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ON SOME QUEUES OCCURRING IN AN INTEGRATED IRON AND STEEL WORKS*

S. SUGAWARA and M. TAKAHASHI

Yawata Iron and Steel Co., Ltd.

Introduction

In such an integrated iron and steel works as Yawata, there are some problems about the material flow from steel plants to primary-rolling mill plants. Hot steel ingots tapped from a steel making furnace in a steel plant are sent to soaking pits in a primary-rolling mill plant. Immidiately after arriving at soaking pits, they are reheated and soaked for rolling. But since their arrival is irregular, they often must wait for soaking, which results in their cooling off and it takes longer time to reheat them. Therefore, once ingots happen to wait for soaking, others add to them and make a line of waiting ingots. It often grows longer and longer until the cold ingots among them must be removed out of the line. In order to explain this phenomenon exactly, this paper will deal with the following problem;

Under what condition does their waiting line grow large infinitely at soaking pits in case we do not remove the cold ingots ?

Of course, our practical problem is how to schedule and control the material flow from steel plants to primary-rolling mill plants. Several system simulations by Monte-Carlo method have been made to solve this problem, and from their results the controlling center were alredy set up. On the other hand, the above problem intends to study a fundamental principle of the queues and takes notice about the convergency of their simulations.

We owe the methods of this paper to J. Kiefer & J. Wolfowitz [1] and

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D. V. Lindley [3], but the properties of our problem are considerably different from those of them.

Formulation of the probrem

We now formulate problem in the terminology of queueing theory.

1) We shall use the term of customer and server for ingots and sorking pits respectively. Define; a unit of customer is one heat of ingots tapped from a steel making furnace and that of server is pits occupied by one heat of ingots. We assume that there are $s\,(\geqq 1)$ serves, $M_1, M_2, \cdots, M_s$, in this system.

2) The ith customer arrives at time t_i with $t_i \leqq t_{i+1}$, $t_0=0$. Let $g_i=t_i-t_{i-1}$ for all $i \geqq 1$. We assume that the g_i are independent random variables with identical probability distributions and the mean, Eg_1, is finite.

3) The length of time which ingots spend in pits to be reheated and soaked is determined by their temperature when they are charged in soaking pits. Therefore we assume that the service time of a customer is a monotone-increasing function, $f(x)$, of his waiting time, x, and

$$f(0)>0, \quad \lim_{x\to+\infty} f(x) \overline{\overline{\rightarrow}} f(\infty)<\infty .$$

Here, we neglect the influence of the rolling mill which refuses a queue between soaking pits and a primary-rolling mill. Since it makes the waiting time longer than that of our monel, the assertion of this paper remains usefull in the case the queue is refused.

4) If a customer arrives when at least one server is free, he is immediately attend to. But if he arrives when none of the server is free, he waits in a queue. His service is begun as soon as at least one server is free and all customers, arrived before him, have been or are being served.

Under such conditions, we shall deal with the problem according to J. Kiefer and J. Wolfowitz [1] as follows. Let u_{ij} be the time at which the jth server, M_j, finishes serving the last of those among the first $(i-1)$ customer which it serves. Let $u'_{ij}=o \vee u_{ij}$. Let w_{is} be the quantities u'_{ij}, $\cdots$, u'_{is} arranged in order of increasing size. Then, w_{i1} is the waiting time of the ith customer.

Write $\boldsymbol{w}_i=(w_{i1},\cdots,w_{is})$, then, since $t_{i+1}=t_i+g_{i+1}$, $\boldsymbol{w}_{i+1}$ is obtained from $\boldsymbol{w}_i$ as follows: Subtract g_{i+1} from every component of $(w_{i1}+f(w_{i1}), w_{i2},\cdots,w_{is})$. Rearrange the resulting quantities in ascending order and replace all negative quantities by zero. The ensuing result is $\boldsymbol{w}_i$.

Let $S \Leftarrow \{\boldsymbol{x}=(x_1,\cdots,x_s)\in R^s$ such that $0\leqq x_1\leqq\cdots\leqq x_s\}$ and $\boldsymbol{x},\boldsymbol{y}\in S$. For $i\geqq 1$, let

$$F_i(\boldsymbol{x}|\boldsymbol{y}) \Leftarrow P\{\boldsymbol{w}_i\leqq \boldsymbol{x}|\boldsymbol{w}_1=\boldsymbol{y}\}$$

where, for $\boldsymbol{a},\boldsymbol{b}\in R^s$, $\boldsymbol{a}\leq\boldsymbol{b}$ implies that every coodinate of $\boldsymbol{a}$ is not greater than the corresponding coordinate of $\boldsymbol{b}$.

Let 0 be the origin in space R^s and $F_i(\boldsymbol{x}) \Leftarrow F(\boldsymbol{x}|0)$. Then it holds $F_{i+1}(\boldsymbol{x})\leqq F_i(\boldsymbol{x})(i\geqq 1)$. Write $\bar{x}_1=(x_1,\infty,\cdots,\infty)$, $F_i^*(x_1)=F_i(\bar{x}_1)$, where F_i^* is the distribution function of the waiting time of the ith customer.

Then we can easily derived the following lemma from $w_{is}-w_{i1}\leqq f(\infty)<\infty$ and [1].

Lemma. There exist $\lim_{i\to\infty}F_i(\boldsymbol{x}) \Rightarrow F(\boldsymbol{x})$ for every $\boldsymbol{x}\in S$ and $\lim_{i\to\infty}F_i^*(x_1) \Rightarrow F^*(x_1)$ for $x_1\geqq 0$ and the following equalitity holds: $F(\bar{x}_1)=F^*(x_1)$ where $\bar{x}_1=(x_1,\infty,\cdots,\infty)$. Now our aim of this paper is to prove the following theorem, especially, ii-b).

Theorem. If Conditions 1), 2), 3) and 4) are satisfied then it follows.

i) If $sEg_1>f(\infty)$, $F(\boldsymbol{x})$ is a distribution function. Therefore $F^*(x_1)$ is also a distribution function.

ii) In the case, $sEg_1<f(\infty)$, there are two case:

ii-a) If $P\{sg_1<f(0)\}=0$, then $F^*(x_1)\equiv 1$ for $x_1\geq 0$

ii-b) If $P\{sg_1<f(0)\}>0$, $F(x)$ is not a distribution function and

$$F(\boldsymbol{x})\equiv 0,\ F^*(x_1)\equiv 0,\ F(\boldsymbol{x}|\boldsymbol{y})\equiv\lim_{i\to\infty}F_i(\boldsymbol{x}\,|\,\boldsymbol{y})\equiv 0$$

Proof

i) Let R_i be the service time of the ith customer and $R_i'\equiv f(\infty)$. Then,

$$R_i=f(w_{i1})\leqq f(\infty)=R_i'<sEg_1\,.$$

Let $\boldsymbol{w}_i'$, F_i', F' be the same functions of $\{g_j\}$ and $\{R_j'\}$ that $\boldsymbol{w}_i$, F_i, F are of

$\{g_i\}$ and $\{R_j\}$. It follows by induction $w_i' \geqq w_i$ $(i=1, 2)$. On the other-hand, $\rho = ER_1'/sEg_1 = f(\infty)/sEg_1 < 1$, so that the theorem of J. Kiefer and J. Wolfowize is applicable for $\{w_i'\}$, *i.e.*

$$\lim_{x\to(\infty,\cdots,\infty)} F'(x) = 1 .$$

From $F_i'(x) \leqq F_i(x) \leqq 1$, it follows, $F'(x) \leqq F(x) \leqq 1$.
Therefore, $1 = \lim_{x\to(\infty,\cdots,\infty)} F'(x) \leqq \lim_{x\to(\infty,\cdots,\infty)} F(x) \leqq 1$.
Hence, $\lim_{x\to(\infty,\cdots,\infty)} F(x) = 1$.

ii-a) In this case, it is easily seen that, with probability one, $w_{i1} = 0 (i=1, 2, \cdots)$. It follows from this, $F_i^*(x_1) \equiv 1 (i=1, 2, \cdots)$, then $F^*(x_1) \equiv 1$.

ii-b) We shall now prove that, if $sEg_1 < f(\infty)$ and $P\{sg_1 < f(0)\} > 0$, then $F(x) \equiv 0$.

Let $[a]$ be the largest integer $\leqq a$ and for some $c > 0$, define,

$$f'(x) = c[f(x)/c], \quad g_i' = c[g_i/c] + c \quad (i=1, 2, \cdots) .$$

Then $g_i' \geqq g_i$ and $f'(x) \leq f(x)$. Let w_i' be the same function of $\{g_j'\}$ and f' that w_i is of $\{g_j\}$ and f. Then it is seen by induction that

$$w_i' \leqq w_i \quad (i=1, 2, \cdots)$$

and if c is sufficiently small, $SEg_1' < f'(\infty)$ and $P\{g_1' < f'(0)\} > 0$.
Therefore, it is sufficient to prove ii-b) for this process, $\{w_i'\}$. We shall write $w_i = w_i'$, $g_i = g_i'$, $f(x) = f'(x)$ in the remainder of this section.
Let $w_1 = 0$. From the definition of w_i, the process, $\{w_i\}$, is a Markov chain with stationary transition probabilities.

Case 1 We show $F(x) \equiv 0$ if $P\{g_1 > f(\infty)\} > 0$.

Let A be the set of origin 0 and all points which can be reached by the process $\{w_i\}$ with positive probability.
This chain is not bounded *i.e.* for arbitrary point $M = (M_1c, \cdots, M_sc)$, there exists a point of A, $(k_1c, \cdots, k_sc)$ such that $k_i > M_i$ $(i=1, 2, \cdots, s)$ since $P\{sg_1 < f(0)\} > 0$. And this chain is irreducible and apperiodic since $P\{g_1 > f(\infty)\} > 0$, *i.e.*, for all point $(h_1c, \cdots, h_sc)$ of A,

$$P\{w_{i+j}=(0, \cdots, 0) \text{ for some } j\geqq 1 | w_i=(h_1c, \cdots, h_sc)\}>0,$$

$$P\{w_{i+1}=(0, \cdots, 0)| w_i=(0, \cdots, 0)\}\geqq P\{g_1>f(\infty)\}>0.$$

Then, from the theorm of Feller [2] XV. 6, our chain belongs to one of the following two cases:

(1) Either the states are all transient or all null state: in this case, $F(x)\equiv 0$.

(2) Or else, all state are ergodic.

Since, we show that all state of our chain are transient.

From $f(\infty)<sEg_1$, there exists a point $(m_1{}^0c, \cdots, m_s{}^0c)$ of A such that

$$f(ic)<sEg_1 \quad \text{for all } i\geqq m_1{}^0.$$

The state $(m_1{}^0c, \cdots, m_s{}^0c)$ is transient if there exists a point $(m_1c, \cdots, msc)$ such that $m_j\geqq m_j{}^0$, $s\geqq j\geqq 1$ and

$$P\{w_{i+kj}>m_j{}^0c \quad \text{for all } k\geqq 1 \text{ and } s\geqq j\geqq 1 | w_{ij}=m_jc \;\; s\geqq j\geqq 1\}>0.$$

From $w_{is}-w_{i1}\leqq f(\infty)$, $i\geq 1$, it is sufficient to show that

$$P\{\sum_{j=1}^{s} w_{i+kj}>\sum_{j=1}^{s} m_j{}^0c+2sf(\infty) \text{ for all } k\geqq 1 | w_{ij}=m_jc \;\; s\geqq j\geqq 1\}>0. \quad (*)$$

Let $\{u_j\}$, $\{U_k\}$ be sequences of random variables as follows:

$$u_j=f(m_1{}^0c)-sg_{i+j}, \qquad U_k=\sum_{i=1}^{k} u_j \qquad j, k=1, 2, \cdots,$$

Then, the u_j are independent random variables with identical probability distributions and the mean, Eu_1, is positive.

The above inequality is correct if

$$P\{U_k>\sum_{j=1}^{s}(m_j{}^0-m_j)c+2sf(\infty) \qquad \text{for all } k\geqq 1\}>0. \qquad (**)$$

Now, by the strong law of large numbers, for any positive ε there exists an N such that

$$P\{U_k<0 \qquad \text{for all } k>N\}1-\frac{\varepsilon}{2}.$$

On the other hand, there exist a point $(m_1c, \cdots, m_sc)$ of A such that

$$P\{U_k>\sum_{j=1}^{s}(m_j{}^0-m_j)c+2sf(\infty) \text{ for all } N\geqq k\geqq 1\}>1-\frac{\varepsilon}{2}.$$

Therefore we obtain

$$P\{U_k > \sum_{j=1}^{s} (m_j^0 - m_j)c + 2sf(\infty) \text{ for all } k \geqq 1\} > 1 - \epsilon ,$$

where ϵ is arbitray positive number and (**), also (*), holds.

Case 2. We now suppose $P\{g_1 > f(\infty)\} = 0$.

Let n_0, n be positive integers such that

$$n_0 c \leqq f(\infty) , \qquad P\{g_1 = n_0 c\} > 0 \quad \text{and} \quad nc > f(\infty) .$$

Let $\{g_j''\}$ be independently and identically distributed random variables with the following distribution:

$$P\{g_1'' = n_0 c\} = P\{g_1 = n_0 c\} - \epsilon ,$$
$$P\{g_1'' = nc\} = \epsilon ,$$
$$P\{g_1'' = ic\} = P\{g_1 = ic\} \qquad i \neq n_0, n .$$

Here ε is a small positive number and $P\{g_1 = n_0 c\} - \epsilon > 0$.

We choose ϵ so small that $f(\infty) < sEg_1''$.

Let w_i'', F_i'', F'' are same functions of $\{g_j''\}$ and f that w_i, F_i, F are of $\{g_j\}$ and f. Then it is easily seen that

$$F_i(\boldsymbol{x}) \leqq F_i''(\boldsymbol{x}) \qquad \text{for every } \boldsymbol{x}$$
$$F(\boldsymbol{x}) \leqq F''(\boldsymbol{x}) \qquad \text{for every } \boldsymbol{x}$$

Therefore, it is sufficient to show $F''(\boldsymbol{x}) \equiv 0$. and the result of case 1 is applicable for this process $\{w_i''\}$.

Case 1 and Case 2 together show that $F(\boldsymbol{x}) \equiv 0$ if $sEg_1 < f(\infty)$ and $P\{sg_1 < f(0)\} > 0$.

Now, let $\boldsymbol{x}, \boldsymbol{y} \in S$. It is easily seen that

$$F_i(\boldsymbol{x} | \boldsymbol{y}) \leqq F_i(\boldsymbol{x}) \qquad \text{for} \quad \boldsymbol{x}, \boldsymbol{y} \in S.$$

Hence

$$F(\boldsymbol{x} \mid \boldsymbol{y}) \equiv \lim_{i \to \infty} F(\boldsymbol{x} \mid \boldsymbol{y}) \leqq F(\boldsymbol{x}) \equiv 0.$$

Then the proof of the theorem is completed.

Note 1. In our theorem the case $sEg_1 = f(\infty)$ remains untouched, sence in this case it seems that there needs some additional conditions on $f(x)$.

Note 2. The above proof of ii-b) also shows that for any sequence $\{g_1, g_2, \cdots\}$, we can find an N such that $w_i > x$ for any $x \equiv S$ and $n > N$, with probability one.

Then, large queues build up, never to disappear.

Conclusion

In Yawata Works, there needs essentially an on-line controlling system for the ingots flow and we have to remove the cold ingots of the waiting line, since the assumption of is-b) are almost satisfied.

On the other hand, in a general iron and steel works, it almost holds $sEg_1 < f(\infty)$ but the g_i are not strictly independent. Therefore, in practical, we would rather replace $P\{sg_1 < f(0)\} > 0$ by the condition that a waiting time w_{i1} can cross over the critical point x_0 such that

$$f(x_0) > sEg_1 .$$

In Tobata Works in our company, facilities and equipments of steel plants and primary-rolling mill plants are arranged in a direct line to assure the smooth flow of ingots and w_{i1} rarely crosses over the critical point x_0.

In this case we do not need a particular controlling system for the ingots flow.

Thus, our problem belongs to one of these two case, Yawata or Tobata type. Almost all works which have been built recently belong to the latter. But in the case of Yawata type, not only the thermo-efficiency but also the production rate of primary rolling mills depends on the amount of cold ingots which occur before soaking pits since we must remove them in order to prevent them from increasing infinitely. Therefore, it is very important how to schedule and control the ingot flow to ensure its smoothness.

A study on the design of its controlling system and system simulation for its purpose in Yawata Works will be presented in the near future.

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