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SCHEDULE OF ROOMING ASSIGNMENT

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I. Introduction

The practical need of finding a broadcast producer or director the best path to walk from room to room in a central broadcast station has paused a problem of scheduling the rooming assignment to be solved for the minimum time and labour that he has to pay.

Handling this type of problem is further expected to provide information to a machine layout in machine shop.

The problem can be interpreted as:

"Assign the adequate number, 1 to n, to each of n locations, so as to minimize the total spent distance under the condition that the mutual distance among these locations are known and the frequency of using i—j route are all given."

This problem, then, may be understood as a problem of selecting a permutation that minimizes the characteristics — the total walking distance—among the possible $n!$ permutations.

The most acute point of this combinatorial problem is to choose an efficient rule of finding the optimal solution starting from an arbitral initial assignment.

II. Process to Solution

(1) Adjustment of Information and Formulation of the Problem.

The total frequency in passing $i \to j$ route is represented by p_{ij} and P_0 is defined as the matrix having p_{ij} as its (i,j) element. Matrix P is then prepared defining $P = P_0 + P'_0$.

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Next the distance between i and j locations, with any one of n locations being assigned, is represented by s_{ij} and called matrix S having s_{ij} as its (i,j) element. Naturally $S=S'$.

Then the meaning of the product $T=PS$ is examined.

First, take the diagonal element of T,

$$t_{ii}=\sum_{k=1}^{n} p_{ik}\, s_{ki}$$

where p_{ik} represents the frequency in i-k route being taken since the frequency constitutes of summation $i\rightarrow k$ and $k\rightarrow i$ route. Thus distance between i and k locations, $s_{ik}=s_{ki}$, is multiplied by this frequency and summed up with k varying all possible locations, whereby yielding the expected distance—*i.e.*, likely the weighted mean distance—when a route passes through the particular location i.

Secondly, $t_{ij}=\sum p_{ik}\, s_{kj}$ is considered and understood to be an interchanging of i and j locations, in comparison with t_{ii} and where substituting s_{kj} in place of s_{ki} is performed. That is, this is the expected path length passing through a new i location in case i and j numbers are interchanged. t_{ji} is thus interpreted to be the expected path length passing through a new j location. Hence, the effect of (i,j) interchange may first be investigated by comparing $t_{ii}+t_{jj}$ and $t_{ij}+t_{ji}$ as illustrated in Fig 1, the actual effect, however, should include

$$U=P\Pi S \qquad (u_{ij}=p_{ij}\times s_{ij})$$

in order to prevent from missing i-j route. Thus the effect of such a (i,j) interchange is investigated by comparing

$$f_{ii}+f_{jj} \text{ and } f_{ij}+f_{ji}$$

provided with $F=T+U$,

where $T=PS$ and $U=P\Pi S$.

Note that

$$\max_{(i,j)}\{f_{ii}+f_{jj}-(f_{ij}+f_{ji})\}\leq 0$$

indicates the case any interchange will not be effect, and that

$$\max_{(i,j)}\{f_{ii}+f_{jj}-(f_{ij}+f_{ji})\}=\{f_{ii}+f_{jj}-(f_{ij}+f_{ji})|(i,j)=(i_0,j_0)\}>0$$

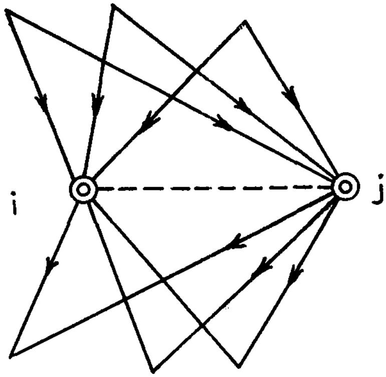

Fig. 1

indidcates the case the best effective interchange will be (i_0, j_0). Hence similar process may be recurred, after applying the (i_0, j_0) interchange, for obtaining the solution, whereby the best means of modification is requested to be found in order to minimize the calculation labour.

(2) Modification based upon Interchange.

After applying the (i_0, j_0) interchange, the new matrix $T=PS$ has the new matrix S having its row and column (i_0, j_0) interchanged respectively in the previous matrix S_0. Tne new matrix $T=PS$ has to be identified from previous $T_0=PS_0$ by relationship

$$T=PS=PCS_0C$$

where $c_{ij}=\delta_{ij}$

(except $c_{i_0i_0}=0,\quad c_{j_0j_0}=0,\quad c_{i_0j_0}=1,\quad c_{j_0i_0}=1$)

may be recognized.

That is, the new matrix T is the multitude of P and S_0 with the form respective i_0 column and j_0 column being interchanged.

Therefore, the calculation

$$PCS_0C-PS_0=(\alpha)$$

is done as follows.

$$\alpha_{ij}=\sum_{k(\neq i_0,\, j_0)} p_{ik}\, s_{kj}+p_{ij_0}\, s_{i_0j}+p_{ii_0}\, s_{j_0j}$$

$$-(\sum_{k(\neq i_0,\, j_0)} p_{ik}\, s_{kj} + p_{ii_0}\, s_{i_0 j} + p_{ij_0}\, s_{j_0 j}) \quad (j \neq i_0,\, j_0)$$

$$= -\{p_{ii_0}(s_{i_0 j} - s_{j_0 j}) - p_{ij_0}(s_{i_0 j} - s_{j_0 j})\}$$

$$= -(p_{ii_0} - p_{ij_0})(s_{i_0 j} - s_{j_0 j})$$

$$\alpha_{ii_0} = \sum_{k(\neq i_0,\, j_0)} p_{ik}\, s_{kj_0} + p_{ij_0}\, s_{i_0 j_0} + p_{ii_0}\, s_{j_0 j_0}$$

$$-(\sum_{k(\neq i_0,\, j_0)} p_{ik}\, s_{ki_0} + p_{ii_0}\, s_{i_0 i_0} + p_{ij_0}\, s_{j_0 i_0})$$

$$= -\sum_k p_{ik}\, s_{ki_0} + \sum_k p_{ik}\, s_{kj_0}$$

$$+ p_{ii_0}\, s_{i_0 i_0} + p_{ij_0}\, s_{j_0 i_0} - p_{ii_0}\, s_{i_0 j_0} - p_{ij_0}\, s_{j_0 j_0}$$

$$+ p_{ij_0}\, s_{i_0 j_0} + p_{ii_0}\, s_{j_0 j} - p_{ii_0}\, s_{i_0 i_0} - p_{ij_0}\, s_{j_0 i_0}$$

$$= -\sum_k p_{ik}\, s_{ki_0} + \sum p_{ik}\, s_{kj_0} - (p_{ii_0} - p_{ij_0})(s_{i_0 j_0} - s_{j_0 j_0}) \;.$$

And similarly, from

$$\alpha_{ij_0} = -\sum_k p_{ik}\, s_{kj_0} + \sum_k p_{ik}\, s_{ki_0} - (p_{ii_0} - p_{ij_0})(s_{i_0 i_0} - s_{j_0 i_0})$$

PCS_0C may be calculated from decreasing the initial matrix PS_0 by

$$(p_{ii_0} - p_{ij_0}) \otimes (s_{ii_0} - s_{ij_0})$$

then interchanging the column $(i_0,\, j_0)$.

Further, the modification of $U = P\Pi S$ may be done by calculating

$$U - U_0 = (\beta)$$

$$\beta_{ij} = 0 \quad (i \neq i_0,\, j_0\,;\; j \neq i_0,\, j_0\,;\; \begin{matrix} i = i_0,\, j_0 \\ j = i_0,\, j_0 \end{matrix})$$

$$\beta_{i_0 j} = p_{i_0 j}\, s_{j_0 j} - p_{i_0 j}\, s_{i_0 j} = -p_{i_0 j}\,(s_{i_0 j} - s_{j_0 j})$$

$$\beta_{j_0 j} = -p_{j_0 j}\,(s_{j_0 j} - s_{i_0 j})$$

$$\beta_{ii_0} = p_{ii_0}\, s_{ij_0} - p_{ii_0}\, s_{ii_0} = -p_{ii_0}\,(s_{ii_0} - s_{ij_0})$$

$$\beta_{ij_0} = p_{ij_0}\, s_{ii_0} - p_{ij_0}\, s_{ij_0} = -p_{ij_0}\,(s_{ij_0} - s_{ii_0})$$

Now the process is straight forward by repeating the similar calculation on $PCS_0C + U$ as before step.

III. Numerical Example

Five locations are considered as shown in Fig. 2 and required to assign 1 to 5 numbers to each location, where each distance between

1—2, 1—3, 1—5, 2—3, 3—4, 3—5 and 4—5 is given as 1 and 1—4 and 2—5 is given as $\sqrt{3}$.

Also given are the frequency in use of a route as

$1\to3\to4$ to be 1/2

$1\to4\to2$ to be 1/3

and $2\to5$ to be 1/6

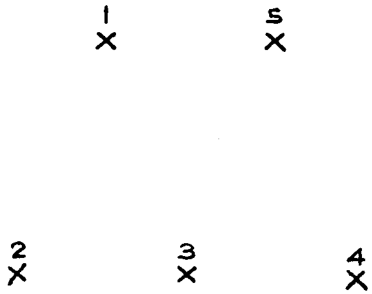

Fig. 2

Calculations start with

$$P_0=\begin{pmatrix} 0 & 0 & 1/2 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 1/6 \\ 0 & 0 & 0 & 1/2 & 0 \\ 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P=P_0+P_0'=\begin{pmatrix} 0 & 0 & 1/2 & 1/3 & 0 \\ 0 & 0 & 0 & 1/3 & 1/6 \\ 1/2 & 0 & 0 & 1/2 & 0 \\ 1/3 & 1/3 & 1/2 & 0 & 0 \\ 0 & 1/6 & 0 & 0 & 0 \end{pmatrix}$$

$$S=\begin{pmatrix} 0 & 1 & 1 & \sqrt{3} & 1 \\ 1 & 0 & 1 & 2 & \sqrt{3} \\ 1 & 1 & 0 & 1 & 1 \\ \sqrt{3} & 2 & 1 & 0 & 1 \\ 1 & \sqrt{3} & 1 & 1 & 0 \end{pmatrix}$$

$$T=PS=\begin{pmatrix} 0 & 0 & 1/2 & 1/3 & 0 \\ 0 & 0 & 0 & 1/3 & 1/6 \\ 1/2 & 0 & 0 & 1/2 & 0 \\ 1/3 & 1/3 & 1/2 & 0 & 0 \\ 0 & 1/6 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & \sqrt{3} & 1 \\ 1 & 0 & 1 & 2 & \sqrt{3} \\ 1 & 1 & 0 & 1 & 1 \\ \sqrt{3} & 2 & 1 & 0 & 1 \\ 1 & \sqrt{3} & 1 & 1 & 0 \end{pmatrix}$$

$$=\begin{pmatrix} 1.08 & 1.17 & 0.33 & 0.5 & 0.83 \\ 0.75 & 0.96 & 0.5 & 0.17 & 0.33 \\ 0.87 & 1.5 & 1 & 0.85 & 1 \\ 0.83 & 0.83 & 0.67 & 1.75 & 1.41 \\ 0.17 & 0 & 0.17 & 0.33 & 0.29 \end{pmatrix}$$

$$U=P\Pi S=\begin{pmatrix} 0 & 0 & 0.5 & 0.58 & 0 \\ 0 & 0 & 0 & 0.67 & 0.29 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0.58 & 0.67 & 0.5 & 0 & 0 \\ 0 & 0.29 & 0 & 0 & 0 \end{pmatrix}$$

$$F=T+U=\begin{pmatrix} 1.08 & 1.17 & 0.83 & 1.08 & 0.83 \\ 0.75 & 0.96 & 0.5 & 0.84 & 0.62 \\ 1.37 & 1.5 & 1 & 1.37 & 1 \\ 1.41 & 1.5 & 1.17 & 1.75 & 1.41 \\ 0.17 & 0.29 & 0.17 & 0.33 & 0.29 \end{pmatrix}$$

then $f_{ii}+f_{jj}$ is compared with $(f_{ij}+f_{ji})$ as

$$\begin{array}{cccc} \begin{matrix}2.04\\(1.92)\end{matrix} & \begin{matrix}2.08\\(2.2)\end{matrix} & \begin{matrix}2.83\\(2.49)\end{matrix} & \begin{matrix}1.37\\(1)\end{matrix} \\ & \begin{matrix}1.96\\(2)\end{matrix} & \begin{matrix}2.71\\(2.34)\end{matrix} & \begin{matrix}1.25\\(0.91)\end{matrix} \\ & & \begin{matrix}2.75\\(2.54)\end{matrix} & \begin{matrix}1.29\\(1.17)\end{matrix} \\ & & & \begin{matrix}2.04\\(1.74)\end{matrix} \end{array}$$

by forming

$$f_{ii}+f_{jj}-(f_{ij}+f_{ji})$$

$$\begin{array}{cccc} 0.08 & -0.12 & 0.34 & \boxed{0.37} \\ & -0.04 & \boxed{0.37} & 0.34 \\ & & 0.21 & 0.12 \\ & & & 0.3 \end{array}$$

from such considerations, (1,5) interchange is first approached

$(p_{i1}-p_{i5})\otimes(s_{i1}-s_{i5})=$

	-1	$1-\sqrt{3}$	0	$\sqrt{3}-1$	1
0	0	0	0	0	0
−1/6	0.17	0.12	0	−0.21	−0.17
1/2	−0.5	−0.36	0	0.36	0.5
1/3	−0.33	−0.24	0	0.24	0.33
0	0	0	0	0	0

Hence PCS_0C, before the column (1,5) is interchanged, becomes

$T-(p_{i1}-p_{i5})\otimes(s_{i1}-s_{i5})$

$$=\begin{pmatrix} 1.08 & 1.17 & 0.33 & 0.5 & 0.83 \\ 0.75 & 0.96 & 0.5 & 0.17 & 0.33 \\ 0.87 & 1.5 & 1 & 0.87 & 1 \\ 0.83 & 0.83 & 0.67 & 1.75 & 1.41 \\ 0.17 & 0 & 0.17 & 0.33 & 0.29 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.17 & 0.12 & 0 & -0.12 & -0.17 \\ -0.5 & -0.36 & 0 & 0.36 & 0.5 \\ -0.33 & -0.24 & 0 & 0.24 & 0.33 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$=\begin{pmatrix} 1.08 & 1.17 & 0.33 & 0.5 & 0.83 \\ 0.58 & 0.84 & 0.5 & 0.29 & 0.5 \\ 1.37 & 1.86 & 1 & 0.5 & 0.5 \\ 1.16 & 1.07 & 0.67 & 1.5 & 1.08 \\ 0.17 & 0 & 0.17 & 0.33 & 0.29 \end{pmatrix}$$

whereby applying the interchange (1, 5)

$$PCS_0C=\begin{pmatrix} 0.83 & 1.17 & 0.33 & 0.5 & 1.08 \\ 0.5 & 0.84 & 0.5 & 0.29 & 0.58 \\ 0.5 & 1.86 & 1 & 0.5 & 1.37 \\ 1.08 & 1.07 & 0.67 & 1.5 & 1.16 \\ 0.29 & 0 & 0.17 & 0.33 & 0.17 \end{pmatrix}$$

on the other hand, U is calculated as

$$\begin{array}{rll|ccccc|l} & \beta_{i1} & & & 0 & 0 & 0.24 & & \beta_{i5} \\ i=2 & 0\times(1-\sqrt{3}) & = & 0 & & & & 0.12 & =1/6\times(\sqrt{3}-1) \\ 3 & 1/2\times(1-1) & = & 0 & & & & 0 & =\ 0\times(1-1) \\ 4 & 1/3\times(\sqrt{3}-1) & = & 0.24 & & & & 0 & =\ 0\times(1-\sqrt{3}) \\ & & & & 0.12 & 0 & 0 & & \end{array}$$

$$
\therefore \quad U=\begin{pmatrix} 0 & 0 & 0.5 & 0.58 & 0 \\ 0 & 0 & 0 & 0.67 & 0.29 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0.58 & 0.67 & 0.5 & 0 & 0 \\ 0 & 0.29 & 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} & 0 & 0 & 0.24 & \\ 0 & & & & 0.12 \\ 0 & & & & 0 \\ 0.24 & & & & 0 \\ & 0.12 & 0 & 0 & \end{pmatrix}
$$

(The blank space in the second term represents zero.)

$$
=\begin{pmatrix} 0 & 0 & 0.5 & 0.33 & 0 \\ 0 & 0 & 0 & 0.67 & 0.17 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0.33 & 0.67 & 0.5 & 0 & 0 \\ 0 & 0.17 & 0 & 0 & 0 \end{pmatrix}
$$

hence

$$
PCS_0C+U=\begin{pmatrix} 0.83 & 1.17 & 0.33 & 0.6 & 1.08 \\ 0.5 & 0.84 & 0.5 & 0.29 & 0.58 \\ 0.5 & 1.86 & 1 & 0.5 & 1.37 \\ 1.08 & 1.07 & 0.67 & 1.5 & 1.16 \\ 0.29 & 0 & 0.17 & 0.33 & 0.17 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0.5 & 0.33 & 0 \\ 0 & 0 & 0 & 0.67 & 0.17 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0.33 & 0.67 & 0.5 & 0 & 0 \\ 0 & 0.17 & 0 & 0 & 0 \end{pmatrix}
$$

$$
=\begin{pmatrix} 0.83 & 1.17 & 0.83 & 0.83 & 1.08 \\ 0.5 & 0.84 & 0.5 & 0.96 & 0.75 \\ 1 & 1.86 & 1 & 1 & 1.37 \\ 1.41 & 1.74 & 1.17 & 1.5 & 1.16 \\ 0.29 & 0.17 & 0.17 & 0.33 & 0.17 \end{pmatrix}
$$

comparison $(f_{ii}+f_{jj})$ with $(f_{ij}+f_{ji})$ after (1, 5) interchange is shown as

$$
\begin{array}{cccc}
1.67 & 1.83 & 2.33 & 1 \\
(1.67) & (1.83) & (2.24) & (1.37) \\
 & 1.84 & 2.34 & 1 \\
 & (2.36) & (2.7) & (0.92) \\
 & & 2.5 & 1.17 \\
 & & (2.17) & (1.5) \\
 & & & 1.67 \\
 & & & (1.5)
\end{array}
$$

$f_{ii}+f_{jj}-(f_{ij}+f_{ji})$

0	0	0.09	−0.37
	−0.52	−0.36	0.08
		0.33	−0.33
			0.17

At this state the new PS is identical with the previous PCS_0C, so that

$$PS=\begin{pmatrix} 0.83 & 1.17 & 0.33 & 0.5 & 1.08 \\ 0.5 & 0.84 & 0.5 & 0.29 & 0.58 \\ 0.5 & 1.86 & 1 & 0.5 & 1.37 \\ 1.08 & 1.07 & 0.67 & 1.5 & 1.16 \\ 0.29 & 0 & 0.17 & 0.33 & 0.17 \end{pmatrix}$$

then the next matrix should be subtructed from the above one (at the step (col. 1, col. 5), (row 1, row 5) for matrix S is already inversed.)

$(p_{i3}-p_{i4})\otimes(s_{i3}-s_{i4})$ equals

	0	−1	−1	1	$1-\sqrt{3}$
1/6	0	−0.17	−0.17	0.07	−0.12
−1/3	0	0.33	0.33	−0.33	0.24
−1/2	0	0.5	0.5	−0.5	0.36
1/2	0	−0.5	−0.5	0.5	−0.36
0	0	0	0	0	0

the results is as

$$\begin{pmatrix} 0.83 & 1.34 & 0.5 & 0.33 & 1.2 \\ 0.5 & 0.5 & 0.17 & 0.62 & 0.34 \\ 0.5 & 1.36 & 0.5 & 1 & 1 \\ 1.08 & 1.57 & 1.17 & 1 & 1.52 \\ 0.29 & 0 & 0.17 & 0.33 & 0.17 \end{pmatrix}$$

interchanging (col. 3, col. 4)

$$\begin{pmatrix} 0.83 & 1.34 & 0.33 & 0.5 & 1.2 \\ 0.5 & 0.5 & 0.62 & 0.17 & 0.34 \\ 0.5 & 1.36 & 1 & 0.5 & 1 \\ 1.08 & 1.57 & 1 & 1.17 & 1.52 \\ 0.29 & 0 & 0.33 & 0.17 & 0.17 \end{pmatrix} \tag{1}$$

other hand, for U

$$U=\begin{pmatrix} 0 & 0 & 0.5 & 0.33 & 0 \\ 0 & 0 & 0 & 0.67 & 0.17 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0.33 & 0.67 & 0.5 & 0 & 0 \\ 0 & 0.17 & 0 & 0 & 0 \end{pmatrix}$$

the next matrix should be subtracted (for matrix S, row (1, 5) and col. (1, 5) have already been interchanged.)

$$\begin{aligned} &\beta_{13}=1/2\times(0-0)=0 & &\beta_{14}=1/3\times(1-1)=0 \\ &\beta_{23}=0\times(1-2)=0 & ; \quad &\beta_{24}=1/3\times(2-1)=0.33 \\ &\beta_{53}=0\times(1-\sqrt{3})=0 & &\beta_{54}=0\times(\sqrt{3}-1)=0 \end{aligned}$$

so that

$$(\beta)=\begin{pmatrix} & & 0 & 0 & \\ & & 0 & 0.33 & \\ 0 & 0 & & & 0 \\ 0 & 0.33 & & & 0 \\ & & 0 & 0 & \end{pmatrix}$$

(blank means zero)

accordingly the new U matrix is

$$\begin{pmatrix} 0 & 0 & 0.5 & 0.33 & 0 \\ 0 & 0 & 0 & 0.67 & 0.17 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0.33 & 0.67 & 0.5 & 0 & 0 \\ 0 & 0.17 & 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.33 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.33 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$=\begin{pmatrix} 0 & 0 & 0.5 & 0.33 & 0 \\ 0 & 0 & 0 & 0.33 & 0.17 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0.33 & 0.33 & 0.5 & 0 & 0 \\ 0 & 0.17 & 0 & 0 & 0 \end{pmatrix} \qquad (2)$$

therefore $T+U$, eq. (1)+eq. (2), is

$$\begin{pmatrix} 0.83 & 1.34 & 0.83 & 0.83 & 1.2 \\ 0.5 & 0.5 & 0.62 & 0.5 & 0.5 \\ 1 & 1.36 & 1 & 1 & 1 \\ 1.41 & 1.9 & 1.5 & 1.17 & 1.52 \\ 0.29 & 0.17 & 0.33 & 0.17 & 0.17 \end{pmatrix}$$

so that the comparison $(f_{ii}+f_{jj})$ with $(f_{ij}+f_{ji})$, after (3, 4) interchanging, becomes

1.33 (1.84)	1.83 (1.83)	2 (2.24)	1 (1.49)
	1.5 (2)	1.65 (2.4)	0.67 (0.67)
		2.17 (2.5)	1.17 (1.33)
			1.34 (1.69)

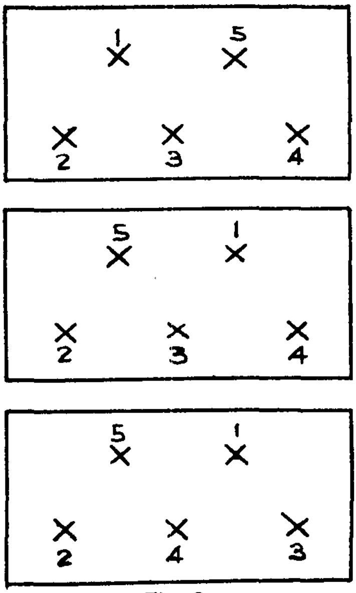

Fig. 3

$(f_{ii}+f_{jj})-(f_{ij}+f_{ji})$

-0.5	0	-0.24	-0.49
	-0.5	-0.77	0
		-0.33	-0.16
			-0.35

Therefore we conclude that there exists no other effective interchange than the previously examined except for the equivalent ordering that can be attained by the interchanges (1,3) and (2,5).

Such conclusions are illustrated by Fig. 3 and the identity of these interchanges are easily verified from considering the given frequencies

$1\to3\to4$ to be $1/2$
$1\to4\to2$ to be $1/3$
and $2\to5$ to be $1/6$.

IV. Discussions

The particularity of this algorism could be arranged in the following step

i. starting from arbitrarily ordering and forming PS_0, U
ii. we are able to select the most efficient interchange by comparing $(f_{ii}+f_{jj})$ with $(f_{ij}+f_{ji})$
iii. on the case of interchanging, we are need to modify the matrix PS and U in accordance with new ordering.
iv. However, modified $PS(=PCS_0C)$ and U are not calculated directly in forming the required matrix F but are calculated as old PS_0 plus correction term, and same to U, which established in cheap labour.

For last step we are need to state in more detail. If direct calculation of PCS_0C is performed,

n^3 multiplications

and n^3 additions should be required, whereas in this process merely $1/n$ of procedures are required as:

n^2+4n multiplications

and $2n^2+4n$ additions.

Further, the characteristics assumed in this problem are distances between each location, but these can be replaced with time required to transforming or any other, the best suitable characteristics in actual case.

V. Application to Digital Computers and Experience

This process of calculation was applied to IBM-7090 machine. The number of FORTRAN statement is approximately 100 resulting compile of 2/100 hr. and the illustrated example of 5×5 was executed by 1/100 hr.

The core size used by 50×50 is 23,435 cores, so capability may raise more. For an actual case of central broadcast station was 45×45 and executed by 2/100 hr.

VI. Application to a Travelling Salesman Problem

A travelling salesman problem having n locations is solved by selecting the minimum total distance route connecting all n locations. Assuming

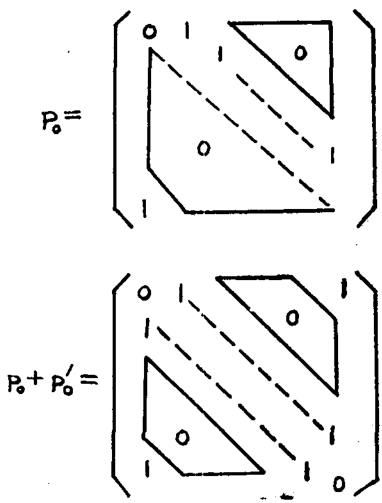

Fig. 4

probability 1 is accepted for n locations with route of

$$1 \to 2 \to \cdots\cdots \to n \to 1$$

the problem may be solved in application of this process. (Fig. 4)

(NUMERICAL EXAMPLE)

A travelling salesman problem having seven locations in a form of regular hexangular vertices and its center as shown in Fig. 5. At first, suppose an assignment of the Fig. 5 is selected, then

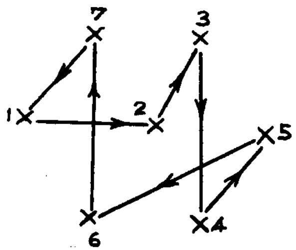

Fig. 5

$$S=\begin{pmatrix} 0 & 1 & \sqrt{3} & \sqrt{3} & 2 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ \sqrt{3} & 1 & 0 & \sqrt{3} & 1 & 2 & 1 \\ \sqrt{3} & 1 & \sqrt{3} & 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1 & 0 & \sqrt{3} & \sqrt{3} \\ 1 & 1 & 2 & 1 & \sqrt{3} & 0 & \sqrt{3} \\ 1 & 1 & 1 & 2 & \sqrt{3} & \sqrt{3} & 0 \end{pmatrix}$$

$$P=\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$T=PS, \quad U=P\Pi S, \quad F=T+U$

$$F=\begin{pmatrix} 2 & 2 & 2 & 3 & 2.73 & 2.73 & 2 \\ 2.73 & 2 & 2.73 & 3.46 & 3 & 3 & 2 \\ 2.73 & 2 & 2.73 & 2.73 & 2 & 2 & 3 \\ 2.73 & 2 & 2.73 & 2.73 & 2 & 3.73 & 2.73 \\ 3.73 & 2 & 3.73 & 2 & 2.73 & 2.73 & 3.73 \\ 3 & 2 & 2 & 3 & 3.46 & 3.46 & 3.46 \\ 2 & 2 & 3.73 & 2.73 & 3.73 & 2.73 & 2.73 \end{pmatrix}$$

$f_{ii}+f_{jj}-(f_{ij}+f_{ji})$

−0.73	0	−2	−0.73	−0.27	0.73
	0	−0.73	−0.27	0.46	0.73
		0	−0.27	**2.19**	−1.27
			1.46	−0.56	0
				0	−2
					0

then we take (3,6) exchange and calculating

$(p_{i3}-p_{i6})\otimes(s_{i3}-s_{i6})$

$p_3(s_{i3}-s_{i6})$

$p_6(s_{i6}-s_{i3})$

respectively, we can get modified PS and U resulting in

$$F=PS+U=\begin{pmatrix} 2 & 2 & 2.73 & 3 & 2.73 & 2 & 2 \\ 2 & 2 & 2 & 2.73 & 3.73 & 3.73 & 2.73 \\ 2.73 & 2 & 2 & 2 & 2 & 2.73 & 3 \\ 3 & 2 & 2.73 & 2 & 2.73 & 3 & 3.46 \\ 3.46 & 2 & 3 & 2.73 & 2 & 2.73 & 3 \\ 3 & 2 & 3.46 & 3 & 2.73 & 2 & 2.73 \\ 2.73 & 2 & 3 & 3.46 & 3 & 2.73 & 2 \end{pmatrix}$$

$(f_{ii}+f_{jj})-(f_{ij}+f_{ji})$

0	−1.46	−2	−2.19	−1	−0.73
	0	−0.73	−1.73	−1.73	−0.73
		−0.73	−1	−2.2	−2
			−1.46	−2	−2.92
				−1.46	−2
					−1.46

therefore we conclude that there are no other effective interchange than (1,2) and (2,3) which yields equivalent results to (3,6).

The final result is shown in Fig. 5 from which it is understood that the route of the minimum total distance is aquired by the single interchange of (3,6).

The equivalent routes by means of (1,2) and (2,3) interchanges are easily checked by seeing Fig. 6.

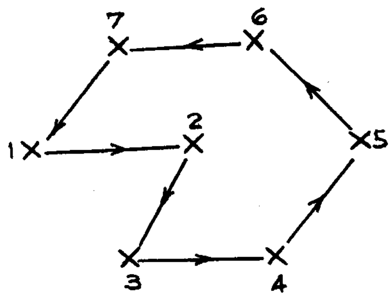

Fig. 6

VII. Conclusion

As the algorithm is based upon the connection of interchange the final result has the possibility dropping in the local optimum relating to the initial permutation.

These difficulties are inherent to the combinatorial problem and need more study I think.

Reference

Gordon C. Armour and Elwood S. Buffa, " A heuristic algorithm and simulation approach to relative location of facilities ", Management Science, Vol. 9, No. 2.