

CRITICAL PATH METHOD IN A LEARNING EFFECT

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(Received July 4, 1966)

1. Introduction

The analytical development of the critical path method under the influence of a Learning Effect is given and a simple example is presented. A Critical Path is usually fixed, namely it can never be changed without consuming some resources, here however, the Learning Effect is considered as one of the resources which can accelerate the activities which compose the network of a project. Learning Effect is based on the learning model where the cumulative average activity durations are log-linear with respect to the number of an activity performance.

2. Cumulated Average Duration per Activity

Activities records for past-performed project is the starting point. The following are obtained:

t_1 =duration to perform the first activity.

X_u =number of activity performance.

A_c =cumulated average duration of activity for number X_u .

Experience has shown that this curve has an equation of the form:

$$(1) \quad A_c = \frac{t_1}{X_u^n}.$$

From Eq. (1):

$$\begin{aligned} (A_c)(X_u^n) &= t_1 \\ \log A_c + n \log X_u &= \log t_1 \\ (2) \quad \log A_c &= -n \log X_u + \log t_1 \end{aligned}$$

plotting A_c vs X_u on log-log paper produces Fig. 1 from Eq. (2)

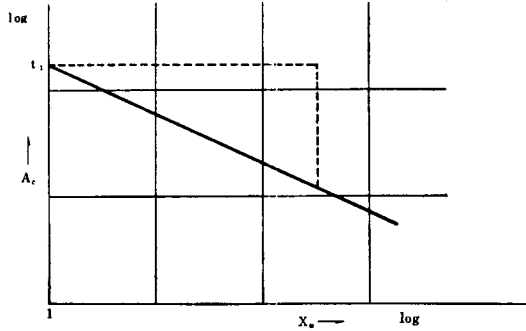


Fig. 1.

$$(3) \quad n = \frac{\log t_1 - \log A_c}{\log X_u}$$

$-n$ is the slope of the straight line in Fig. (1)

2.2. Total durations vs cumulated performance number

Let T_c = cumulated total duration 1st through x -th performance inclusive :

$$(4) \quad \begin{aligned} T_c &= (A_c) \cdot (X_u) \\ T_c &= (t_1 X_u^{-n}) \cdot X_u \\ T_c &= t_1 X_u^{1-n} \end{aligned}$$

$$(5) \quad \log T_c = (1-n) \log X_u + \log t_1$$

Values of T_c for various values of X_u is shown in Fig. 2 and it is apparent that T_c vs X_u is a straight line on log-log paper with slope $(1-n)$.

2.3. Durations per individual activity vs cumulated of performance

Let T_x = durations per individual activity for the X_u -th

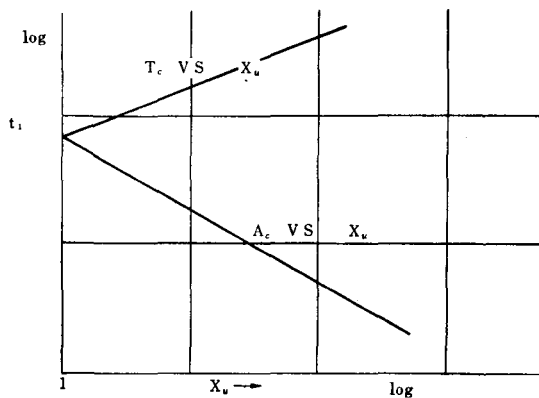


Fig. 2.

$$\begin{aligned}
 T_x &= t_1 X_u^{1-n} - t_1 (X_u - 1)^{1-n} \\
 T_x &= t_1 [X_u^{1-n} - (X_u - 1)^{1-n}] \\
 \log T_x &= \log t_1 + \log [X_u^{1-n} - (X_u - 1)^{1-n}] \\
 \log T_x &= \log t_1 + \log \{ \text{antilog} [(1-n)(\log X_u) \\
 &\quad - \text{antilog} [(1-n) \log (X_u - 1)] \}
 \end{aligned}$$

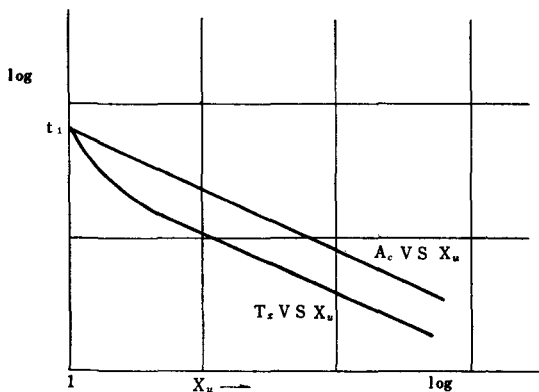


Fig. 3.

The curve is shown in Fig. 3. It is noted that where X_u is greater than 10, the curve very closely approximates a straight line parallel to the A_c vs X_u curve.

From Eq. (6)

$$\begin{aligned}
 T_x &= t_1 X_u^{1-n} \left[1 - \left(\frac{X_u - 1}{X_u} \right)^{1-n} \right] \\
 T_x &= t_1 X_u^{1-n} \left[1 - \left(1 - \frac{1}{X_u} \right)^{1-n} \right] \\
 &= t_1 X_u^{1-n} \left\{ 1 - \left[1 - \frac{(1-n)}{X_u} - \frac{(1-n)n}{X_u^2 2!} \right. \right. \\
 &\quad \left. \left. - \frac{(1-n)n(n+1)}{X_u^3 3!} \dots \right] \right\} \\
 T_x &= t_1 X_u^{1-n} \left[\frac{(1-n)}{X_u} \right] \cdot \left[1 + \frac{n}{X_u^2 2!} + \frac{n(n+1)}{X_u^3 3!} + \dots \right] \\
 (7) \quad T_x &\cong \frac{t_1(1-n)}{X_u^n}
 \end{aligned}$$

2.4. Percentage designation for A_c vs X_u

The particular A_c vs X_u curve plotted in Fig. 2 has a slope $-n = -0.322$ and it is known as an "80% Curve" By this designation is meant that whenever X_u is doubled, the new value of A_c is 80% of its previous value. In general, let

P = Percentage

X_i = A discrete value of X_u

$$P = 100 [t_1 / (2 X_i)^n \div t_1 / X_i^n] = 100 [1/2]^n$$

3. Changeable Critical Path

Activity (i, j) has the normal time D_{ij} and normal cost M_{ij} crash time d_{ij} and the cost m_{ij} and the actual time y_{ij} satisfies

$$(8) \quad d_{ij} \leq y_{ij} \leq D_{ij}$$

Usually critical path which expends the longest time will never be

changed without putting some resources. Here the critical path whose activity has the learning percentage L_{ij} % will be changed.

Supposing a project has six events and seven activities shown in Fig. 4 each activity duration is presented in Table 1 but these durations are measured when the project was performed 100 times.

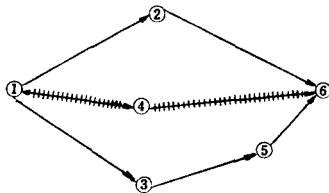


Fig. 4.

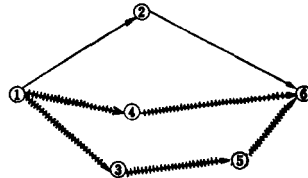


Fig. 5.

Proceeding the number of the project performed, the critical path will be changed. The first critical path is 1—4—6 because total duration of this path is 21 days which is the longest compared with the other two paths. Then another path whose events are 1, 3, 5, 6 becomes a critical path shown in Fig. 5 at the same time the first critical path has still the function as the critical path this state is called the 2nd period of the project.

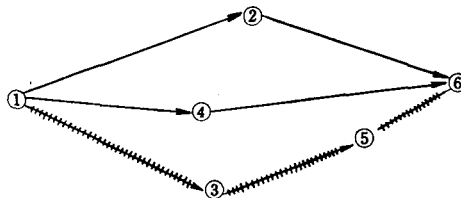


Fig. 6. 3rd. period C.P.

Then only the new critical path whose events are 1, 3, 5, 6 remains as the critical path in the 3rd period, shown in Fig. 6. The critical

Table 1.

Activity	F. Event	R. Event	Duration	Learning %
A	1	2	3	100
C	1	3	10	90
B	1	4	8	65
D	2	6	16	85
E	4	6	13	70
F	3	5	6	75
G	5	6	4	80

path in the 4th period is shown in Fig. 7 and there are two paths that is 1—2—6, 1—3—5—6. The last period state is shown in Fig. 8 the critical path 1—2—6 is only the path which will never be changed.

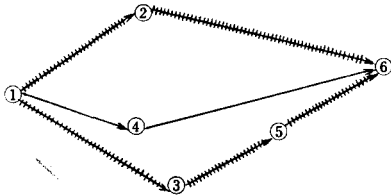


Fig. 7. 4th period C.P.

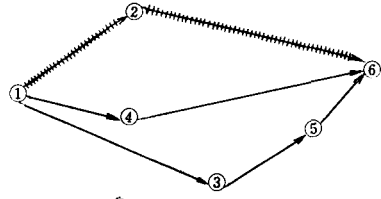


Fig. 8. 5th period C.P.

In this way, the critical path is changed seven times simply because each activity has the learning effect. These learning effects for a each path are shown in Fig. (9) on log-log paper. At A point, path (1—4—6) is the critical path and the total duration of the path is 21 days, at B point (2nd period) two critical paths appears that is 1—4—6, and 1—3—5—6. The 3rd period exist between B and C. At C point the forth period and then the fifth period begins the next.

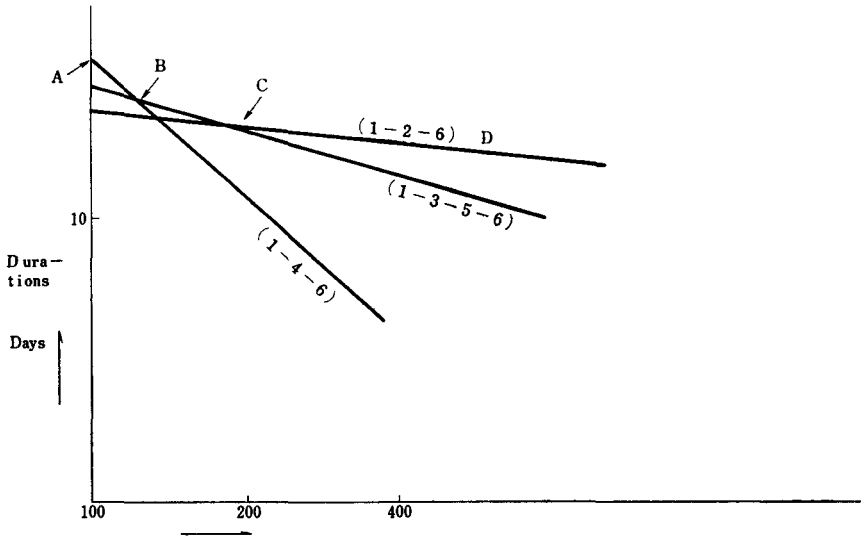


Fig. 9. Cumulated number of Activity Performed

4. Least Costly Schedule

It would be desirable to have some method for determining the least costly schedule for a given period of the project. The cost and duration of an each activity are written in Table 2.

Table 2.

Activity	Normal		Crash		Cost Slope
	<i>D</i>	<i>M</i>	<i>d</i>	<i>m</i>	
A	3	50	2	100	50
B	8	120	7	150	30
C	10	100	8	180	40
D	16	80	10	200	20
E	13	40	9	80	10
F	6	140	4	260	60
G	4	100	4	100	—

Cost unit : ¥ 10,000.

Table 3 shows the total project cost at the first period of the project and from it the IV plan is the least costly one, that is ¥14,200,000. However, this IV plan is not always the best plan, because each activity has the learning effect. For example, at the fifth period of the project the least costly schedule will be the plan whose duration is 16 days

Table 3.

Plan	I	II	III	IV	V	VI
Duration	21	20	19	18	17	16
Project cost	630	640	690	730	790	900
Indirect cost	900	820	740	690	660	590
Total cost	1,530	1,460	1,430	1,420	1,450	1,490

Cost unit: ¥ 10,000.

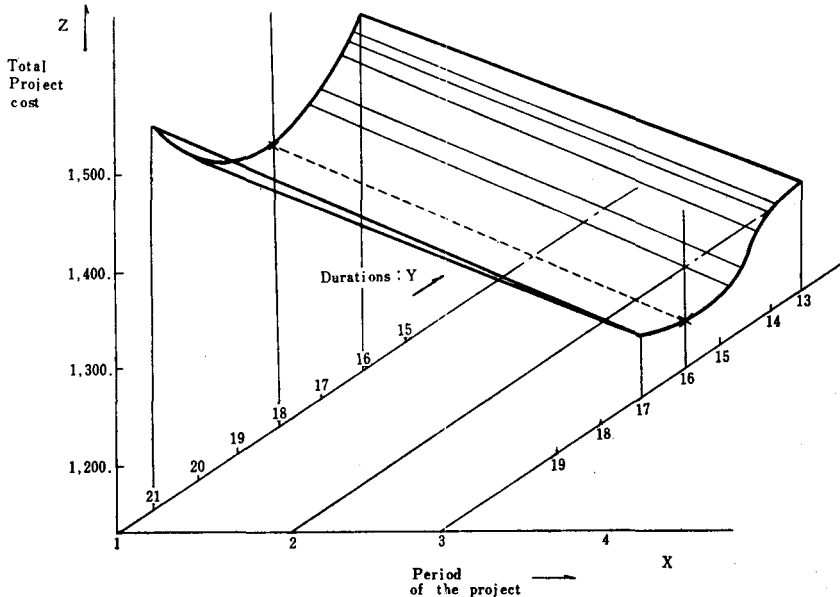


Fig. 10. The least costly schedule for the project

and the total cost is ¥11,540,000. These data are derived from Table 5 which results from the Table 4.

Fig. 10 shows the least costly shedule curve as the dotted line on the surface.

Table 4.

Activity	Normal		Crash		Cost slope c_{ij}
	D_{ij}	M_{ij}	d_{ij}	m_{ij}	
A	3.0	50	2.0	100	50
B	5.0	75	4.0	105	30
C	9.0	90	7.0	170	40
D	14.0	70	8.0	190	20
E	9.0	28	5.0	68	10
F	5.0	116	3.0	236	60
G	3.0	75	3.0	75	—

Unit: Days. ¥ 10,000.

5. Conclusion

Based on the analysis of the Learning Effect whose model is log-linear, the following conclusions may be drawn from the simple example whose activities are seven which has the learning effect each other.

1. The Critical Path of the project is changeable without consuming some resources.
2. The Critical Path of the project will be changed by the effect of learning of each activity which composes the network of the project.
3. The least costly shedule of the project is also changeable with respect to the number of the project performance.

It should be recognized that the conclusions strictly speaking apply only to the project whose activities have the learning effect whose model is log-linear, but other model will be all right to predict the least costly plan of the project if the model characteristic is known.

For practical purposes, this Critical Path Method under the influence of the learning effect can apply in the field of tunnel construction, assembly job, and so on.

Table 5.

Plan	I	II	III	IV	V
Duration	17	16	15	14	13
Project cost	504	564	624	704	794
Indirect cost	660	590	540	520	500
Total cost	1,164	1,154	1,164	1,224	1,294

Cost unit: ¥ 10,000.

Duration: days.