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APPLICATION OF OPERATIONS RESEARCH TO ROAD TRAFFIC

(Estimation of Utilization Rates of Roads

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The following is an attempt to estimate by the application of the information theory the rate of vehicular traffic on each of several roads that have both the same starting and terminal points.

1. Road Traffic and Information Theory

For convenience, this study is made on two roads, a and b, that run between A and B. The number of cars that travel between A and B is denoted by n, the number of cars that travel on Road a by n_a, the number of cars that travel on Road b by n_b, and the probabilities for a and b by P_a and P_b, respectively. When n $(=n_a+n_b)$ is sufficiently large, the corollaries are $P_a=n_a/n$ anc $P_b=n_b/n$. In this case, P_a and P_b indicate the utilization rates of the two roads.

Supposing that one of the two roads—Road b, for example—is a toll road, the cars (numbering n) sometimes choose it because this road, even they have to pay it costs them the toll, enables them to reach the destination in a shorter time, while sometimes they choose Road a because they need not pay the toll even it takes a longer time to reach the destination. Thus their choice between the two roads is not definite, and this accounts for the complicated phenomena that ensue.

When the "not definite" degree is large it means the entropy (H) (in this case $H=-P_a \log P_a - P_b \log P_b$) is large. In other words, the behaviors of the cars, when considered for the whole, are extremely varied.

Supposing that the cars traveling between A and B have no information whatsoever concerning Road a and b, there is no other way than to presume that the rate of vehicular traffic on each road is 50 : 50, which is supposed to be the actual case.

If, on the other hand, the information is available that Road a needs t_a minutes to go and Road b t_b minutes, such information naturally has something to bear upon the rate of traffic on each road, resulting in the change of the rate. And, on this basis, can the P_a and P_b values be estimated.

Since the average entropy is $H=-P_a \log P_a - P_b \log P_b$, it becomes necessary to compute P_a and P_b at such values as will bring to the maximum the amount of information, viz., $H/\bar{t}$ per average time required for all cars, with $\bar{t}=n_a/n\times t_a+n_b\times n_b/n\times t_b.=P_at_a+P_bt_b$

That is to say, such P_a and P_b values as will bring $(-P_a \log P_a - P_b \log P_b)/\bar{t}$ to the maximum on condition of $P_a+P_b=1$ are the rates of traffic on both roads that are estimated on the basis of the mere information that Road a needs t_a minutes to go and Road b t_b minutes.

The equation is:

$$\partial/\partial P_i\{H/\bar{t}+\lambda(P_a+P_b)\}=0$$
$$P_i=P_a,\ P_b$$
$$P_a+P_b=1$$

λ is a Lagrange multiplier, and, when 2 is the base, $P_i=2^{-t_i\times H/\bar{t}}$.

In the case of $2^{-H/\bar{t}}=W^{-1}$, $P_a+P_b=1$ is instrumental to $W^{-t_a}+W^{-t_b}=1$.

The positive root that satisfies the equation given above is found in in $H/\bar{t}=\log W$. This leads to $P_i=2^{-t_i \log W}$. The P_a and P_b values thus obtained acount for rates of vehicular traffic on Road a and Road b.

Calculations and Observations

Given in the following are the calculations and observations conducted on the basis of the theory shown in (1) above for the roads kept in operation by the Japan Highway public Corporation. Although, in the case cited in (1) above, the traveling time is the sole information available for the calculation, the four elements, viz., traveling time, traveling expense, driving comfort, and protection of freight, are used as information available to the drivers in the case of Matsue Highway, Sangu Highway, Kamigo Bridge, and Keiyo Highway, of which the result of surveys conducted concerning the reason and rate of travel appears in Table 1.

The parameters are computed (see Table 2) for time ratio in the case of traveling time, for traveling expense ratio in the case of traveling expense (which includes the toll in the case of the toll road), for the values of 10 for sand road, of 1 for toll road, and of 2 for free, paved road in the case of driving comfort (since no appropriate data available), and for inverse ratio of the rate of pavement in the case of the protection of freight, in order to arrive at the P values for each toll road according to reasons of travel.

The P values thus computed multiplied by the reason-of-travel ratios appearing in Table 1 are the overall estimation of utilization rates that are found in Table 3.

The observations are based on the results of O.D. surveys at the road side. Evident in Table 3, it shows, generally speaking, the calculated

Table 1 Reason-of-Travel Ratios

	Saving of Time	Saving of Expense	Driving Comfort	Protection of Freight	Total
Matsue Highway	0. 176 0. 134 0. 125	0. 216 0. 479 0. 299	0. 608 0. 218 0. 353	0. 0. 169 0. 223	1. 000 // //
Sangu Highway	0. 311 0. 222 0. 213	0. 225 0. 256 0. 237	0. 464 0. 246 0. 285	0. 0. 276 0. 265	1. 000 // //
Kamigo Bridge	0. 269 0. 260 0. 255	0. 233 0. 418 0. 299	0. 498 0. 185 0. 285	0. 0. 137 0. 161	1. 000 // //
Keiyo Highway	0. 455 0. 293 0. 397	0. 120 0. 209 0. 168	0. 425 0. 272 0. 290	0. 0. 226 0. 145	1. 000 // //

Note: In each group of figures, the top one is for small passenger cars, the middle for ordinary trucks, and the bottom for small trucks.

Table 2 Values of Information Elements

	Matsue Highway						Sangu Highway						Kamigo Bridge						Keiyo Highway					
	Small Trucks		Ordinary Trucks		Small Passenger Cars		Small Trucks		Ordinary Trucks		Small Passenger Cars		Small Trucks		Ordinary Trucks		Small Passenger Cars		Small Trucks		Ordinary Tracks		Small Passenger Cars	
Traveling time (min)	9	11	8	14	7	14	12	29	12	24	12	24	20	29	20	29	17	26	15	35	17	35	15	29
Ditto, ratio	5	6	4	7	1	2	1	2	1	2	1	2	2	3	2	3	2	3	1	2	1	2	1	2
Traveling expense (yen)	105	112	211	173	111	127	226	192	398	374	232	204	217	258	419	512	223	281	310	192	398	368	216	200
Ditto, ratio	9	10	5	4	9	10	10	9	10	9	10	9	4	5	4	5	4	5	10	9	10	9	1	1
Rate of pavemens (%)	100	38	100	38	100	38	100	61	100	61	100	61	100	0	100	0	100	0	100	100	100	100	100	100
Ditto, inverse ratio	3	8	3	8	3	8	3	5	3	·5	3	5	1	10	1	10	1	10	1	1	1	1	1	1
Driving comfort	1	10	1	10	1	10	1	10	1	10	1	10	1	Over 10	1	Over 10	1	Over 10	2	10	2	10	2	10

Note: In each group of figures, in left one is for toll roads and the right for free roads.

Table 3. Comparison of Overall Estimated Utilization Rates and Observed Values.

	Time	Expense	Comfort	Freight	Overall	Observed Value	x^2-Test
Matsue Highway							
Small passenger cars	618	518	834	—	728	720	Non-significant
Ordinary trucks	597	462	834	664	595	334	Significant
Small trucks	531	518	834	664	663	678	Non-significant
Sangu Highway							
Small passenger cars	618	482	834	—	787	862	Non-significant
Ordinary trucks	618	482	834	588	627	660	//
Small trucks	618	482	834	588	640	760	//
Kamigo Bridge							
Small passenger cars	618	538	1,000	—	790	900	Significant
Ordinary trucks	570	538	1,000	834	672	670	Non-significant
Small trucks	570	538	1,000	834	725	810	//
Keiyo Highway							
Small passenger cars	618	500	755	—	665	758	Non-significant
Ordinary trucks	618	482	755	500	600	410	Significant
Small trucks	618	482	755	500	618	617	Non-significant

and observed values are comparatively well balanced though in the case of ordinary trucks, the observed values are sharply below the calculated values in many cases. This comes true in the case of toll roads, whatever method of estimation adopted. This seems attributable to the fact that for the majority of ordinary trucks the toll payment is out of consideration from the first. On the viewpoint of informatiom theory, this can be interpreted to illustrate that some definite segment of vehicular traffic never uses the toll roads. Contrary is the case with passenger cars and buses that usually run on the tourist roads and, hence, they are constant and willing choosers of the toll roads.

The aforementioned study yet to be perfected in many respects, proves that with the application of the information theory it becomes practical to determine simultaneously the different utilization rates of roads, however large the number of roads is, and that except in the case of some specific types of vehicles, the calculated and observed values are comparatively well balanced.

Abstract

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A Flow Network with High Rəliability (*Keiei-Kagaku, 9, No. 1*)

In a connected and undirected network, flows between any two nodes are possible. Wowever, if edge failures occur, the situation is changed. For afailure of cut-edge, some of flows become impossible. This means that a disconnected network is obtained by the removal of the failed edge. For many important networks, such as the communication network, the effect of this kind of failure is serious. To protect the network from the effect, it is considered that some redundant edges are added previously. In this report, the method how to obtain the optimal reliable network from the given network is discussed. Here, a reliable network means a network in which flows between any two nodes are possible even if an edge in the network is failed.

The problem discussed is an follows: A connected and undirected network $[X, A]$ is given arbitrarily, where X and A represent the sets of nodes and edges respectively. It is possible to add an edge (x_i, x_j) to $[X, A]$ at the cost of c_{ij}. If we choose a suitable set of edges B, the network $[X, A \cup B]$ become a reliable notwork. The total cost for B is

$$C(B) := \sum_{(x_i, x_j) \in B} c_{ij} . \qquad (1)$$

The problem is to find the optimal reliable network which minimizes the equation (1).

This problem is solved along the following line. To begin with, we examine the notwork $[X, A]$ whether any loops are involved in it. If some loops are involved, the condensing procedure is made. This procedure is roughly stated such that a loop of network is condensed into a single node. And, as far as loops are involved, this condensing procedure is continued. Then, a tree $[X_T, A_T]$ is obtained finally. If this tree consists of a single node, the original network $[X, A]$ is a reliable network. If not, $[X, A]$ is not a reliable network. In the latter case, we try to find the optimal reliable network $[X_T, A_T \cup B_T]$ for the tree $[X_T, A_T]$. If this is done, the optimal reliable network for $[X, A]$ is easily obtained from $[X_T, A_T \cup B_T]$. Thus, the problem is to reduced to find the optimal reliable network only for trees. It is shown that this problem can be reduced to some kind of the shortest path problem. Finally, an example is shown.

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