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ON CERTAIN KINDS OF ORDERINGS

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Introduction

Let $C_1, \cdots, C_t$ be a sequence of classes, each having $n_1, \cdots, n_t$ members, and the total number of their members be n. Suppose that each C_i has a linear order between its members but we do not give beforehand any order between members of different classes.

We wish to distribute N articles impartially to these classes, allotting $N_1, \cdots, N_t$ to $C_1, \cdots C_t$, respectively. When distributions are repeated over and over again, it would be desirable to be impartial in such a way that the distributions should satisfy the following conditions:

(1) $$\frac{N_i}{N} \to \frac{n_i}{n} \quad \text{according as } N \to \infty .$$

(2) The expectation of the deviation of N_i from $\frac{n_i}{n}N$, i.e. $E\left(\left|N_i - \frac{n_i}{n}N\right|\right)$ does not increase infinitely with N, but is limited to a small range of values, as possible.

Under these leading ideas, Prof. Katuzi Ono has devised the following two plans in distributing scholarships among the several groups of student.

The one is a '*way of ordering*' by which whole members of all classes are arranged in a linear order and the articles are to be distributed in this order. (*Rule* (A)).

The other is a '*way of determining allotments*' of C_i in proportion to n_i (*Rule* (B)) up to decimal corrections which would be taken into account next time.

I have checked these plans and have been able to prove mathematically the existence of solutions for both Rules. In the present paper, I will introduce these plans and discuss the properties of each solution together with its existence proof.

Rule (A)

Let $f_i(j)$ be the number denoting the order of the j-th member C_{ij} in the class C_i satisfying the following conditions:

A_1. $$\frac{n}{n_i}(j-1) < f_i(j) \leqq \frac{n}{n_i} j$$

A_2. If the number $f_i(j)$ is not determined by A_1 exactly, the ambiguous part should be determined by lot among the candidate members C_{ij} each with a chance to be selected in proportion to n_i.

Rule (B)

Since the number N, n, n_i, and N_i might change every time, we denote them by $N(k)$, $n(k)$, $n_i(k)$, and $N_i(k)$, respectively, for k-th distribution.

Let $r_i(k)$ be the term of correction of C_i, and $M_i(k)$ be the *standard allotment* defined by

$$M_i(1) = r_i(k) + \frac{n_i(k)}{n(k)} N(k)$$

$$r_i(k) = 0$$

$$r_i(k+1) = M_i(k) - N_i(k) \quad \text{for} \quad k \geqq 1.$$

Then we determine $N_i(k)$ by the following conditions:

B_1. (1) $N_i(k)$ is a non-negative integer satisfying $\sum_i N_i(k) = N(k)$.

(2) $M_i(k) - 1 < N_i(k) < M_i(k) + 1$.

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B_2. In case where $N_i(k)$ can not be determined by B_1, we select by lot the classes for which $N_i(k)$ would be larger than $M_i(k)$. Any class C_i taking part in this lot should have a chance to be selected in proportion to the decimal part of each $M_i(k)$.

§ 1. The Existence of Solution for Rule (A)

1.1. The matrix representation of the problem

We can represent the problem in the form of matrix uniquely (Table 1). In each column of this table, we have n_i pairs of numbers such as

$$\left(j_i, \left[\frac{n}{n_i}(j_i-1)\right]+1\right), \cdots, \left(j_i, \left[\frac{n}{n_i} j_i\right]\right)$$

for $j_i=1, \cdots, n_i$ $\left(\left[\frac{n}{n_i} j\right]\right.$ denotes the greatest integer which is not larger

Table 1.

$n_1=2$ C_1	$n_2=4$ C_2	$n_3=4$ C_3	$n_4=5$ C_4
(1. 1)	(1. 1)	(1. 1)	(1. 1)
(1. 2)	(1. 2)	(1. 2)	(1. 2)
(1. 3)	(1. 3)	(1. 3)	(1. 3)
(1. 4)	(2. 4)	(2. 4)	(2. 4)
(1. 5)	(2. 5)	(2. 5)	(2. 5)
(1. 6)	(2. 6)	(2. 6)	(2. 6)
(1. 7)	(2. 7)	(2. 7)	(3. 7)
(2. 8)	(3. 8)	(3. 8)	(3. 8)
(2. 9)	(3. 9)	(3. 9)	(3. 9)
(2. 10)	(3. 10)	(3. 10)	(4. 10)
(2. 11)	(3. 11)	(3. 11)	(4. 11)
(2. 12)	(4. 12)	(4. 12)	(4. 12)
(2. 13)	(4. 13)	(4. 13)	(5. 13)
(2. 14)	(4. 14)	(4. 14)	(5. 14)
(2. 15)	(4. 15)	(4. 15)	(5. 15)

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than $\left.\frac{n}{n_i}j\right)$, respectively, for each j_i, the last pair should be underlined. These underlined pairs are called *nodes.*

The prescription A_1 of Rule (A) states that in every k-th row of this table we have to select one and only one element (j_i, k) and put $f_i(j)=k$. A_2 prescribes the step that is to be taken when we can pick out one from two or more columes.

The matrix representation of this problem has the following properties.

1°. *Let the total number of nodes up to the k-th row be N_k, and it follows that*

(1) $\qquad N_k \leqq k \quad \text{for} \quad 1 \leqq k \leqq n-1,$

(2) $\qquad N_n = n.$

(proof)

Case (1): If we notice the element $(j_i, k+1)$ which lies in the $k+1$-th row of the column C_i, the number of nodes up to the k-th row of this column is j_i-1. Thus

$$\frac{n}{n_i}(j_i-1)<k+1,$$

$$N_k=\sum_i(j_i-1)<(k+1)\frac{\sum_i n_i}{n}=k+1,$$

therefore, $N_k \leqq k$.

For case (2), there is n_i nodes in each column C_i, therefore, in total,

$$N_n=\sum_i n_i=n.$$

1.2. Full count matrix

Now we proceed to define the *full count matrix.* Let the 1-type full count matrix be the matrix which has only one element and only one node.

For $k \geqq 2$, the k-type full count matrix is defined by

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1) $N_k = k$,

2) $N_\alpha < \alpha$ for $\alpha = 1, \cdots, k-1$,

3) It consists of k-rows and k-columns except for the nodeless columns.

2°. *Matrix having k-rows and k-nodes is decomposable into a finite number of* $1 \sim k$*-type full count matrices, and the total sum of its numbers is k.*

(proof)

If the matrix is full count, then its type number is k. If it is not full count,

$$N_\alpha \leqq \alpha \quad \text{for} \quad \alpha = 1, \cdots, k-1,$$

and there is β as in

$$N_\beta = \beta \text{ and } 1 \leqq \beta \leqq k-1, \text{ according to } 1°.$$

We arrange these β's in the order of their magnitude as follows:

$$\beta_1, \cdots, \beta_\lambda,$$

where λ is finite and holds

$$1 \leqq \beta_t - \beta_{t-1} \leqq k-1$$

for $t = 1, \cdots, \lambda+1$ and $\beta_0 = 0$, $\beta_{\lambda+1} = k$.

Therefore, except for the nodeless columns the matrix is decomposed into $\lambda+1$ full count matrices of $(\beta_t - \beta_{t-1})$-type, and the total sum of its type numbers is

$$\sum_{t=1}^{\lambda+1} (\beta_t - \beta_{t-1}) = k.$$

1.3. Construction of solutions

3°. *In a k-type full count matrix, we can determine a nuuber of linear order* $f_i(j)$, *satisfying the Rule (A), from 1 to k for all i and j.*

(proof)

We can prove this by complete induction with respect to k. The case $k=1$ is trivial.

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Now, assuming that the assertion holds for $1 \sim k$-type full count matrices, we will show that it also holds for the case of $k+1$-type. According to the definition of full count matrix, there is no node in the first row, so we can choose the element of the 1-st order by A_2. Then, we strike off the corresponding node and the 1-st row, and we obtain a matrix having k-rows and k-nodes. This matrix is decomposable into a finite set of matrices, the types of which are not greater than k, by 2°. According to the assumption of induction, we can determine the linear ordering in each matrix. Thus, connecting these linear orderings

Table 2.

$n_1=2$ C_1	$n_2=3$ C_2	$n_3=15$ C_3	(row)	$n=20$ (node)	
(1. 1)	(1. 1)	(1. 1)!	1	1	#
(1. 2)	(1. 2)	(2. 2)!	1	1	#
(1. 3)	(1. 3)	(3. 3)!!	1	0	
(1. 4)	(1. 4)!	(3. 4)	2	1	
(1. 5)	(1. 5)	(4. 5)!	3	1	
(1. 6)	(1. 6)	(5. 6)!	4	2	#
(1. 7)	(2. 7)	(6. 7)!!	1	0	
(1. 8)!	(2. 8)	(6. 8)	2	1	
(1. 9)	(2. 9)	(7. 9)!	3	1	
(1. 10)	(2. 10)	(8. 10)!	4	2	#
(2. 11)	(2. 11)!!	(9. 11)	1	0	
(2. 12)	(2. 12)	(9. 12)!	2	1	
(2. 13)	(2. 13)	(10. 13)!	3	2	#
(2. 14)	(3. 14)	(11. 14)!	1	1	#
(2. 15)!!	(3. 15)	(12. 15)	1	0	
(2. 16)	(3. 16)	(12. 16)!	2	1	
(2. 17)	(3. 17)	(13. 17)!	3	1	
(2. 18)	(3. 18)	(14. 18)!	4	1	
(2. 19)	(3. 19)!	(15. 19)	5	0	
(2. 20)	(3. 20)	(15. 20)!	6	3	#

(#—full count mark, (j,k)!—k is the C_{ij}'s order, and !!—selected by lot.)

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in a line, we obtain a liner ordering for the $k+1$-type full count matrix.

4°. *For the whole matrix of this problem, we can give a linear ordering by A_1 and A_2.*

(proof)

The whole matrix has n-nodes and n-rows, therefore, by 2°, it is decomposable into a finite set of $1\sim n$-type full count matrices, and the sum of their type numbers is n. In any one of these matrices, we can give an ordering whose last order is its type number, by 3°. Connecting these orderings in a line, we obtain a whole ordering.

1.4. Numerical example

Table 2 shows the procedure to give an ordering of ties kind for $n_1=2$, $n_2=3$, $n_3=15$, and $n=20$.

§2. The General Solution for Rule (B)

2.1. Notations

Only in case $M_i(k)>0$, we denote its decimal part by $d_i(k)$, i.e.

$$d_i(k)=M_i(k)-[M_i(k)].$$

Furtheremore, we put

$$R(k)=N(k)-\sum_{i(M_i(k)>0)} M_i(k),$$

where $\sum_{i(M_i(k)>0)}$ means the summation for such i's satisfying the condition $M_i(k)>0$.

2.2. Lemmas

From the condition (1) of B_1, we can easily see the following properties.

1°. $-1<r_i(k)<1$.

2°. $M_i(k)>-1$.

3°. $\sum_i r_i(k)=0$, *for any* k.

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(proof)

We can prove this by complete induction with respect to k.

In case $k=1$, holds

$$\sum_i r_i(1)=0$$

by definition of $r_i(k)$.

For case $k'\geqq 1$, let us assume $\sum r_i(k)=0$. Then we have

$$\begin{aligned}\sum_i r_i(k'+1)&=\sum_i (M_i(k')-N_i(k'))\\&=\sum_i M_i(k')-\sum_i N_i(k')\\&=\sum_i r_i(k')+\sum_i \frac{n_i(k')}{n(k')}N(k')-N(k')\\&=\sum_i r_i(k')\\&=0\end{aligned}$$

The following properties follow immediately from 3°.

4°. $\sum_i M_i(k)=N(k)$.

5°. $\sum_{i(M_i(k)>0)} M_i(k)\geqq N(k)$.

2.3. Construction of solution for Rule (B).

In this section, we discuss the case where $N_i(k)$ is determined by the condition B_1 of Rule (B) only, at first.

6°. *If* $M_i(k)\leqq 0$, *then* $N_i(k)=0$.

(proof)

By (2) of B_1 and 2°, we have

$$-2<M_i(k)-1<N_i(k)<M_i(k)+1\leqq 1,$$

therefore, we have $N_i(k)=0$.

7°. *If* $M_i(k)>0$ *and* $d_i(k)=0$, *then* $N_i(k)=M_i(k)$.

(proof)

There is non-negative integer satisfying the condition (2) of L_1 except $M_i(k)$.

Next, we treat the case where $N_i(k)$ is not determined uniquely by

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the condition B_1 of Rule (B).

8°. *If* $M_i(k)>0$ *and* $d_i(k)>0$, *then*

$$N_i(k)=[M_i(k)] \text{ or } [M_i(k)]+1.$$

(proof)

By assumption, we obtain

$$M_i(k)-1<[M_i(k)]<[M_i(k)]+1<M_i(k)+1.$$

Then, $N_i(k)$ satisfying the condition (2) of B_1 might be $[M_i(k)]$ or $[M_i(k)]+1$

9°. (1) $R(k)\geqq 0$,

(2) *The number of classes* C_i *for which* $d_i(k)>0$ *holds is not less than* $R(k)$.

(proof)

(1) By 7° and 8°,

$$[M_i(k)]=N_i(k) \quad \text{or} \quad N_i(k)-1.$$

Therefore,

$$\sum_{i(M_i(k)>0)}[M_i(k)]\leqq\sum_{i(M_i(k)>0)}N_i(k)=N(k)$$

i.e.

$$R(k)=N(k)-\sum_{i(M_i(k)>0)}[M_i(k)]\geqq 0.$$

(2)

$$\begin{aligned} R(k)&=N(k)-\sum_{i(M_i(k)>0)}[M_i(k)] \\ &\leqq\sum_{i(M_i(k)>0)}M_i(k)-\sum_{i(M_i(k)>0)}[M_i(k)] \quad \text{(by 5°)} \\ &=\sum_{i(d_i(k)>0)}d_i(k) \\ &\leqq \text{(the number of } C_i\text{'s for which } d_i(k)>0 \text{ holds).} \end{aligned}$$

The identity holds only in the case where all $d_i(k)$'s are equal to zero and consequently $R(k)$ is equal to zero, too.

10°. *From among the classes* C_i *for which* $M_i(k)>0$ *and* $d_i(k)>0$ *hold, we can select by lot* $R(k)$ *classes. For these classes holds* $N_i(k)=[M_i(k)]+1$, *and for the rest holds* $N_i(k)=[M_i(k)]$. *Any class* C_i *which takes part in this lot should have a chance to be selected in proportion to* $d_i(k)$.

(proof)

By definition of $R(k)$,

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$$N(k)=\sum_{i(M_i(k)>0)}[M_i(k)]+R(k)$$
$$=\sum_{i(d_i(k)=0)}[M_i(k)]+\sum_{i(d_i(k)>0,\ \text{for selected ones})}([M_i(k)]+1)$$
$$+\sum_{i(d_i(k)>0,\ \text{for others})}[M_i(k)].$$

The latter deformation is possible by 9°.

Existence of a complete solution can be shown by 6°, 7° and 10°.

2.4. Rule (B′)

The following Rule (B′) is equivalent to Rule (B).

Rule (B′)

(1) $\sum_i N_i(k)=N(k)$.

(2) If $M_i(k)\leqq 0$, then $N_i(k)=0$.

(3) If $M_i(k)>0$ and $d_i(k)=0$, then $N_i(k)=M_i(k)$.

(4) Among the classes C_i for which $M_i(k)>0$ and $d_i(k)>0$ hold, we select $R(k)$ classes by lot, for which $N_i(k)=[M_i(k)]+1$ holds, and $N_i(k)=[M_i(k)]$ holds for others. Any class C_i which takes part in this lot should have a chance to be selected in proportion to $d_i(k)$.

(proof)

If there is a solution of (B), it satisfies the condition (1)~(4) of (B′), and conversely a solution of (B′) satisfies (B_1—(1)), according to the last section 2.3.

Now we have only to prove that the condition (B_1—(2)) is satisfied for a solution of (B′). The condition B_2 immediately follows (B′—(4)).

As (B_1—(2)) is equivalent to

$$\text{(a)} \qquad -1<r_i(k)<1,$$

we can prove this by complete induction.

In case $k=1$, (a) is trivial.

In case $k\geqq 1$, we assume that (a) holds for k.

By definition

$$M_i(k)=r_i(k)+\frac{n_i(k)}{n(k)}N(k)>-1,$$

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$$r_i(k+1)=M_i(k)-N_i(k).$$

If $-1<M_i(k)\leqq 0$, we obtain $N_i(k)=0$ from (B′)—(2). Therefore

$$-1<r_i(k+1)=M_i(k)\leqq 0.$$

If $M_i(k)>0$,

$$N_i(k)=[M_i(k)] \quad \text{or} \quad [M_i(k)]+1,$$

must hold according to (B′—(3)) and (4), so

$$r_i(k+1)=d_i(k) \quad \text{or} \quad d_i(k)-1.$$

Table 3.

	C_1	C_2	C_3	C_4	$R(k)$
n_i/n	2/15	4/15	4/15	5/15	
$N_i(1)$	1	0	0	0	1
$r_i(2)$	−13/15	4/15	4/15	5/15	
$M_i(2)$	−11/15	8/15	8/15	10/15	
$N_i(2)$	×	1	0	0	1
$r_i(3)$	−11/15	−7/15	8/15	10/15	
$M_i(3)$	−9/15	−3/15	12/15	15/15	
$N_i(3)$	×	×	0	1	0
$r_i(4)$	−9/15	−3/15	12/15	0	
$M_i(4)$	−7/15	1/15	16/15	5/15	
$N_i(4)$	×	0	1	0	0
$r_i(5)$	−7/15	1/15	1/15	5/15	
$M_i(5)$	−5/15	5/15	5/15	10/15	
$N_i(5)$	×	0	0	1	1
$r_i(6)$	−5/15	5/15	5/15	−5/15	
$M_i(6)$	−3/15	9/15	9/15	0	
$N_i(6)$	×	1	0	×	1
$r_i(7)$	−3/15	−6/15	9/15	0	
$M_i(7)$	−1/15	−2/15	13/15	5/15	
$N_i(7)$	×	×	1	0	1
.	.	.	.	.	.
.	.	.	.	.	.

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Since $0 \leqq d_i(k) < 1$, we obtain

$$-1 < r_i(k+1) d_i(k+1) < 1 .$$

2.5. Numerical example

In practical application of this problem Rule (B′) is more convenient than Rule (B). Table 3 is a special case where $N(k)$ is always equal to one.

§ 3. Properties of Solutions

3.1. Rule (A) is best possible

1°. *For our conditions, the solution of Rule (A) is surely best possible.*

(proof)

Let N articles be distributed and N_i articles be alloted to every C_i, according to Rule (A). For our purpose, it would be sufficient to show

$$\left| N_i - \frac{n_i}{n} N \right| < 1 .$$

If $N_i = k$, this means that the distribution has finished up to C_{ik} but not $C_{i\, k+1}$. Therefor eweobtain

$$\frac{n}{n_i}(k-1) < f_i(k) \leqq N < f_i(k+1) \leqq \frac{n}{n_i}(k+1)$$

i.e.
$$-1 < N_i - \frac{n_i}{n} N < 1 .$$

3.2. Rule (B) is a desirable one

For Rule (B), we discuss only such case that $n_i(k)$ and $n(k)$ are equle to constants n_i and n respectively.

We first discuss the differences between two allotments; one is a total sum of allotments for several times, and the other is an allotment of a distribution which is performed simultaneously insted of several times. We use asterisks referring to this distribution, such as $N_i^*(k)$, $M_i^*(k)$, etc.

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2°. *Let,* $\cdots, N(k+\lambda-1)$ *be a sequence of numbers of articles for* λ *times distributions beginning with k-th distribution, and let us put*

$$N^*(k)=\sum_{j=0}^{\lambda-1}N(k+j).$$

Then we have

$$N_i^*(k)-1\leqq\sum_{j=0}^{\lambda-1}N_i(k+j)\leqq N_i^*(k)+1.$$

(proof)

$$\text{(b)}\quad M_i^*(k)=r_i(k)+\frac{n_i}{n}N^*(k)$$

$$=r_i(k)+\frac{n_i}{n}N(k)+\sum_{j=1}^{\lambda-1}\frac{n_i}{n}N(k+j)$$

$$=M_i(k)+\sum_{j=1}^{\lambda-1}(M_i(k+j)-r_i(k+j))$$

$$=M_i(k)+\sum_{j=1}^{\lambda-1}(M_i(k+j)-M_i(k+j-1)+N^i(k+j-1))$$

$$=M_i(k+\lambda-1)+\sum_{j=1}^{\lambda-1}N_i(k+j-1)$$

$$\text{(c)}\quad =r_i(k+\lambda)+\sum_{j=0}^{\lambda-1}N_i(k+j)$$

Since $-1<r_i(k+\lambda)<1$, we obtain

$$\text{(d)}\quad M_i^*(k)-1<\sum_{j=0}^{\lambda-1}N_i(k+j)<M_i^*(k)+1.$$

1) If $-1<M_i^*(k)\leqq 0$, then $N_i^*(k)=0$. Thus we have by (d)

$$N_i^*(k)-1<\sum_{j=0}^{\lambda-1}N_i(k+j)<N_i^*(k)+1.$$

2) If $M_i^*(k)>0$, then

$$\sum_{j=0}^{\lambda-1}N_i(k+j)=[M_i^*(k)]\quad\text{or}\quad[M_i^*(k)]+1$$

holds by (d). Therefore

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$$N_i^*(k)-1<\sum_{j=0}^{\lambda-1}N_i(k+j)\leqq N_i^*(k)+1$$

or
$$N_i^*(k)-1\leqq\sum_{j=0}^{\lambda-1}N_i(k+j)<N_i^*(k)+1.$$

From 1) and 2) above, we get

$$N_i^*(k)-1\leqq\sum_{j=0}^{\lambda-1}N_i(k+j)\leqq N_i^*(k)+1.$$

In the special cases that the total number of articles equal a multiple of n, we have the following.

3°. *If* $\sum_{j=0}^{\lambda-1}N(1+j)=m\cdot n$, *then* $N_i(1+j)=m\cdot n_i$.

(proof)

In the proof of 2° of this section, let us put $k=1$. Then we have

$$\begin{aligned} M_i^*(1)&=r_i(1)+\frac{n_i}{n}\sum_{j=0}^{\lambda-1}N(1+j) && \text{by (b)}\\ &=m\cdot n_i\\ &=r_i(\lambda+1)+\sum_{j=0}^{\lambda-1}N_i(1+j) && \text{by (c)}\end{aligned}$$

Since $\sum_{j=0}^{\lambda-1}N_i(1+j)$ and $m\cdot n_i$ are both positive integers, we have $r_i(\lambda+1)=0$.

By 2° and 3° we know that the difference between the two kinds of allotment is almost 1, and that the devition due to the dealing of fractions will be canceled if the number of articles is just a multiple of n.

Now, we have the following conclusion.

4°. *For Rule* (*B*), *the deviation of* N_i *from* $\frac{n_i}{n}N$ *is limited to a sufficiently small range of values*

$$\left|N_i-\frac{n_i}{n}N\right|<1,$$

that is best possible.

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(proof)

By (d) in the proof of 2°, we obtain

$$M_i^*(1)=\frac{n_i}{n}N$$

and

$$M_i^*(1)-1<N_i<M_i^*(1)+1.$$

Therefore,

$$-1<N_i-\frac{n_i}{n}N<1.$$

Remark

In the special case, where one article is given after the other each time, the solution by Rule (B) gives an ordering. Thus we can use the Rule (B) as a way of giving ordering. We have illustrated this in Table 3 for numerical example of Rule (B).

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