

## A PRACTICAL METHOD FOR TRUCK DESPATCHING PROBLEM

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### Abstract

This paper presents a method for solving the truck despatching problem which may be considered as the generalization of the travelling-salesman problem. The method developed is simpler and an advantageous one over the existing method given by G.B. Dantzig and H.J. Rāmser, and later on modified by G. Clarke and J.W. Wright to advantage. It does not restrict the size of the problem to be solved and also is independent of the data concerned, i.e., the variation in the input data does not effect the solution. It makes maximum possible use of the capacity of the carrier and considers modification to the problem. It is hoped that it presents a practical means for solving any truck despatching problem irrespective of its size and data. A numerical example is used as an illustration.

### Introduction

The truck despatching problem, which is frequently encountered in some of the organisations, may be considered as the generalization of the travelling-salesman problem. In the latter, we find a route of minimum length which starts from one point, visits all the other given points just

once and returns to the starting point. The present problem is one of finding a set of delivery routes from a central depot to a large number of delivery points. The quantity demanded at each delivery point is known in units which also define the capacity of the trucks to be used in carrying the loads. A truck can visit several points, which depends upon its capacity and the quantity demanded at the visiting points, before returning to the central depot from where it starts. The object is to prepare a schedule of runs which minimizes the total distance covered by the whole fleet.

The truck despatching problem was first considered by Dantzig and Ramser [1]. They suggested an approach which used linear programming ideas. Their approach is iterative and is such that which minimizes the inter pair distances between the points but does not give necessarily optimal solution. Later on their method was modified to advantage by Clarke and Wright [2] and they presented a working solution to the truck despatching problem. Though their modified method also does not give necessarily optimal solution but based upon their experience provides a better scheduling than that which the usual despatch office can provide.

The present paper develops a method which is simpler and provides a practical means for solving the truck despatching problem irrespective of its size and data concerned. It also considers some modifications to the problem.

### **Formulation of the Problem**

Let  $P_1, P_2, P_3, \dots, P_n$  be the delivery points with  $q_1, q_2, q_3, \dots, q_n$  as the demands, respectively, to be serviced from a central depot  $P_0$ . All inter distances  $d_{ij}$  ( $i, j=0, 1, 2, 3, \dots, n$ ) between the points are given and assumed to be symmetric. There are large number of trucks available and for the present, suppose each of capacity  $C$ . Later on we shall consider the case where the capacities of all the trucks available are not necessarily the same. If the capacity of a truck satisfies

$$C \geq \sum_{i=1}^n q_i,$$

the problem is the same as the travelling-salesman problem since the carrier can serve every delivery point on one trip which links all the given points. Thus in truck despatching problem it is assumed that the capacity of the carrier satisfies the relation

$$C \ll \sum_{i=1}^n q_i.$$

Mathematically the truck despatching problem may be stated as follows: Given the following:

- (1) A set of  $n$  points  $P_i$  ( $i=1, 2, 3, \dots, n$ ) to which deliveries are made from a point  $P_0$ , called the central depot.
- (2) A symmetric matrix  $D=[d_{ij}]$ , called the distance matrix and which specifies the distances  $d_{ij}$  between every pair of points ( $i, j=0, 1, 2, \dots, n$ ).
- (3)  $q_i$  ( $i=1, 2, 3, \dots, n$ ) are the quantities of the amount to be delivered at the points  $P_i$  ( $i=1, 2, 3, \dots, n$ ).
- (4)  $C$  is the capacity of the carrier available and where,

$$C > \max q_i (i=1, 2, 3, \dots, n),$$

$$\text{but } C \ll \sum_{i=1}^n q_i.$$

- (5) If  $x_{ij}=x_{ji}=1$  is interpreted to mean that points  $P_i$  and  $P_j$  are adjacent ( $i, j=0, 1, 2, \dots, n$ ) and if  $x_{ij}=x_{ji}=0$  means that the points are not adjacent in any of the tour, then obviously

$$\sum_{i=0}^n x_{ij} = \sum_{j=0}^n x_{ji} = 1.$$

The problem is to find those values of  $x_{ij}$  which make the total distance

$$T = \sum_{i,j=0}^n d_{ij} x_{ij}$$

a minimum subject to the conditions specified in (2) to (5).

It should be noted that the number of carriers is not involved in the problem. Also more than one product can be delivered to each delivery point. There may be a difficulty in practice of mixing the products, if the products are of liquid type. But this difficulty can be easily removed by compartmentation of the trucks.

### **Modification in the Formulation**

In formulating the problem Dantzig and Ramser [1] did not take into consideration the maximum use of the truck capacity. In order to take it into consideration, besides the conditions specified in (2) to (5), we shall also put up that there is a penalty cost per unit volume of the capacity of the truck which is not used and which also depends upon the distance covered in any of the trip. If we assume that there is a transportation cost per unit capacity per unit distance, then the problem is not only to minimize the transportation cost but also to penalty cost. Though the present method does not consider the penalty cost specifically but is such that which makes the maximum possible use of the capacity of the carrier and thus automatically minimizes the penalty cost. This cost can be reduced to a great extent by using the carriers of different capacities as will be seen later on and which also suggests a better solution.

### **The Method**

The method for solving the truck despatching problem can be conveniently separated into the following heads:

#### **Finding the number of trips**

The first step to solve the truck despatching problem is to decide in how many trips the delivery can be made to all given points. This number depends upon the capacity of the carrier and the total delivery which is to be made for a given problem. Thus the number of trips is

given by

$$N = \frac{\sum_{i=1}^n q_i}{C} \text{ if } C \text{ is a perfect divisor of } \sum_{i=1}^n q_i, \\ = N_0 + 1 \text{ otherwise,}$$

where,  $N_0$  denotes the greatest integer part of  $\frac{\sum_{i=1}^n q_i}{C}$ .

### Grouping the delivery points

The given points  $P_i$  ( $i=1, 2, 3, \dots, n$ ) are divided into  $N$  groups. The number of points in a group may differ from another one since it depends upon the deliveries to be made. The systematic way of forming these  $N$  groups is to start with the first point  $P_1$ . Corresponding to point  $P_1$  we consider the second row of the distance matrix  $D$  and find which point  $P_i$  ( $i=2, 3, 4, \dots, n$ ) is nearer to  $P_1$ . Now if the sum  $q_1 + q_i \leq C$ , then  $P_i$  ( $i=2, 3, 4, \dots, n$ ) is the second element which is included in the first group  $G_1$ . If  $q_1 + q_i < C$ , then from the given matrix  $D$ , find which of the point is nearer to  $P_i$ , where  $P_i$  is the second selected point, and include that point in  $G_1$ , if the sum of the deliveries corresponding to the included points in  $G_1$  is not greater than  $C$ . This process of inclusion of one more point in  $G_1$  is continued until the sum of the deliveries corresponding to the included points does not exceed  $C$ . As we want to make use the maximum capacity of the carrier, the point included in  $G_1$  at the end or at the beginning may be interchanged with the another one which is the next nearer point to the last included point, if its inclusion makes the sum of the deliveries more near to  $C$ . In a similar fashion we form the other groups  $G_2, G_3, G_4, \dots, G_N$ .

In case  $N = N_0 + 1$ , there may be a group consisting only one delivery point. The selection of this point must be such that the delivery to be made to this point should not be less than half the delivery of the carrier and its distance from central depot  $P_0$  should not be more than half the distance of the maximum distant point from  $P_0$ . Though there may be several point having this property, but the point which is to be

selected must ensure the maximum possible use of the capacity of the carrier in all the trips.

### Finding the optimal tours

Having grouped the delivery points, the truck despatching problem may now be thought of as solving the  $N$  symmetric travelling-salesman problems. In each of the trip the carrier starts from central point  $P_0$ , so it is common to all the groups  $G_1, G_2, G_3, \dots, G_N$ . The symmetric distance matrix  $D$  is separated into  $N$  sub matrices corresponding to each of the  $G_1, G_2, G_3, \dots, G_N$  and written in such a manner that the first row of each sub-matrices denotes the distances from  $P_0$  to all the points in the group. Now the simplest procedure for finding the optimal tours in each case can be separated into the following parts:

#### (a) Reduction of the distance matrix

The distance matrices obtained corresponding to each of the group  $G_1, G_2, G_3, \dots, G_N$  are considered separately. First subtract the minimum element of each row of the distance matrix from the elements of row itself to get the row reduced distance matrix. Now subtract the minimum element of each column of the row reduced distance matrix from the elements of column itself to get the doubly reduced distance matrix. Thus the doubly reduced distance matrices corresponding to each of the group  $G_1, G_2, G_3, \dots, G_N$  will contain at least one zero element in each of their rows and columns.

The rest of the operations are made separately on each of the doubly reduced distance matrices. In further for doubly reduced distance matrix we shall write distance matrix.

#### (b) Selection of the point to be served at the end of the trip

To select the point to be served at the end of the trip we consider the 1st column of the distance matrix which corresponds to the point  $P_0$  and we choose the first three minimum elements in it. To make the best choice out of them, the following rules strictly followed:

1. Choose the other one minimum element in each of the three rows in which the first three chosen elements, of the 1st column of

distance matrix, lie. Corresponding to each later chosen element, find the sum of the other first three minimum elements in each of the three columns in which the later chosen elements lie. Find which sum is smallest. Then out of the first three chosen elements of the first column, the one which corresponds to the smallest sum is selected for inclusion in the optimal tour. The selected element is enclosed in a small circle and the row in which the selected element lies and the 1st column  $P_0$  are crossed off which means that the element of this row and the column  $P_0$  will not be considered further. The row, in which the selected element lies, corresponds to the point  $P_i$  ( $i=1, 2, 3, \dots, n$ ) which is to be served at the end of the trip. The column which corresponds to the selected point is also crossed off.

If any two sums or all the three sums taken above are equal, then to make a better choice, add the fourth minimum element of each column to the corresponding sum so as to make them unequal. The addition of the higher minimum element is continued until the equal sums become unequal. Then the element, of the 1st column, which corresponds to the smaller sum, is the better choice.

2. If any of the two chosen elements, of the 1st column of the distance matrix, lie in two adjacent rows, then the middle one is not selected and comparison for better choice is made only in the rest two.

3. If the any two sums taken in step 1 are equal and the two chosen minimum elements which correspond to the two chosen minimum elements of the 1st column of the distance matrix, lie in the same column, then choose the next minimum element in each of the two respective rows and do the same process as in step 1.

### **(c) Selection of the remaining points in an optimal manner**

The selection of the remaining points is made according to the following steps:

1. Corresponding to the point  $P_0$  we consider the 1st row of the distance matrix. In this row we consider only those two elements, of the two columns which bear the same numbers as the rows in which

the two chosen elements in 1(b) which have not been selected for inclusion in the optimal tour, lie. Find the sum of the other first three minimum elements in each of the two columns in which the considered elements lie (remember that the elements of the crossed off row and columns will not be taken into consideration). Then the element of the first row which corresponds to the greater sum is selected for inclusion in the optimal tour. It is enclosed in a small circle and the corresponding column and the 1st row are crossed off.

If the two taken sums are equal and the corresponding elements of the 1st row are not equal, then smaller element is the better choice for inclusion. If the two elements of the first row are also equal, then add the fourth minimum element of each column to the corresponding sum so as to make the sums unequal. The addition of higher minimum element is continued until the equal sums become unequal. Then the element which corresponds to the greater sum is selected for inclusion.

2. Now we consider the row which corresponds to the point to which the column in which the last selected element of the 1st row lies. Choose the minimum element of this row. Find the sum of the other first three minimum elements of the column in which the chosen element lies. Now choose the next minimum element of the same row but the chosen element must be either the first or second or third minimum element of the column in which it lies (remember that the elements of the crossed off rows and columns will not be taken into consideration). Find the sum of the other first three minimum elements of the column in which the second chosen element lies. If there remains any other element of the same row which is the first minimum element of the column in which it lies, then also find the sum of the other first three minimum elements of the column in which it lies. Now the element, of the row under consideration, which corresponds to the greatest sum is selected for inclusion in the optimal tour. The selected element is enclosed in a small circle and the corresponding row and column, in which the selected element lies, are crossed off.



If the two sums or two chosen elements or both are equal, then the same rule is applied as in step 1(c). The process of considering the row corresponding to the point to which the column, in which the last selected element lies, corresponds, is continued until there remain three columns uncrossed.

3. At the end of the step there remain four rows uncrossed. Now consider the row which is to be considered. Choose the minimum element of this row and find the sum of the other two elements which are the only available elements of the column in which the chosen element lies. Then choose the next minimum element of the same row and find the sum of the other two elements of the column in which the second chosen element lies. The element, of the row under consideration, which corresponds to the greater sum is selected for inclusion in the optimal tour and the corresponding row and column are crossed off.

At this stage the row, which corresponds to the point to be served at the end of the trip, is considered as uncrossed. Now consider the row which is to be considered and select an element for inclusion in the optimal tour by applying the same rule as above and cross off the corresponding row and column in which the selected element lies. Finally the element, which lies in the row to be considered and in the column which corresponds to the point to be served at the end of the trip, is selected for inclusion in the optimal tour. Thus we have found the optimal tour which can be represented in the same order in which we have considered the rows, starting from the 1st row. This process of finding the optimal tour is done with each of the doubly reduced distance matrices corresponding to  $G_1, G_2, G_3, \dots, G_N$ .

While applying the above procedure, if it is found that there is only one single element, in the row under consideration, which is either the first or second or third minimum element of the column in which it lies, then it is selected for inclusion without any comparison. If at any subsequent stage it is found that there is no element, in the row under

consideration, which is either the first or second or third minimum element of the column in which it lies, then it is an indication that either the selection of the point to be served at the end of the trip is wrong or the selection of the element in the first row is wrong. In case both the said selections are found correct, which may happen very rarely, then experience to date shows that the only way to remove this difficulty is to choose the another element, of the 1st row, which was in the comparison of better choices.

4. The last step is to find the sum of the elements, of the original distance matrix, which correspond to the elements enclosed in the circles in each of the doubly reduced distance matrix corresponding to each group  $G_1, G_2, G_3, \dots, G_N$ . Thus the total optimal distance to be covered by the whole fleet is given by

$$T = \sum_{r=1}^N T_r,$$

where,  $T_r$  denotes the length of the optimal tour for the group  $G_r$  ( $r=1, 2, 3, \dots, N$ ).

### Illustrative Example

The method developed can be best illustrated by solving a numerical example. The example given here was also solved by Dantzig and Ramser [1], but by their approach they could not get the optimal solution. There are twelve given points  $P_1, P_2, P_3, \dots, P_{12}$  which are serviced from a central depot  $P_0$ . The inter distances between the points are given in Table 1. The diagonal elements are not defined as they represent the distances from a point to itself. The first column of the same table headed by  $Q$  gives the amounts of deliveries which are made to each point, respectively. The capacity of the carrier to be used is 60000.

It can be verified that the number  $N$  of the trips, in which the delivery can be made to all the points, is 4.

To divide the all given delivery points into 4 groups, the said procedure is applied and the four groups obtained are

Table 1

$Q$	$d_{ij}$	$P_0$	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_7$	$P_8$	$P_9$	$P_{10}$	$P_{11}$	$P_{12}$
	$P_0$	—	9	14	21	23	22	25	32	36	38	42	50	52
1200	$P_1$	9	—	5	12	22	21	24	31	35	37	41	49	51
1700	$P_2$	14	5	—	7	17	16	23	26	30	36	36	44	46
1500	$P_3$	21	12	7	—	10	21	30	27	37	43	31	37	39
1400	$P_4$	23	22	17	10	—	19	28	25	35	41	29	31	29
1700	$P_5$	22	21	16	21	19	—	9	10	16	22	20	28	30
1400	$P_6$	25	24	23	30	28	9	—	7	11	13	17	25	27
1200	$P_7$	32	31	26	27	25	10	7	—	10	16	10	18	20
1900	$P_8$	36	35	30	37	35	16	11	10	—	6	6	14	16
1800	$P_9$	38	37	36	43	41	22	13	16	6	—	12	12	20
1600	$P_{10}$	42	41	36	31	29	20	17	10	6	12	—	8	10
1700	$P_{11}$	50	49	44	31	31	28	25	18	14	12	8	—	10
1100	$P_{12}$	52	51	46	39	29	30	27	20	16	20	10	10	—

given by

$G_1=(P_1, P_2, P_3, P_4)$ , with total delivery to be made=5800

$G_2=(P_6, P_8, P_9)$ , with total delivery to be made=5100

$G_3=(P_7, P_{10}, P_{11}, P_{12})$ , with total delivery to be made=5600

$G_4=(P_5)$ , with total delivery to be made=1700

The reader verify that these combinations of the points make the maximum possible use of the capacity of the carrier and so automatically minimize the penalty

Table (a<sub>1</sub>)

$d_{ij}$	$P_0$	$P_1$	$P_2$	$P_3$	$P_4$
$P_0$	—	9	14	21	23
$P_1$	9	—	5	12	22
$P_2$	14	5	—	7	17
$P_3$	21	12	7	—	10
$P_4$	23	22	17	10	—

Table (b<sub>1</sub>)

$d_{ij}$	$P_0$	$P_6$	$P_8$	$P_9$
$P_0$	—	25	36	38
$P_6$	25	—	11	13
$P_8$	36	11	—	6
$P_9$	38	13	6	—

Table (c<sub>1</sub>)

$d_{ij}$	$P_0$	$P_7$	$P_{10}$	$P_{11}$	$P_{12}$
$P_0$	—	32	42	50	52
$P_7$	32	—	10	18	20
$P_{10}$	42	10	—	8	10
$P_{11}$	50	18	8	—	10
$P_{12}$	52	20	10	10	—

Table (a<sub>2</sub>)

$d_{ij}$	$P_0$	$P_1$	$P_2$	$P_3$	$P_4$
$P_0$	—	0	5	12	14
$P_1$	4	—	0	7	17
$P_2$	9	0	—	2	12
$P_3$	14	5	0	—	3
$P_4$	13	12	7	0	—

Table (b<sub>2</sub>)

$d_{ij}$	$P_0$	$P_6$	$P_8$	$P_9$
$P_0$	—	0	11	13
$P_6$	14	—	0	2
$P_8$	30	5	—	0
$P_9$	32	7	0	—

Table (c<sub>2</sub>)

$d_{ij}$	$P_0$	$P_7$	$P_{10}$	$P_{11}$	$P_{12}$
$P_0$	—	0	10	18	20
$P_7$	22	—	0	8	10
$P_{10}$	34	2	—	0	2
$P_{11}$	42	10	0	—	2
$P_{12}$	42	10	0	0	—

Table ( $a_3$ )

$d_{ij}$	$P_0$	$P_1$	$P_2$	$P_3$	$P_4$
$P_0$	—	0	5	12	Ⓜ
$P_1$	Ⓞ	—	0	7	14
$P_2$	5	Ⓞ	—	2	9
$P_3$	10	5	Ⓞ	—	0
$P_4$	9	12	7	Ⓞ	—

Table ( $b_3$ )

$d_{ij}$	$P_0$	$P_6$	$P_8$	$P_9$
$P_0$	—	Ⓞ	11	13
$P_6$	0	—	Ⓞ	2
$P_8$	16	5	—	Ⓞ
$P_9$	Ⓜ	7	0	—

Table ( $c_3$ )

$d_{ij}$	$P_0$	$P_7$	$P_{10}$	$P_{11}$	$P_{12}$
$P_0$	—	0	Ⓜ	18	18
$P_7$	Ⓞ	—	0	8	8
$P_{10}$	12	2	—	0	Ⓞ
$P_{11}$	20	Ⓜ	0	—	0
$P_{12}$	20	10	0	Ⓞ	—

cost. The point  $P_0$  is common to all the four groups and therefore is not written in the formation of the groups.

Corresponding to each of the three groups  $G_1, G_2, G_3$ , the distance matrices abstracted from Table 1 are given in Table ( $a_1$ ), Table ( $b_1$ ), and Table ( $c_1$ ), respectively. The row reduced matrices of these matrices are given in Table ( $a_2$ ), Table ( $b_2$ ), and Table ( $c_2$ ) and the doubly reduced distance matrices in Table ( $a_3$ ), Table ( $b_3$ ), and Table ( $c_3$ ), respectively.

Now the said procedure is applied to find the optimal tours. First we consider the doubly reduced distance matrix given in Table ( $a_3$ ).

To select the point of  $G_1$  which is to be served at the end of the trip, we choose the first three minimum elements of the column  $P_0$  of Table ( $a_3$ ). These elements are

$$d_{1,0}=0, d_{2,0}=5, \text{ and } d_{4,0}=9.$$

As  $d_{1,0}$  and  $d_{2,0}$  are in two adjacent rows and  $d_{2,0}$  is in the middle of  $d_{1,0}$  and  $d_{4,0}$ , therefore comparison is made only in  $d_{1,0}$  and  $d_{4,0}$ .

The other minimum element of the row  $P_1$  is  $d_{1,2}=0$ , and the sum of the other first three minimum elements of the column  $P_2$  is given by

$$d_{3,2}+d_{0,2}+d_{4,2}=0+5+7=12.$$

The other minimum element of the row  $P_4$  is  $d_{4,3}=0$ . The sum of the other first three minimum elements of the column  $P_3$  is given by

$$d_{2,3}+d_{1,3}+d_{0,3}=2+7+12=21.$$

Now as the first taken sum is smaller, therefore the element  $d_{1,0}$  is selected for inclusion in the optimal tour. It is enclosed in a small circle and the row  $P_1$  and the columns  $P_0$  and  $P_1$  are crossed off.

Now in the 1st row, we consider the element  $d_{0,2}=5$  and  $d_{0,4}=11$ . Corresponding to these elements the sum of the other two elements, which are the only available elements in uncrossed rows, in each of the corresponding column is given by

$$S_{0,2}=d_{3,2}+d_{4,2}=0+7=7$$

and

$$S_{0,4}=d_{3,4}+d_{2,4}=0+9=9.$$

As  $S_{0,4} > S_{0,2}$ , therefore the element  $d_{0,4}=11$  is selected for inclusion in the optimal tour and the corresponding row  $P_0$  and column  $P_4$  are crossed off.

Next we consider the row  $P_4$ . At this stage the row  $P_1$  is considered as uncrossed. The first minimum element of row  $P_4$  is  $d_{4,3}=0$ . The corresponding sum of the elements of column  $P_3$  is given by

$$S_{4,3}=d_{2,3}+d_{1,3}=2+7=9.$$

The next minimum element of the row  $P_4$  is  $d_{4,2}=7$ . The corresponding sum of the elements of column  $P_2$  is given by

$$S_{4,2} = d_{3,2} + d_{1,2} = 0 + 0 = 0.$$

As  $S_{4,3} > S_{4,2}$ , therefore the element  $d_{4,3}$  is selected for inclusion in the optimal tour and the corresponding row  $P_4$  and the column  $P_3$  are crossed off.

Next we consider row  $P_3$  and as the only column  $P_2$  is uncrossed, the element  $d_{3,2} = 0$  is selected for inclusion in the optimal tour and corresponding row  $P_3$  and column  $P_2$  are crossed off.

Finally in the row  $P_2$ , the element  $d_{2,1} = 0$  is selected for inclusion in the optimal tour.

Thus the optimal tour obtained is  $P_0, P_4, P_3, P_2, P_1, P_0$ .

The length of this tour = 54.

Next we find optimal tours for the groups  $G_2$  and  $G_3$  and for which we consider the doubly reduced distance matrices given in Table ( $b_3$ ) and Table ( $c_3$ ), respectively. Proceeding in a similar way as for  $G_1$ , the optimal tours obtained for  $G_2$  and  $G_3$  are given by

$$P_0, P_6, P_8, P_9, P_0, \text{ with tour length} = 80$$

and

$$P_0, P_{10}, P_{12}, P_{11}, P_7, P_0 \text{ with tour length} = 112.$$

The optimal tour for the group  $G_4$  is obviously

$$P_0, P_5, P_0 \text{ with tour length} = 44.$$

Therefore optimal distance to be covered by the whole fleet is given by

$$T = 54 + 80 + 112 + 44 = 290.$$

Dantzig and Ramser [1] also believed that 290 units was the optimal solution for the present example.

### Carriers with Different Capacities

Based upon the experience in the investigation it is found that if we use the carriers of different capacities, we can still reduce the distance to be covered by the whole fleet. This is because some time

a point being nearer to a point is not included in the same group due to the capacity restriction and as we want to make use of maximum possible capacity of the carrier. Thus the use of carriers of different capacities facilitates to group the delivery points in a better way both for using the maximum capacity of the carriers and as well as to reduce the total distance to be covered by the whole fleet.

While using the carriers of different capacities, we start the grouping of delivery points with the carrier of maximum capacity and the same criterion is used as given in the method for this purpose. Having grouped the delivery points into suitable number of groups which depends upon the total delivery to be made and the capacities of the carriers to be used, the same procedure as given in the method is applied to find the optimal tour for each group.

For the example solved in this paper if we use the three carriers of capacities 6000, 6100, 6200, respectively, then to make the maximum possible use of the capacity of the whole fleet and to reduce the distance to be covered, the delivery points may be divided according to the said rule in the following groups:

$$G_1=(P_9, P_{10}, P_{11}, P_{12}), \text{ with delivery to be made}=6200,$$

$$G_2=(P_5, P_6, P_7, P_8), \text{ with delivery to be made}=6100,$$

$$G_3=(P_1, P_2, P_3, P_4), \text{ with delivery to be made}=5800.$$

The central depot  $P_0$  is common to all the three groups. The optimal tours obtained for these groups, respectively, are

$$P_0, P_{10}, P_{12}, P_{11}, P_9, P_0, \text{ with tour length}=112,$$

$$P_0, P_6, P_8, P_7, P_5, P_0, \text{ with tour length}=78,$$

and

$$P_0, P_1, P_2, P_3, P_4, P_0, \text{ with tour length}=54.$$

Thus the optimal tour length to be covered by the whole fleet is 244.



### Conclusion

A method for truck despatching problem has been developed. The method given here is simpler as compared to already existing methods. It gives satisfactory solution within a reasonable time by hand computation and is also independent of the data concerned, i.e., the variation in the input data does not affect the solution. At every step an element is selected for inclusion in the optimal tour and the corresponding row and column are crossed off. This prevents the other  $2n-1$  elements for further inclusion as the possible choices. Thus all feasible combinations, of the elements of the distance matrix, are considered implicitly and explicitly. During the investigation several problems were solved and in each case satisfactory solution was obtained. It is hoped that the method provides a practical means for solving any truck despatching problem irrespective of its size and data.

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