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SOME CONSIDERATIONS ON PREVENTIVE MAINTENANCE POLICIES WITH NUMERICAL ANALYSIS

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§1. INTRODUCTION

In the previous papers [3, 4], the authors proposed a new type of preventive maintenance policy which was named as Policy III to maintain a complex system of equipments, and noted that it has some practical usefulnesses.

Especially, in [4], it was shown that an optimal type of preventive maintenance policy from a practical view point becomes type III. Moreover, an optimal policy of type III may have robustness for varying the mean life time. This fact was illustrated numerically in [3] when the life time distribution is of Weibull type:

$$\begin{aligned} F(x) &= 1-\exp\{-\alpha(x-\gamma)^{\beta}\} \qquad && (x>\gamma) \\ &= 0 && (x\leq\gamma) \end{aligned} \tag{1.1}$$

Above discussions were limited only for the cases where the object function is the limiting efficiency and the location parameter γ of the Weibull distribution is zero. However, since these discussions were already extended to the case where the object function is taken as the maintenance cost rate [4], some numerical examples for the cost rate case will be illustrated. Since the case of $\gamma>0$ may not be neglected in many practical uses of that policy, γ need not constraint to zero in our numerical computations. Thus, in this paper some numerical examples will be given for which the parameters are selected systematically.

These examples will show that the policy is considerably tough for varying the parameters α and γ, (where β is assumed to be fix) while the Policy II cannot be so tough in those cases. Furthermore, that the optimal policies of type III are more efficient than the corresponding optimal policies of type II, will be shown by these examples. An analytical treatment on asymptotic behavior of this fact will be added in the last section.

For the preparation of this paper, the various computations were made with an electronic computor. So, the contents of a series of computations will be explained in § 3.

§ 2. EXTENSION OF SOME FORMULAS IN THE PREVIOUS PAPERS

In the previous papers, the concrete expressions of the optimal replacement time, the maintenance cost rate, the limiting efficiency, the interval reliability and so on, are given under the assumption that the life time distributions of the systems are of Weibull type with zero location parameter.

However, in many practical cases, it may be recognized that the failure distribution is of a mixed Weibull type as cited already in [4] (Remark 6. 3). But, since many systems will be run in the factory (i.e., 'aging') during the period corresponding with the catastrophic failure, one can see that the part of catastrophic failure of the life time distributions of the systems are truncated in their practical uses. Moreover, when the aging procedure has been done completely, one may neglect the number of failures before γ from the count of failures. Thus, it may be considered that the life time distribution is of a simple Weibull type with a non-zero location parameter γ in usual cases. Hence, deleting the (chance) failures before the time γ, Policy III based on a simple Weibull type life distributions with a non-zero location parameter can be applied. In discussions of Remark 6. 3 in [4], "Policy III does not depend on γ", should be replaced by "Policy III depends on γ", because when the maintenance cost rate (or limiting efficiency) is accepted as the object function to be minimized (or maximized), this statement is not valid.

Thus, this paper introduces the extension formulas of the maintenance cost rate and the limiting efficiencies for Policy II and III to the non-zero location parameter case. It is assumed that the life time distributions of system are Weibull type with the identical parameters β and γ. Then, the extended formulas can be expressed readily as follows:

$$C^{(2)}(t)=[(T_m+C_m)\alpha(t-\gamma)^\beta+C_s+T_s]/[t+\alpha(t-\gamma)^\beta T_m+T_s] \quad (t>\gamma)$$
$$=[C_s+T_s]/[t+T_s] \quad (t\leq\gamma) \qquad (2.1)$$

$$C^{(3)}(k)=[(k-1)(T_m+C_m)+T_s+C_s]\Big/\left[\mu\beta/B\left(\frac{1}{\beta},\ k\right)+\gamma+(k-1)T_m+T_s\right] \qquad (2.2)$$

$$E_{ff\infty}^{(2)}(t)=t/[t+\{\alpha(t-\gamma)^\beta\}T_m+T_s] \qquad (2.3)$$

$$E_{ff\infty}^{(3)}(k)=\left[\mu\beta/B\left(\frac{1}{\beta},\ k\right)+\gamma\right]\Big/\left[\mu\beta/B\left(\frac{1}{\beta},\ k\right)+\gamma+(k-1)T_m+T_s\right] \qquad (2.4)$$

using the notations used in [4]. Hence, the replacement times for the optimal policy of type II or III may be obtained from these formulas.

Since no discussion was made in [3] on the maintenance cost rate of Policy II, the expression (2.1) is quite new, but its deduction is almost similar to that of (2.2). Therefore the detailed discussion on the deduction was omitted in this paper.

§ 3. OUTLINES OF NUMERICAL COMPUTATIONS

A series of numerical computations are listed as follows:

i) Optimal replacement time (k_0) for Policy III.

For the sake of effective use of Policy III, some numerical tables of the optimal replacement time k_0 were prepared. Assuming $C_s=C_m=0$, since maximization of the limiting efficiency is equivalent to minimization of the maintenance cost rate, the tables are presented for the maintenance cost rate only. If it is assumed that the family of the life time distributions of the systems is given by $\{F_i(x)\}$ such that

Table 1. k_0 for $\mu=\alpha^{-1/\beta}\cdot\Gamma\left(1+\frac{1}{\beta}\right)$

C_m	ρ	T_s	γ	OPTIMAL REPLACEMENT TIME (k_0)									
				1	2	3	4	5	6	7	8	9	10
1	10	1	0				∞	36.571	13.298	8.593	6.547	5.391	4.643
1	10	1	1				∞	28.444	10.343	6.683	5.092	4.193	3.611
1	10	1	3				∞	12.190	4.432	2.864	2.182	1.797	1.547
1	10	1	5	.285	.533	1.066	3.657	∞					
1	10	1	10	3.142	5.866	11.733	40.228	∞					
1	10	1	15	6.000	11.200	22.400	76.800	∞					
1	10	1	20	8.857	16.533	33.066	113.371	∞					
1	10	1	30	14.571	27.200	54.400	186.514	∞					
1	10	1	40	20.285	37.866	75.733	259.657	∞					
1	10	1	50	26.000	48.533	97.066	332.800	∞					
1	10	2	0					∞	17.731	9.547	6.789	5.391	4.540
1	10	2	1					∞	13.298	7.160	5.092	4.043	3.405
1	10	2	3					∞	4.432	2.386	1.697	1.347	1.135
1	10	2	5	.500	.888	1.600	3.637	∞					
1	10	2	10	3.000	5.333	9.600	21.942	∞					
1	10	2	15	5.500	9.777	17.600	40.228	∞					
1	10	2	20	8.000	14.222	25.600	58.514	∞					
1	10	2	30	13.000	23.111	41.600	95.085	∞					
1	10	2	40	18.000	32.000	57.600	132.657	∞					
1	10	2	50	23.000	40.888	73.600	168·228	∞					
1	10	4	0						∞	14.321	7.638	5.391	4.256
1	10	4	1						∞	9.547	5.092	3.594	2.837
1	10	4	3	OPTIMAL $k=7$									
1	10	4	5	.800	1.333	2.133	3.657	8.126	∞				
1	10	4	10	2.800	4.656	7.465	12.800	28.444	∞				
1	10	4	15	4.800	8.000	12.800	21.942	48.761	∞				
1	10	4	20	6.800	11.333	18.133	31.085	69.079	∞				
1	10	4	30	10.800	18.000	28.800	49.371	109.714	∞				
1	10	4	40	14.800	24.666	39.466	67.657	150.349	∞				
1	10	4	50	18.800	31.333	50.133	85.942	190.984	∞				
1	10	7	0							∞	15.276	5.391	3.405
1	10	7	1							∞	5.092	1.797	1.135
1	10	7	3	.461	.727	1.066	1.567	2.438	4.432	14.321	∞		
1	10	7	5	1.076	1.696	2.488	3.657	5.688	10.343	33.417	∞		
1	10	7	10	2.615	4.121	6.044	8.881	13.815	25.119	81.156	∞		
1	10	7	15	4.153	6.545	9.600	14.100	21.942	39.896	128.895	∞		
1	10	7	20	5.697	8.969	13.155	19.330	30.069	54.672	176.634	∞		
1	10	7	30	8.769	13.818	20.266	29.779	46.323	84.225	272.111	∞		
1	10	7	40	11.846	18.666	27.377	40.228	62.577	113.777	367.589	∞		
1	10	7	50	14.923	23.515	34.488	50.677	78.831	143.330	463.067	∞		
1	10	10	0	OPTIMAL $k=10$									
1	10	10	1	.250	.380	.533	.731	1.015	1.477	2.386	5.092	∞	
1	10	10	2	.750	1.142	1.600	2.194	3.047	4.432	7.160	15.276	∞	
1	10	10	5	1.250	1·909	2.666	3.657	5.079	7.388	11.934	25.460	∞	

$$F_i(x)=1-\exp\{-\alpha_i(x-\gamma)^\beta\} \qquad (x>\gamma)$$
$$=0 \qquad (x\leqq\gamma) \qquad (3.1)$$
$$(i=1, 2, \cdots, p)$$

then, the discussions in §2 of [3] may hold substituting $T_s+\gamma$ for T_s, and $C_s-\gamma$ for C_s in (2.7) based on Theorem 2.2 and Corollary 2.1 in [3]. The tables show the values of μ that is the mean life time after γ and is given by $\mu=\alpha^{-1/\beta}\cdot\Gamma\left(1+\frac{1}{\beta}\right)$ as illustrated in Table 1. In practice firstly we shall estimate γ and μ, then have to find out the position of μ in the corresponding row. Table 1 shows that in a example with the value:

$$C_m=1, \quad C_s=10, \quad T_s=4, \quad \gamma=10, \quad \mu=6,$$

since

$$4.666<\mu=6.0<7.466$$

we can see the optimal replacement time $k_0=3$. Such tables were already constructed in the following cases:

$\beta=4/3$, 2, 3, 4
$C_m=0.01$, 0.1, 1, 5, 10, 50
$\rho=C_s/C_m=1$, 5, 10, 20, 30, 50, 100, 500
$T_s=1$, 2, 3, 4, 5, 7, 10, 20, 30, 40, 50
$\gamma=0$, 1, 3, 5, 10, 15, 20, 30, 40, 50

where we shall assumed that $T_mC_0=1$ without any loss of generality.

A part of our tables will be published in [5] in near future. If published the whole tables, they will consist of about 500 pages.

ii) Interval reliabitity of Policy III

The numerical computations of (3.10) in [3] for some parameters T_m, T_s, α, β, and k were done. It is assumed that $\mu=1$ without any loss of generality. Some considerations upon the results will be given in §6, so we shall not add any discession here.

iii) Limiting efficiency of Policy II ($\beta=2$)

Since the limiting efficiency $Eff_\infty^{(2)}(t)$ is given by (2.3), one can find the optimal replacement period t^* which maximize (2.3) by differentiating it. But, the calculations are tedious except the case $\beta=2$, then, we calculate the limiting efficiency of Policy II for the case $\beta=2$ only.

Firstly we compute t^* for some parameters T_s, μ, γ using the following expression

$$t^*=\sqrt{\gamma^2+\frac{T_0}{\alpha T_m}} \tag{3.2}$$

and the limiting efficiency for t^* Next, for the sake of getting some knowledges concerning with the robustness of Policy II, the limiting efficiency for parameters T_s, μ, γ, and t_0^* which is the optimal replacement period for T_s, μ_0 and γ_0 are computed.

If the limiting efficiency for T_s, μ, γ and t_0^* not so small compared with the one for T_s, μ, γ and t^* which is the optimal replacement period for T_s, μ, γ, one can say that the policy has a high robustness for varying μ and γ. Some discussions from the view point will be given in §4.

iv) Limiting efficiency of Policy III

To find k_0 which maximizes (2.4) has no any difference between the distinct two β's in the difficultness of its numerical computation. But, since we shall compare the robustness or absolute values of the limiting efficiency of Policy III with these of Policy II, our present computations are limited to the case $\beta=2$. And, an analogous computing plan to iii) are done.

v) Maintenance cost rate of Policy II

Based on (2.1), one can calculate the maintenance cost rate of Policy II under the condition $\beta=2$. Of course, without any loss of generality put $T_m=1$. Firstly, t^* which minimize the maintenance cost rate for a set of parameters (C_m, ρ, T_s, γ, α) is calculated by the following formula

$$t=\frac{1}{2A}(-B+\sqrt{B^2+4A(A\gamma^2+B\gamma+C_s+T_s)}) \tag{3.3}$$

where

$$A=\alpha(T_m+C_m)=\alpha(1+C_m),$$

$$B=2\alpha(C_mT_s-C_sT_m)=2\alpha C_m(T_s-\rho),$$
$$\rho=C_s/C_m.$$

Then, $C_\infty^{(2)}(t^*)$ for the corresponding set of parameters and for some set of parameters which departure from it on α and γ are calculated.

vi) Maintenance cost rate of Policy III

Using (2.3) and the statement in i), analogous computations to v) are possible. Combining the results in v) and vi) the comparison of robustness between Policy II and III for varying γ and μ (or α) may be done numerically. The following section will be devoted to do this.

§4. ROBUSTNESS OF POLICY III COMPARED WITH THAT OF POLICY II

At preventive maintenance policy of type III, whether a minimal repair ought to be done or an overhaul to be done is decided not by the total running time but by counting the number of minimal repairs. Thus, we can see intuitively that Policy III may be robust for the variation of the location parameter γ.

In general, if the life time distribution has a location parameter $\gamma(>0)$, do not encounter with trouble in time interval $(0, \gamma)$ and hence have no minimal repair. Thus the location parameter γ will effect the decision of the optimal replacement period of maintenance policy. But the estimation of location parameter γ is usually difficult. In such a circumstance, the total running time t^* till the next overhaul in Policy II will be effected directly by acccuracy of the estimate $\hat{\gamma}$ of γ. More precisely, the bias of the estimate $b_\gamma=\hat{\gamma}-\gamma$ is interpreted as deviation of t^* from the optimal one. This fact will be illustrated through some graphs of the limiting efficiencies and the maintenance cost rates (Fig. 1~4).

On the other hand, in Policy III, as the number of times of minimal repairs are only counted and the number of times of failures occuring in $[\gamma, \hat{\gamma}]$ (or $[\hat{\gamma}, \gamma]$) may be expected rather small, Policy III that use the

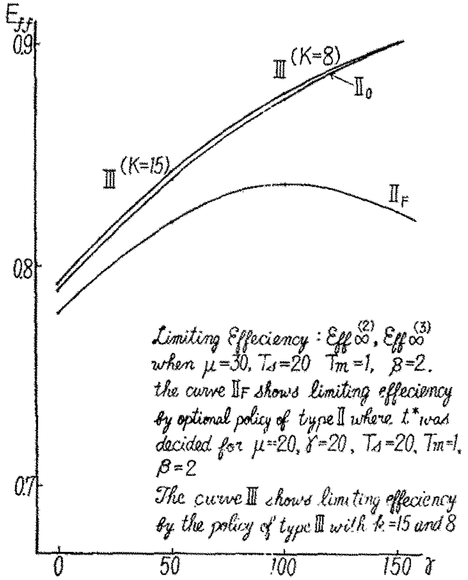

Fig 1 (a)

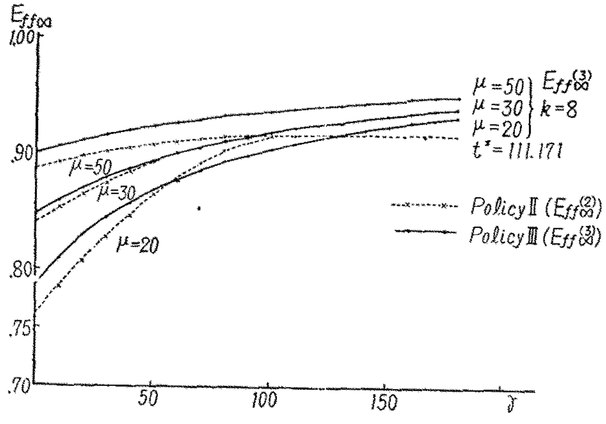

Fig 1 (b)
Ts=10, Tm=1, β=2

optimal replacement times is rather tough for variation of γ. This property will be desirable in partical cases.

Now, henceforth, we observe the numerical results obtained in § 3, and discuss on toughness of Policy III.

As were given in § 2, the limiting effeciency or maintenance cost rate is decided for a set of parameters (μ(or α), γ, T_s, T_m, C_s, C_m and β), and as was noted in § 3 we have computed that quantities under the assumption $C_0T_m=1$ and β is fixed. Based upon the computational results we observe in the following illustrative figures how the limiting effeciency or maintenance cost rate depart from its optimal value when μ or γ varies, while the other parameters are fixed. For example in Fig. 1, the limiting effeciency are plotted as functions of γ;

i) the curve II_F shows the limiting effeciency of Policy II $Eff_{\infty}^{(2)}(\gamma\,;t)$ where t is found as the optimal replacement period for a value of $\gamma_0=20$ and $\mu=20.0$. $T_s=20$, $\beta=2$,

ii) the curve II_0 shows the one of Policy II where the replacement period t^* is decided to maximize the maintenance cost rate for each γ, that is, t is always optimal for varying γ,

iii) the curve III shows the one of Policy III whose k is decided to be $k=8$ optimal for $\gamma=40$ and $k=15$ for $\gamma<40$.

The graphs for various cases by changing the parameters systematically were observed. However, in this section some typical cases selected from the detailed list of § 3 will be only shown because they can be classified according to their tendencies ignoring some difference of magnitudes.

*F*ig. 3. (a), the graph of maintenance cost rate explains the case where $T_s=5$, $C_m=50$ and $C_s=500$. Optimal policy was decided as $k_0=2$ for type III and $t^*=7.082$ for type II. In this graph we can see the following fact.

i) Suppose that location parameter γ is fixed and equal to γ_0, and that the optimal value is t^*. By curve II_F we see that the maintenance cost rate increases markedly for the value γ larger than γ_0. On the other hand the maintenance cost rate for Policy III optimized as $k_0=2$ decreases.

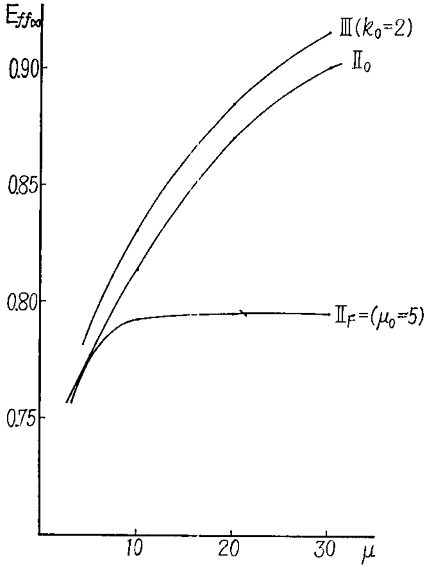

Fig 2 (a)
$\gamma=15,\ T_s=5,\ T_m=1,\ \beta=2$

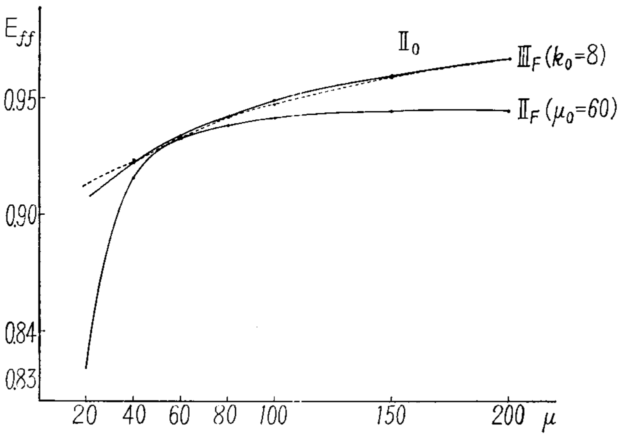

Fig 2 (b)
$\gamma=200,\ T_s=20,\ T_m=1,\ \beta=2$

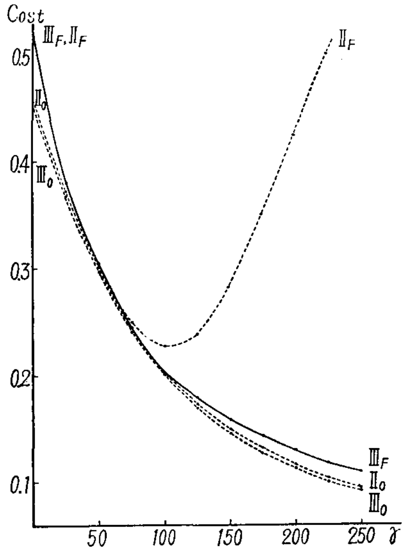

Fig 3 (b)
Ts=5 Cs=20, Cm=0.1, μ=15, β=2

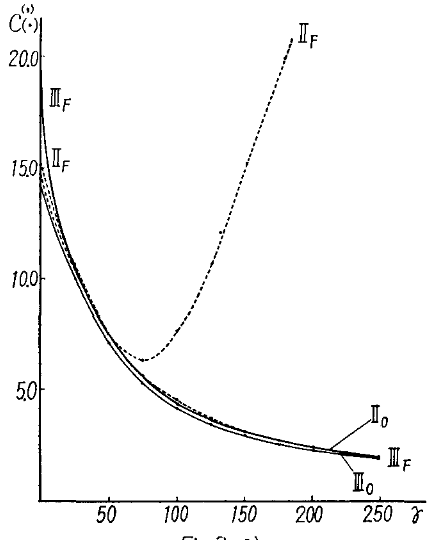

Fig 3 (a)
Maintenance Cost rate
Ts=5, Cs=500, Cm=50, μ=15, β=2

(Compare curves II_F and III.)

ii) The curve III_F is not differ far from the curve III_0, but curve II_F is above the curve II_0 for large γ.

Curve III_F means the graph of the maintenance cost rate as a function of γ where optimal value of k is decided for some fixed value γ or μ, and curve III_0 means the graph where optimal value of k is decided for each γ, that is always optimal for that γ.

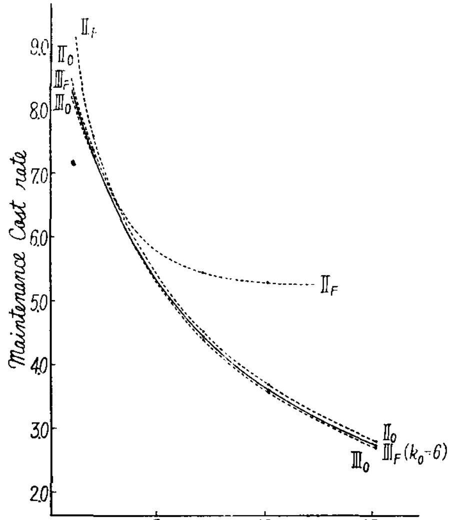

Fig 4.
Maintenance Cost rate
$\gamma=10$, $T_d=1$, $C_d=100$, $C_m=10$, $\beta=2$

From the above consideration one can see the following facts, that is, in Fig. 3, the curve II_F and III teach us some aspects. If one underestimate the location parameter γ as $\hat{\gamma}$ ($<\gamma_0$), and decide the policy of type II by t^* so as to minimize the maintenance cost rate at $\gamma=\hat{\gamma}$, then the maintenance cost rate at $\gamma=\gamma_0$ is large enough if bias $\gamma_0-\hat{\gamma}$ is large. On the other hand, according to Policy III, the cost rate due to the decision $k=k_1$ obtained by estimation of γ to be $\hat{\gamma}$ is not so different from that due to the decision $k=k_2$ obtained by the correct estimation of γ to be γ_0. Thus we can conclude that if we estimate γ to be smaller than γ_0, as sometimes occurs, in order to be in safe side, one must pay immense costs produced by using the policy of type II.

§ 5. A SUPPLEMENTARY REMARK

In the previous paper [3], it was shown that the type III is an optimal policy from a practical view point and has robustness for the system of which mean life time is varying: that is to say, although it seem evident that limiting effeciency of the policy of type III is superior to that of the policy of type II, any rigorous proof was not given. We shall add here a remark that the limiting efficiency of the type III is asymptotically larger than that of type II in the case when the life time distribution of the system is of Weibull type. The proof is at first begun by calculation of the limiting efficiency and then proceed to the evaluation of their expressions in terms of $R=\dfrac{T_s-T_m}{T_m}$ which is supposed to be large in many practical cases.

Theorem.

When the systems of which the life time distribution is of Weibull type with a common shape parameter β and zero location parameter are performed by two maintenance policies of types II and III, we have the following relation for sufficiently large R

$$Eff_{\infty}^{(3)} \geqq Eff_{\infty}^{(3)}.$$

Proof.

We shall denote the limiting effeciency of Policy III which any overhaul may be performed at every k-th failure as $Eff_{\infty}^{(3)}(k)$. This is given as

$$Eff_{\infty}^{(3)}(k)=\frac{\mu\beta}{\mu\beta+(k-1)B\left(k,\frac{1}{\beta}\right)T_m+B\left(k,\frac{1}{\beta}\right)T_s} \tag{5.1}$$

Of course, $Eff_{\infty}^{(3)}(k_0)=Eff_{\infty}^{(3)}$ which corresponds to the optimal procedure of Policy III, where k_0 is the largest integer such as

$$Eff_{\infty}^{(3)}(k-1) \leqq Eff_{\infty}^{(3)}(k) \tag{5.2}$$

and will be given as

$$k_0 = [k_0'] + 1,$$

where k'_0 is the root of the following functional equation

$$Eff_{\infty}^{(3)}(k_0') = Eff_{\infty}^{(3)}(k_0'+1) \tag{5.3}$$

and

$$k_0' = \frac{T_s - T_m}{T_m(\beta-1)}. \tag{5.4}$$

For the Policy II, optimal effeciency is given by

$$Eff_{\infty}^{(2)} = \frac{t^*}{t^* + (\alpha t^{*\beta})T_m + T_s} \tag{5.5}$$

where

$$t^* = \left(\frac{T_s}{\alpha(\beta-1)T_m} \right)^{1/\beta}. \tag{5.6}$$

Thit fact were often treated in previous papers [3], [4].

Hence, inserting t^* into (5.5) we have

$$Eff_{\infty}^{(2)} = \frac{\alpha^{-1/\beta}}{\alpha^{-1/\beta} + \frac{\beta T_s}{\beta-1} \left\{ \frac{(\beta-1)T_m}{T_s} \right\}^{1/\beta}}. \tag{5.7}$$

On the other hand it is clear that from Fig. 5

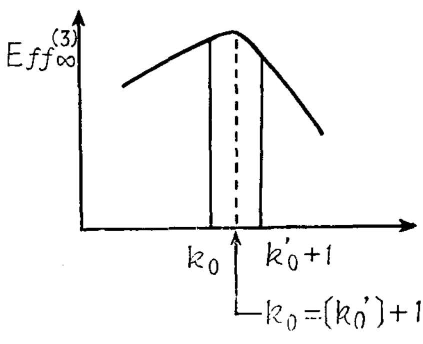

Fig 5

$$Eff_{\infty}^{(3)}([k_0']+1) \geqq Eff_{\infty}^{(3)}(k_0'),$$

because $Eff_{\infty}^{(3)}(k)$ is an unimodal function of k.

Inserting the value of k_0' into (5.1), we have a little less optimal effeciency $Eff_{\infty}^{(3)}(k_0')$, and if we have

$$Eff_{\infty}^{(3)}(k_0') \geqq Eff_{\infty}^{(2)}, \tag{5.8}$$

our proof of the theorem is complete.

To get $Eff_{\infty}^{(3)}(k_0')$ from (5.3) and (5.4),

$$Eff_{\infty}^{(3)}(k_0') = \frac{\mu\beta}{\mu\beta+(k_0'-1)-B\left(k_0', \frac{1}{\beta}\right)T_m + B\left(k_0', \frac{1}{\beta}\right)T_s}$$

$$= \frac{\alpha^{-1/\beta}\Gamma\left(\frac{1}{\beta}\right)}{\alpha^{-1/\beta}\Gamma\left(\frac{1}{\beta}\right)+\left(\frac{T_s-T_m}{T_m(\beta-1)}-1\right)\frac{\Gamma(k_0')\Gamma\left(\frac{1}{\beta}\right)}{\Gamma\left(k_0'+\frac{1}{\beta}\right)}T_m+\frac{\Gamma(k_0')\Gamma\left(\frac{1}{\beta}\right)}{\Gamma\left(k_0'+\frac{1}{\beta}\right)}T_0}$$

$$= \frac{\alpha^{-1/\beta}}{\alpha^{-1/\beta}+\frac{\Gamma(k_0')}{\Gamma\left(k_0'+\frac{1}{\beta}\right)}\left\{\frac{T_s-\beta T_m}{\beta-1}+T_s\right\}}$$

$$= \frac{\alpha^{-1\beta}}{\alpha_{-1/\beta}+\frac{\Gamma(k_0')}{\Gamma\left(k_0'+\frac{1}{\beta}\right)}\cdot\frac{-\beta T_m+\beta T_s}{\beta-1}} \tag{5.9}$$

To prove (5.8), it is sufficient to show that

$$\frac{\Gamma(k_0')}{\Gamma\left(k_0'+\frac{1}{\beta}\right)}\frac{-\beta T_m+\beta T_s}{\beta-1} \leqq \frac{\beta T_s}{\beta-1}\cdot\left(\frac{(\beta-1)T_m}{T_s}\right)^{1/\beta}$$

or

$$\frac{\Gamma\left(\frac{R}{\beta-1}+\frac{1}{\beta}\right)}{\Gamma\left(\frac{R}{\beta-1}\right)} \geqq \frac{R}{(1+R)^{1-1/\beta}} \cdot \frac{1}{(\beta-1)^{1/\beta}}$$

$$=\left(\frac{R}{\beta-1}\right)^{1/\beta}\left(\frac{R}{1+R}\right)^{1-1/\beta} \tag{5.10}$$

where

$$R=\frac{T_s-T_m}{T_m}=r-1, \qquad r=\frac{T_s}{T_m}$$

and

$$\frac{R}{\beta-1}=\frac{T_s-T_m}{T_m(\beta-1)}=k_0'.$$

The left hand side of (5.10) can be expanded as

$$\frac{\Gamma\left(\frac{R}{\beta-1}+\frac{1}{\beta}\right)}{\Gamma\left(\frac{R}{\beta-1}\right)}=\left(\frac{R}{\beta-1}\right)^{1/\beta}\left\{1+\frac{1}{2}\frac{\frac{1}{\beta}\left(\frac{1}{\beta}-1\right)}{\frac{R}{\beta-1}}+O\left(\frac{1}{\left(\frac{R}{\beta-1}\right)^2}\right)\right\}$$

On the other hand, the right hand side of (5.10) is expanded as

$$\left(\frac{R}{\beta-1}\right)^{1/\beta}\left(\frac{R}{1-R}\right)^{1-1/\beta}=\left(\frac{R}{\beta-1}\right)^{1/\beta}\left\{1+\left(\frac{1}{\beta}-1\right)\frac{1}{R}+\frac{\left(\frac{1}{\beta}-1\right)\left(\frac{1}{\beta}-2\right)}{2!}\left(\frac{1}{R}\right)^2+\cdots\right\}$$

comparing the second term, we have

$$\frac{1}{2}\frac{\frac{1}{\beta}\left(\frac{1}{\beta}-1\right)}{\frac{R}{\beta-1}} \geqq \left(\frac{1}{\beta}-1\right)\frac{1}{R} \quad \text{for} \quad \beta\geqq 1,$$

or

$$\frac{\beta-1}{2\beta}\leqq 1$$

Hence, neglecting thэ term $O\left(\frac{1}{R^2}\right)$, (5.10) or (5.8) is valid.

§ 6. A GRAPHICAL DISCUSSION ON THE INTERVAL RELIABILITY OF POLICY III

As were shown in the previous paper [4], the interval reliablity can be calculated expanding in powers $X=\frac{x}{\mu}$ when X is small, and in this case, the tables for the determination of the optimal k which minimizes maintenance cost rate was usefull for that of k which minimizes the interval reliability. But when X is not small, some elaborated computations are necesary. This computations were skectched in § 3, by which we have conclusions that

i) the interval reliability for $X=\frac{x}{\mu}$ (that is, the probability that the system continues to operate under Policy III for the time interval $(t, t+x)$ for some t) continues to have optimal value in $(0, X_1)$ for k, which is optimal for $X=0$, and then k_0-1 becomes optimal in (X_1, X_2) and k_0-2 becomes optimal in (X_2, X_3) and so on;

ii) but as shown in Fig. 5, the interval reliability for k_0 does not different from those of k_0-1 or k_0-2.

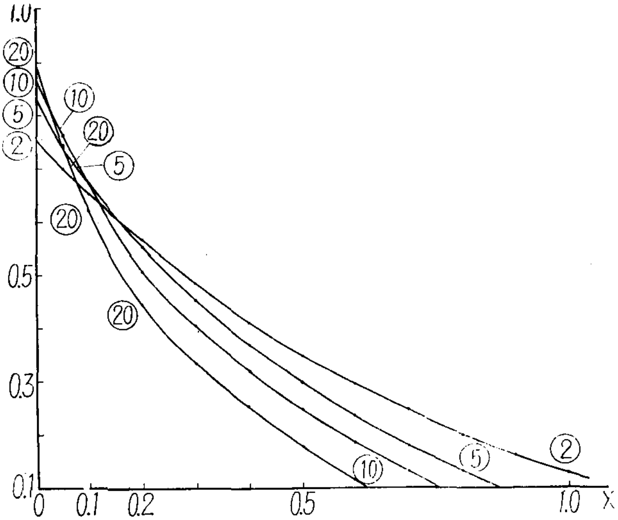

Fig 6 (a)
interval reliability
$T_s=0.3$, $T_m=0.01$, $\beta=2$
the number in ○ shows the valve of k

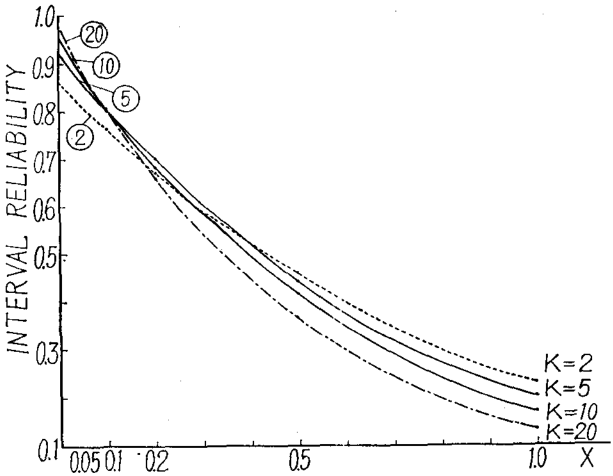

Fig. 6 (b)
interval reliability
$T_s=0.1$, $T_m=0.04$, $\beta=4/3$

Hence, summarizing the above remarks we can say that after we decided k_0, the policy continues to be optimal nearly in $(0, X)$ for some small X. In practical use, X is not so large since the interval reliability is small for large X, and so we may use k_0 as a basis for optimal or near optimal procedure for the interval reliability.

Fig. 6 shows some typical cases by which we concluded as above. And the other cases have the similar shapes except their numerical differences.

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