those commuters}}$$

Measured by this index, area 32 is almost as inconvenient as area 36, and area 35 corresponds to area 40. It is also noticeable that area 31 is more inconvenient than area 46. That is perhaps close to the actual feelings of the residents in those areas.

Fig. 9 predicts what will happen if the number of commuters doubles under the same setup as the previous case.

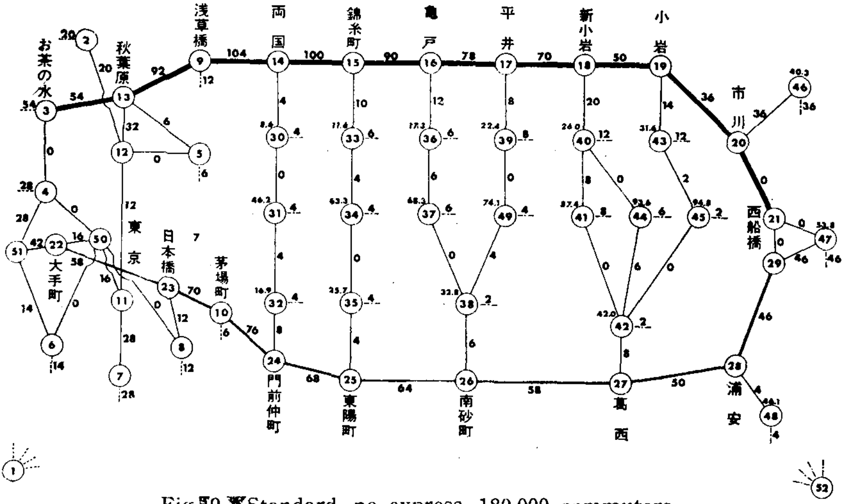

Fig. 9. Standard, no express, 180,000 commuters

Sobu-line now carries 104,000 passengers at one point. This is its "ultimate capacity." The overflowed commuters from the east have to go through the subway line. So now it carries 76,000 rather than 32,000 as we might have expected from the previous figure.

The effect of constructing an express line is seen in Fig. 10. The number of commuters is back on the lower level and is 90,000. It is divided into three parts:—55 thousand on Sobu local, 27 thousand on Sobu express, and only 8 thousand on Tozai-line. Under these circum-

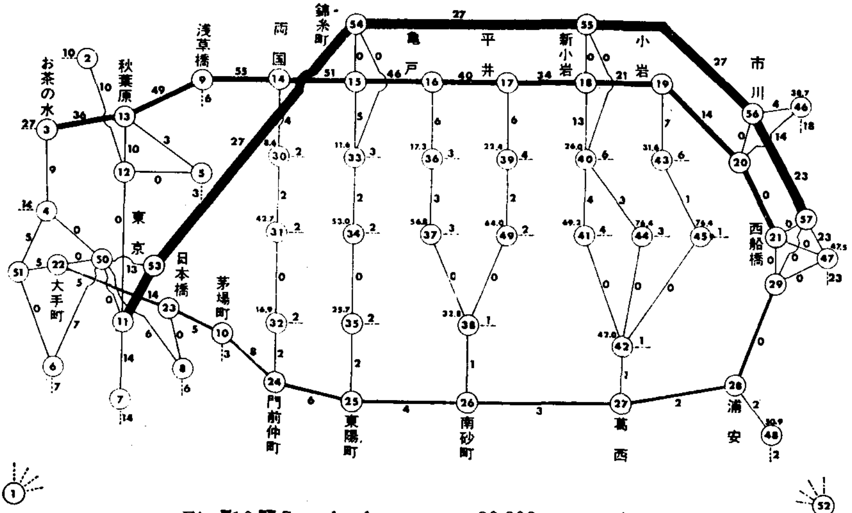

Fig. 10. Standard, express, 90,000 commuters

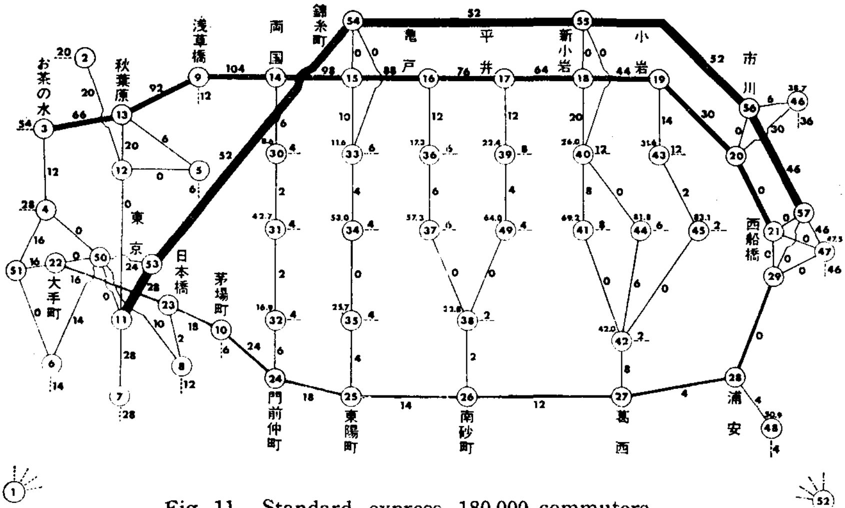

Fig. 11. Standard, express, 180,000 commuters

stances, it can easily be seen that the new subway line is almost useless for carrying the commuters flowing into this system from the east. All those commuters ride the express line. Also all commuters originating in the intermediate areas go upwards to ride Sobu-line.

If the number of commuters doubles, both Sobu local and Sobu express are saturated, and the overflow ride the subway (Fig. 11). However the change of pattern occurs not in the switching of the main flow from the east, but in the switching of the people who live in the intermediate areas.

One reason why the load distribution turns out to be so uneven in each case is that the fare of JNR lines are much cheaper than the fare of subway lines, the ratio being 5:8 approximately.

So one might think that the increase of JNR fare would push the passengers from JNR to subway. The result of the computations along this line is summarized in Fig. 12. Unit of the abscissa is the current fare.

The effect is certainly appreciable. But the JNR fare must go up

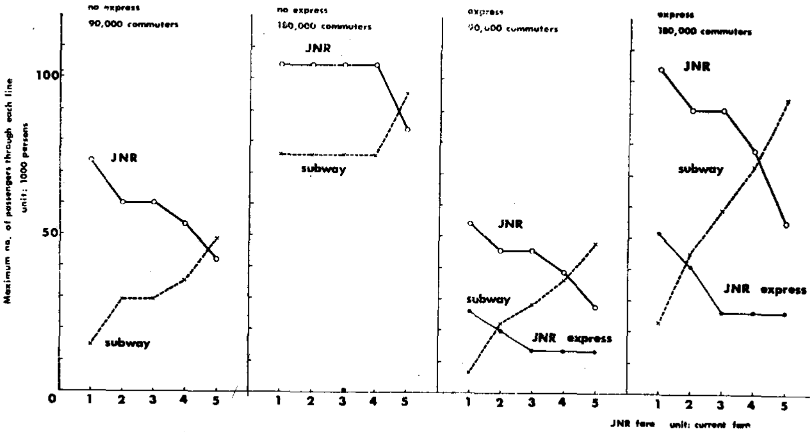

Fig. 12. Summary of the results, I: Effect of JNR fare

to 4 or 5 times the current value before the really even distribution is to be expected.

If we change the route of the proposed new subway line, then the pictures become a little more encouraging.

The load distribution under the "standard" situation is now 65:25 (Fig. 13). The subway carries all the passengers originating in the intermediate areas.

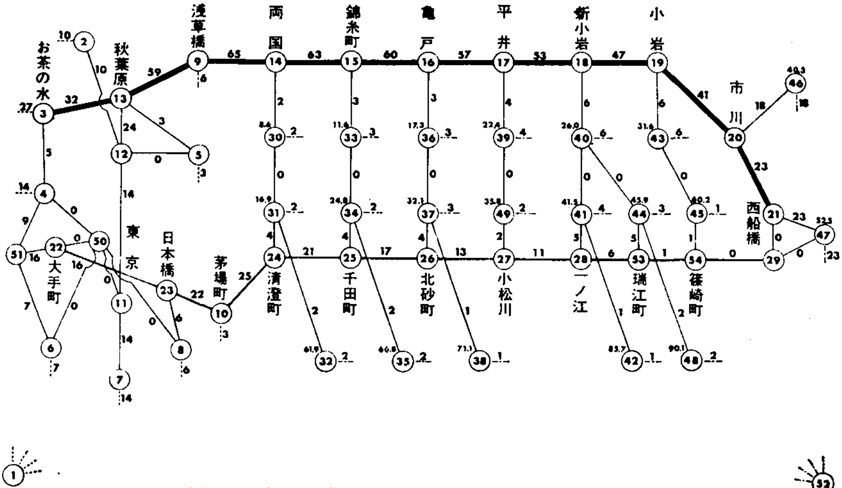

Fig. 13. Alternative subway route, no express, 90,000 commuters

When the number of commuters doubles, the distribution is 104:76 (Fig. 14), the same as in the case of standard subway route.

With the express line, the load distribution is unchanged, except that the load of JNR 65,000 is now divided into 46,000 for Sobu local and 19,000 for Sobu express (Fig. 15). This is a desirable distribution. Even if the number of commuters doubles, the ratio will not change because no line will get saturated.

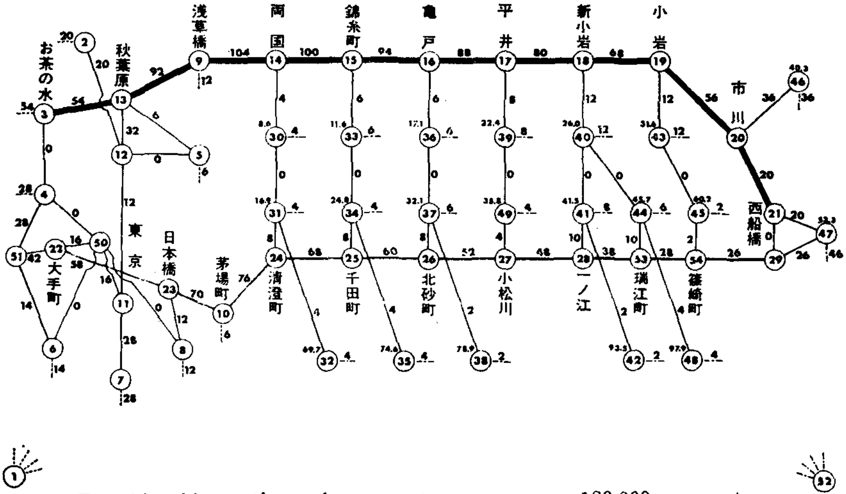

Fig. 14. Alternative subway route, no express, 180,000 commuters

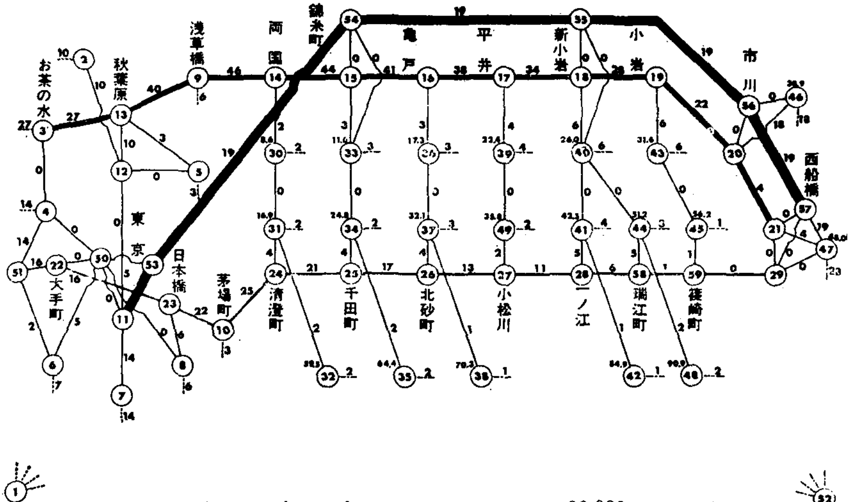

Fig. 15. Alternative subway route, express, 90,000 commuters

The desirable conditions produced by altering subway route are reflected in Fig. 16. The crossing now occurs between 2 and 3.

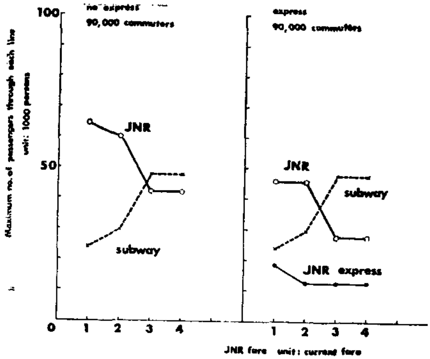

Fig. 16. Summary of the results, II: Effect of JNR fare when alternative subway route is adopted

7. Concluding remarks

The set of computations reported in this paper is just a small example of the use of "network flow model." But, as mentioned before, it suggests how, and to what extent, it can produce useful results in predicting the distribution of passengers among competitive routes.

We feel that it is simple enough to handle by a medium size computer, and yet it is sufficiently robust under various conditions. In particular, it is worth mentioning that this analysis does not require the origin-destination type data, which are hard to obtain, especially when we are evaluating several alternative plans for the future. We hope this kind of technique will be utilized to its full extent to produce a good and well-balanced transportation system in "developing" countries, which practically all countries are in this respect.

8. Acknowledgments

The main part of the work reported in this paper was carried out as a project "Analytical Study of the Commuter Transportation" of the Union of Japanese Scientists and Engineers (JUSE) under the contracts with the Japanese National Railways (JNR). The authors are indebted to Messrs. Yajima, Okudaira, Tsunekawa, Arai, and Takase, who have been other active members of the committee for the project.

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