SOME METHODOLOGIES FOR MODEL CONSTRUCTION

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INTRODUCTION

At the Seventh National Meeting of TIMS, Professor Churchman gave some interesting statistics concerning the nature of publications in the journal "Management Science." The main conclusion was that the vast majority of articles is what may be called "deductive." Certain assumptions are made at the start. These are usually presented in the form of an assumed mathematical model. Then a strictly deductive mathematical process sets in by which certain "solutions," properties, and other findings are produced.

This state of affairs has given great concern to those researchers who believe that mathematical deduction is **not** the essence of management science. In fact, we have seriously neglected other aspects that are more vital both form a practical standpoint and from the standpoint of making management science a science. Foremost among these are the validation of the model and the data problem. The data problem concerns the determination of model parameters. As Professor Churchman has pointed out, we often behave as if these things were the easy part of the job and a matter of course when in fact we know that this is not so.

In this paper I shall only be concerned with model validation and not with the data problem. In passing, however, I should like to say that I consider both problems very nearly the same in that the data problem must also be solved by constructing valid forecasting models or estimating models for the parameters of the master model. We shall leave these submodels aside and consider only the master model from here on.

Our basic question will be: What is it that lends validity to a model relation-

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Invited Paper Presented at the 10th International Meeting of TIMS Tokyo, August 21-24, 1963

ship and makes it compelling? If we can find founding principles of model validity then it is clear that these can be put to immediate practical use in the very process of model construction.

It is sometimes asserted that model validity can be established by statistical tests and statistical fits. Personally, I have never found an instance where this procedure was convincing. True, statistical fitting may be the proper way to determine coefficients and parameters of a model if one is already satisfied about the validity of the basic model structure. But it cannot establish such validity. In particular, it inspires no confidence about the causality of the model relationship which is basic to any decision model. In addition to this philosophical reason there is an important practical reason for not wanting to rely on statistical means. In many decision-making situations, especially the more interesting strategic ones, we simply do not have sufficient data, or the data may not apply directly to the situation. Nevertheless we must build valid decision models. Thus, out of practical necessity we need other foundations of model validity. Statistical significance is not even a necessary condition. We must be able to do without it. In summary, model validity cannot rest upon statistical foundations. It must be built of sterner intellectual stuff.

It is the purpose of this paper to look into the question of what "sterner intellectual stuff" there is and to suggest that here lies a very fruitful area for making management science more of a science than it is.

It is the thesis of this paper that model validity should be made to rest upon a set of logical principles that are logically compelling and convincing in themselves. A number of such principles will be suggested, and their use in model construction will be illustrated by practical examples. It is hoped that the set of principles will be expanded and improved in the future.

COMMITMENT

The first principle may be called the principle of commitment. If a party to a decision-making situation agrees to do certain things in a certain way, and if this commitment can be stated in the form of a model relationship, then the validity of the model relationship may be considered well-founded. A simple example is found in inventory models with price breaks. An integral part of these models is a (mathematically discontinuous) relationship between quantity bought (in physical units) and total price payable to the vendor. Obviously, it has never occurred to anybody to demand that this price-quantity relationship be validated statistically. The fact that the vendor has agreed to deliver under these terms is a perfectly good and compelling foundation of its validity. The past is utterly irrelevant. The agreement can be made or altered at will at any time and will then be valid in the future. This simple logical principle is much more powerful in lending validity than statistical tests could ever be.

Similar examples can be found in many instances. A model may incorporate a relationship between taxes and taxable income. Its validity rests upon the fact that the government has committed itself to certain tax laws. Or it may incorporate a relationship between the sales revenue and the research and development budget of a firm which the management of the firm has committed itself to by policy decision. If a classical Hitchcock transportation model is formulated for, say: shipment by rail, then the model derives its validity, especially its linearity, from the tariff which the railroad has committed itself to. In all these cases the existence of the commitment guarantees the validity of the model relationship for the time of the duration of the commitment.

EQUAL BEHAVIORAL RESPONSE TO EQUAL ENVIRONMENTS

A second principle which is not as clean-cut as the first but has great practical potential may be called the principle of equal behavioral response to equal environments. Especially in marketing we are often interested in models relating sales potential to alternative structures of the product line. Sales potential is, of course, the result of aggregated human behavior. A given structure of the product line may be considered as an environment facing the potential customer population. Clearly, no new product environment is specifically the same as any environment the customer has experienced in the past. This is why historical statistical data are never directly applicable. However, it is often possible to evolve more abstract notions of environment and response that are applicable to both a new situation and past experience. The researcher can then search the domain of past experience in an attempt to find a segment of it that exhibits the same (abstract) environmental conditions as the new situation he is interested in. He can then transplant the environment-response relationship from that segment of experience to the new situation.

An example will illustrate this principle. A company markets a certain type of machine. The speed of the device is considerably smaller than that of most competitive devices whereas the sale price is almost the same. The sales department feels that the inferiority in speed has seriously impaired its performance and that the speed of the device should be changed to meet competition. However, changing the speed requires a major investment. The question before management is whether the additional sales response would be sufficient to profitably recover the investment for the change. If a decision model is to be constructed for this situation, then the core of the model must be a relationship between the company's sales potential on one hand and two alternate values of speed on the other hand. Obviously, it is hopeless to try to extract directly from sales experience a relationship between sales potential and speed. After all, the company has experienced only one value of speed. However, it is possible to give a more abstract characterization of the environment the customer will face after the speed is brought up to competitive standards(and the price remains the same). The environment may be characterized by the notion of cost indifference from the customer's standpoint. If the cutomer has a certain volume of work to do then the cost of doing it will be the same whether he uses the company's device or a competitive device. We may now search sales experience to date in an attempt to find a segment of it that also took place in a cost indifference environment. In fact it may be argued that a certain group of small customers does find itself in a cost indifference situation. For if a customer's volume of work is so small that he cannot fully utilize either the faster or the slower device, then he will need one machine in either case, and since the prices are the same no difference in cost arises. On the other hand, the large customers are definitely not in a cost indifference situation in this sense. If this reasoning is valid, then the acceptance of the slower device should be better with small customers than with large customers. Indeed, analysis of the available field data reveals that the company's market share (in terms of customers) increases steadily as the size of customer decreases (Fig. 1). This is all the more significant since the industry-wide conversion ratio (that is the percentage of potential customers using **any** machine) decreases (Fig. 1). Thus, the market share α_s presently enjoyed by the company in the smallest size group of customers may be considered indicative of the overall market share the company would enjoy if all customers were placed in a cost indifference environment by the contemplated change in speed. The details of the model may be completed on that basis.

Naturally, the example has been simplified but it illustrates how the logically convincing principle of equal behavioral response (in a statistical sense) to equal environments can be utilized in model construction. There is plenty of room for ingenuity—and for mistakes—in defining what constitutes equal environments.

A well-known application of the same principle is the estimation of income elasticity of demand for goods and services. Suppose we are interested in how a change in income structure of the population—perhaps in an underdeveloped country—will affect demand. We have a family budget survey which shows how people in various income classes presently spend their money. The proposed change maps each member of the population from its present into a new income class. We simply assume that he will behave much like those members of the population that previously occupied his new income class. This is precisely the same approach we have used in the previous example. We search out a segment of past experience that took place in the same environment which is now proposed as the new environment of a member of the population, and we base our expectation of response on that segment of experience. In this example the abstract notion of environment is that of income class.

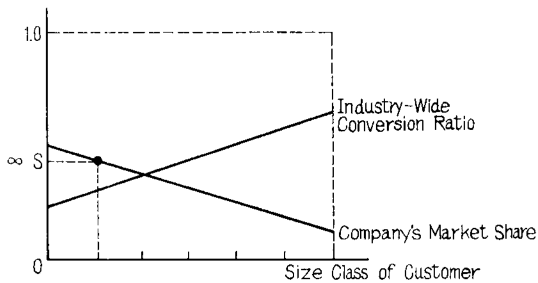

Fig. 1 Sales Experience to Dates

These notions may fruitfully be carried to still higher levels of abstraction. It is sometimes possible to formulate rather general principles of behavior, for example that an entrepreneur who has a choice of two technologies of roughly the same cost (such as clerks and data processing equipment) will usually decide in favor of the more advanced technology. It is clear that such principles are of great importance in scientific investigations of market potentials.

PROPERTIES OF PHYSICAL PROCESSES

A third set of principles has to do with physical processes. For a long time, physical processes have been the best understood. We all accept certain properties of physical processes as valid without question and hence need not hesitate to incorporate them into models.

Let us look at queuing models, inventory models, and models of production and distribution. All of these models deal with basicly physical phenomena. If we abstract from the specific nature of the probability distributions and other data that may be involved, then these models derive their basic validity from a few simple axioms of particle flow or other forms of mechanical flow. Notable among these are the principle of physical conservation and the principle that the same space (such as a service facility) cannot be occupied by two finite particles at the same time. The former gives rise to the basic differential equation describing the behavior of the inventory level in time, the latter gives rise to the basic differential equations of queuing models. We have no trouble accepting the validity of such simple statements concerning physical processes. Hence it is typical that doubts about queuing models are usually concerned with the validity of the probability distributions, that is the data, but not the basic physical queuing mechanism.

Another important physical principle is that of a "technological recipe." It is typical of many technological processes that output quantities and input quantities are linearly related, and that inputs are required in characteristic proportions. We often find statements of such "technological recipes" in linear programming models of oil refining, and again have no trouble accepting the validity of these recipes which reflect well-established knowledge of physical processes. The well-known input-output models of the Leontief type also rest entirely on the notion of technological recipes.

The same can be said of so-called "explosion" models for estimating parts requirements for assembled products.

ESTABLISHED THEORY

Finally, it is clear that we can always incorporate wellestablished theory into our models. If a theory is generally accepted and considered valid, then a model that draws upon it may also be considered valid. In fact a theory is itself a model of a higher class. Let us consider an example from economics. It is well-accepted theory or perhaps just common sense that the market performance of a company depends on three broad sets of factors: the company's own actions, competitors' actions, and general economic conditions. On the basis of this well-accepted theory Weinberg (1) has constructed a flexible model of company performance which contains specific variables descriptive of company actions, competitors's actions, and the general economic environment and shows their simultaneous interaction. According to Weinberg there has been a dozen or so highly successful applications of the model in quite different industries. The model may well derive its success form its strong base of validity which is given by a generally accepted theory of the structure of a company's enviornments.

The role of accepted theory as a basis for model validity is perhaps even more important in a negative sense. If wellaccepted theory says that certain factors are important in a given situation, and if such factors are absent form the model then we have reason to doubt the model's validity. For example I have seen advertising models and models of research and development that relate company sales to virtually just the company's own decision variables. In light of the preceding theory such models cannot be valid except under the most restricted conditions which may be so restricted as to render the model useless for decision-making. A model structure should be complete in light of accepted theory; we may call this the principle of logical completeness.

An extremely important special case is the requirement that models should take account of what may be called theoretical process limits. All too often processes, especially sales trends, are extrapolated into the future without consideration of inherent limits and boundaries that restrict the future course. Perhaps the classical example is the brisk sales picture of a new product which is in fact just displacing an older product. Outstanding companies have been led into overexpansion of capacity because of their failure to recognize the inherent limits of the growth process. The search for meaningful theoretical limits is usually an extremely fruitful endeavor. The establishment of a theoretical limit of a process also helps to make the law of diminishing returns operational in the sense that meaningful return curves can be constructed.

CONCLUSION

The thesis of this paper has been that the validity of models should be made to rest upon logical principles. Specificly, I have proposed the principles of commitment, of equal behavioral response to equal environments, and of physical processes, expecially in the form of " technological recipes ", and the simple laws of physical flow. Furthermore I have emphasized to principle of exploiting generally accepted theory with the special aspects of logical completeness and theoretical limiting concepts. What distinguishes these principles is their wide applicability in quite different areas-they are truly interdisciplinary in character. By reference to practical examples I have tried to show that the notion of logical principles is an operational one and that the regard or disregard of such principles can make the difference between a practically useful model and a useless one. It is hoped that more attempts will be made to identify and unify logical principles of model validity, and that this process will contribute to making management science a science.

REFERENCE

1. Weinberg, R. S., " Multiple Factor Break-Even Analysis: The Application of Operations-Research Techniques to a Basic Problem of Management Planning and Control"