

MICROSCOPIC AND MACROSCOPIC ASPECTS OF SINGLE LANE TRAFFIC FLOW

ROBERT HERMAN AND RICHARD ROTHERY

Research Laboratories, General Motors Corporation, Warren, Michigan

(Received Sept. 20, 1962)

ABSTRACT

A number of specific car following models for single lane flow without passing have been analyzed using a large sample of speed, concentration and flow data obtained at a single observation point in the Holland Tunnel. The results obtained have been compared with actual car following experiments conducted in the same tunnel. It is interesting to note that it is possible to relate the specific car following behavior to the overall flow characteristics although it is not possible to discriminate too well among the various car following models. A comparison is made regarding the flow-concentration characteristics of the Lincoln and Holland Tunnels. In addition an "acceleration noise" (dispersion of the acceleration distribution function) study is reported and results are given concerning the build up of noise in platoons of vehicles. Finally it is pointed out that the "speed noise" or average of the absolute relative speed between adjacent vehicles is a minimum when the flow is a maximum.

I. INTRODUCTION

During the past few years a considerable amount of research has been carried out in the hope of understanding the comparatively simple traffic situation of single lane flow where passing is not permitted. This situation is of practical interest since, for example, it can exist temporarily on dual and multilane highways when the concentration of vehicles is sufficiently high. Furthermore, since the traffic in most vehicular tunnels is regulated, single lane traffic flow exists continuously for all concentrations. It is hoped that the general knowledge gained through studies of this relatively simple situation will provide a possible basis for the understanding of even more complex problems in so far as traffic is concerned.

One of the principal efforts in this area has been that of trying to understand the behavior of the single lane traffic stream from the detailed manner in which individual vehicles follow one another. This approach, which might be termed a microscopic one, has been frequently called the car following model of single lane traffic flow. That this point of view is capable of describing the steady-state stream of traffic was initially indicated by Gazis, Herman and Potts¹ and further substantiated by car following experiments conducted in the Lincoln and Holland Tunnels which were reported by Herman and Potts².

Another principal effort has been concerned with what are termed macroscopic variables. These variables are concerned with the overall motion-properties of the traffic stream. Examples of such variables are flow—the number of vehicles passing a given point on the roadway per unit time; the concentration—the number of vehicles occupying a given length of roadway; and the speed of the traffic stream. The well known paper by Lighthill and Whitman³, oriented from this latter point of view, describes traffic flow as a compressible fluid. Greenberg⁴ has also suggested that the traffic stream might be approximated by a continuous fluid where the speed of the fluid depends only on the concentration.

From all of these studies it has become clear that a considerable amount of accurate experimental data would be needed if the various facets of this problem were to be understood. In the past, experiments had been conducted which provided data in a form so laborious to reduce as to seriously limit their value. Recently, Edie, Foote, Herman and Rothery⁵ have reported a series of observations that consists of a large sample of data obtained in the Holland Tunnel of New York City with new instrumentation. This instrumentation was developed for the purpose of obtaining vehicle information on the roadway in such a form that it can be readily converted to a digital computer format. A detailed description of this instrumentation has already been reported in the literature⁶.

It is the purpose of the present paper to discuss a relatively large sample of data from the microscopic point of view of the follow-the-leader model of single lane traffic and to indicate to what extent it gives a consistent description of the traffic stream.

II. CAR FOLLOWING ANALYSIS

The theory of the car following model of single lane traffic—flow with no passing, attempts to describe the traffic stream by assuming that each driver follows the vehicles ahead in some specific manner. The basic equations of the theory express the idea that each driver—vehicle complex behaves in a manner that can be described by a stimulus-response equation of the form

$$\text{Response } (t+T) = \text{Sensitivity} \times \text{Stimulus } (t). \quad (1)$$

Several such car following models have been proposed.^{1,2,7,8} Common to all of these is that the response has been taken as the vehicle acceleration, since drivers have direct control over this variable. Furthermore, it has been established through numerous experiments that there exists a relatively high correlation between the acceleration of a given vehicle following another and the relative speed between this vehicle and the one ahead. Therefore, to a first approximation the stimulus-response equation which can be employed to describe the manner in which the $(n+1)^{\text{th}}$ vehicle follows the n^{th} vehicle in a single lane of traffic is

$$\frac{d^2 x_{n+1}(t+T)}{dt^2} = \lambda \left[\frac{dx_n(t)}{dt} - \frac{dx_{n+1}(t)}{dt} \right]. \quad (2)$$

In this equation, $dx_n(t)/dt$, and $d^2 x_n(t)/dt^2$ are the speed and acceleration of the n^{th} vehicle at time t . The time lag of the driver-vehicle complex is T and λ is the sensitivity. The essential difference between most of the models that have been proposed is in the functional that has been taken for the sensitivity. Chandler, Herman and Montroll⁹ initially took λ a constant for mathematical simplicity. Later Gazis, Herman and Potts¹ considered a functional where the sensitivity was inversely proportional to the spacing between the vehicles. Thus the further the vehicles are separated, the smaller is the response for a given stimulus. In addition to the spacing effect Edie⁷ suggested a form which also took into account the effect of the absolute speed of the following vehicle by assuming that the sensitivity of the driver varied in proportion to this absolute speed. The sensitivity function was assumed to be proportional to

$$s_{n+1}^{-2}(t) \frac{dx_{n+1}(t+T)}{dt},$$

where $s_n(t)$, the spacing between the vehicles, is given by $[x_n(t) - x_{n+1}(t)]$.

Furthermore, while Greenshields¹⁰ did not suggest a car following model, he did postulate on the basis of relatively limited data a straight line relationship between the concentration and the speed. This relationship can be derived from a car following law as shown by Herman and Potts² if one assumes for the sensitivity the following expression

$$s_n^{-2}(t).$$

where again $s_n(t)$ is the spacing. All four of the functionals discussed above are special cases of a more general function, namely

$$\lambda = \frac{a_{l,m}}{s_n^l(t)} \left[\frac{dx(t+T)}{dt} \right]^m, \tag{3}$$

where $a_{l,m}$ is a constant, called the sensitivity coefficient, whose value depends on the exponents l and m . Using the above expression for the sensitivity a more general equation for car following is expressed by

$$\frac{d^2x_{n+1}(t+T)}{dt^2} = \frac{a_{l,m}}{s_{n+1}^l(t)} \left[\frac{dx_{n+1}(t+T)}{dt} \right]^m \left[\frac{dx_n(t)}{dt} - \frac{dx_{n+1}(t)}{dt} \right]. \tag{4}$$

This equation is, course, only an idealized approximation which attempts to describe the manner in which real driver-vehicle systems interact on a single lane highway with no passing. However, experiments have shown that even the model expressed by Eq. (4) for $l=m=0$, i. e. constant sensitivity, is a relatively good approximation to reality. The simple general car following equation also of considerable interest in that it is relatively easy to obtain a fairly complete description of steady state traffic conditions for particular values of l and m . Gazis, Herman and Rothery⁸ have discussed in detail a number of these steady state equations in the light of available experimental car following data. They have shown that the relationship that results from the integration of Eq. (4) is of the form

$$f_m(x) = a_{l,m} f_l(s) + c'' \tag{5}$$

where a is the steady state speed of the traffic stream, s is the average vehicle spacing (including the vehicle length) and c'' is an appropriate constant consistent with the physical restrictions. The function $f_p(x)$ where p is equal to l or m is defined by

$$f_p(x) = x^{1-p}, \text{ for } p \neq 1. \tag{6}$$

and

$$f_p(x) = \ln x, \text{ for } p = 1. \tag{7}$$

The constant c'' is related to either a "free mean speed", u_f or a "jam spacing" s_j , depending on the particular values of l and m .

Since the average vehicle spacing is the reciprocal of the concentration we can obtain by using Eq. (5) the various relationships between the speed of the traffic stream, the concentration and the flow, q . We note that in the steady state the flow is defined as

$$q = uk. \quad (8)$$

Even though considerably more car following experiments need to be carried out over a wide range of speeds and spacings, three particular models appear to be of significance. These are the reciprocal spacing model for which $l=1$ and $m=0$; the reciprocal-spacing-speed model ($l=2$ and $m=1$); and the reciprocal square spacing model ($l=2$ and $m=0$). It is not altogether clear at present which, if any, yields the best description of single lane traffic. However, it has been shown that these models do exhibit a relative superiority over other models in their correlation with the car following data.

We have analyzed the present data, collected from a point on the roadway, in order to ascertain whether the above three models give a consistent description of traffic from the macroscopic point of view. The equations of steady state that result from the reciprocal spacing model ($l=1, m=0$) are

$$u = a_{1,0} \ln(k_j/k), \quad (9)$$

and the corresponding flow equation

$$q = a_{1,0} k \ln(k_j/k). \quad (10)$$

As mentioned earlier, Greenberg proposed that the traffic stream might be described as a fluid confined to move in one direction and whose speed was assumed to be a function of the concentration only. In this way he obtained the following formula for the speed as a function of the concentration

$$u = c \ln(k_j/k). \quad (11)$$

Equation (11) can be identified with the steady state result in Eq. (9) if we let

$$c = a_{1,0}. \quad (12)$$

The sensitivity coefficient, $a_{1,0}$ and c have the dimensions of speed. This speed is the speed at which the flow is a maximum, frequently referred to as the characteristic speed.

The reciprocal spacing-speed model ($l=2, m=1$) gives the following steady state equations

$$u = u_f e^{-k/k_m} \tag{13}$$

and

$$q = u_f k e^{-k/k_m}, \tag{14}$$

where u_f is the "free mean speed" and k_m is the concentration when the flow is a maximum. In this case, the sensitivity coefficient is given by

$$a_{2,1} = k_m^{-1}. \tag{15}$$

From the inverse square spacing model ($l=2, m=0$) we obtain the steady-state equations

$$u = 2c(1 - k/k_j), \tag{16}$$

and

$$q = 2ck(1 - k/k_j), \tag{17}$$

where again, as in the reciprocal spacing model, c and k_j are the speed at maximum flow and the "jam concentration", respectively. The sensi-

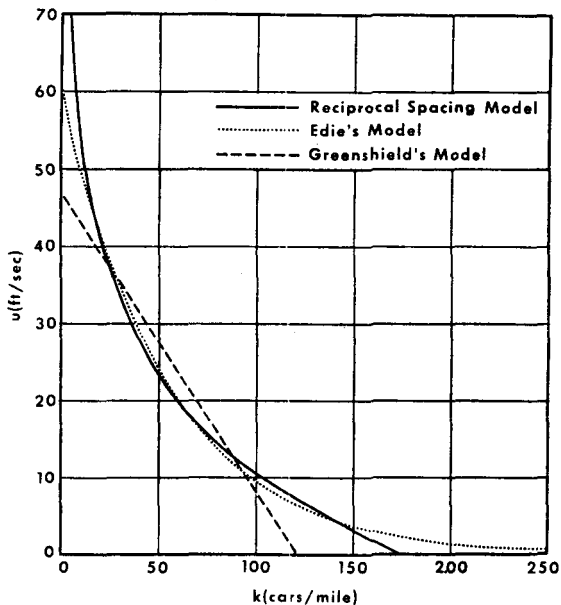


Fig. 1. Speed (ft/sec) versus vehicle concentration (cars/mile). The curves correspond to the steady state speed-concentration relationships for the various indicated models.

tivity coefficient for this model is denoted by $a_{2,0}$ and is given by

$$a_{2,0} = 2ck/j. \quad (18)$$

The speed vs. concentration relations for the three models discussed above are given by Eqs. (9), (13) and (16) and are shown in Fig. 1.

III. COMPARISON OF THEORY AND EXPERIMENTS

These three sensitivity coefficients defined in terms of macroscopic parameters by Eqs. (11), (15) and (18) can be calculated directly from car following experiments involving only two cars*. From actual car following experiments conducted in the past in the Holland Tunnel numerical values for the sensitivity coefficients have been determined. This can be accomplished by making correlation studies of the measured variables appearing in the general car following relationship in Eq. (4) taking into account the fact that there is a time lag which can also be determined. The values obtained are as follows:

$$\begin{aligned} a_{1,0} &= c = 26.8 \text{ ft/sec} = 18.2 \text{ miles/hr.} \\ a_{2,1} &= k_m^{-1} = (123 \text{ cars/mile})^{-1} \\ a_{2,0} &= \frac{2c}{k_j} = 0.57 \text{ ft}^2/\text{sec} \end{aligned} \quad (19)$$

It is also possible to determine these constants according to the macroscopic point of view from the data obtained on many vehicles at a point on the roadway.^{5,**} We have examined from this sample all vehicles whose speeds fall in a small speed range, say Δu at u . These vehicles form a virtual steady state in that each vehicle is essentially travelling with the same speed so that the speed of this particular virtual traffic stream is just u . In this virtual state the concentration is then the sum of the individual spacings of N vehicles moving with this speed divided into N . Thus, the concentration k is given by

$$k = N \left[\sum_{n=1}^N s_n \right]^{-1}, \quad (20)$$

or

$$k = N \left[\sum_{n=1}^N k_n^{-1} \right]^{-1}, \quad (21)$$

* For a detailed account of these experiments see reference 2.

** A more complete discussion of a large sample of data taken in the Holland Tunnel is in preparation by the authors and L. C. Edie and R. S. Foote.

i. e., the concentration of the stream is the harmonic average of the individual concentration of each vehicle, k_n , defined by

$$k_n = \frac{1}{s_n}. \tag{22}$$

TABLE I

Speed (ft/sec)	Average Spacing (ft)	Concentration (cars/mile)	No. of Vehicles
7	40.9	129.0	22
9	42.9	123.2	58
11	48.5	108.9	98
13	51.0	103.5	125
15	57.0	92.6	196
17	59.4	88.8	293
19	62.5	84.6	436
21	65.6	80.4	656
23	68.4	77.2	865
25	74.9	70.5	1062
27	78.1	67.6	1267
29	84.6	62.4	1328
31	88.6	59.6	1273
33	92.4	57.2	1169
35	100.0	52.8	1096
37	107.2	49.3	1248
39	112.3	47.1	1280
41	112.6	43.1	1162
43	143.9	36.7	1087
45	143.4	36.8	1252
47	158.1	33.4	1178
49	181.5	29.1	1218
51	187.3	28.2	1187
53	201.5	26.2	1135
55	238.3	22.2	837
57	250.4	21.1	569
59	282.4	18.7	478
61	257.7	20.5	291
63	294.5	17.9	231
65	334.7	15.8	169
67	342.2	15.4	55
69	401.7	13.1	56

Furthermore, since this set of vehicles forms a virtual steady state, the equation that the flow q is given by $q=uk$ is not a mere definition of k but an exact relation for these virtual states.

This treatment of the data consisting of 24,000 vehicles has been carried out for speed classes incremented every 2 ft/sec. In Table I we have the mean spacing, the corresponding concentration, and the number of vehicles for each speed range. The speed-concentration data is shown

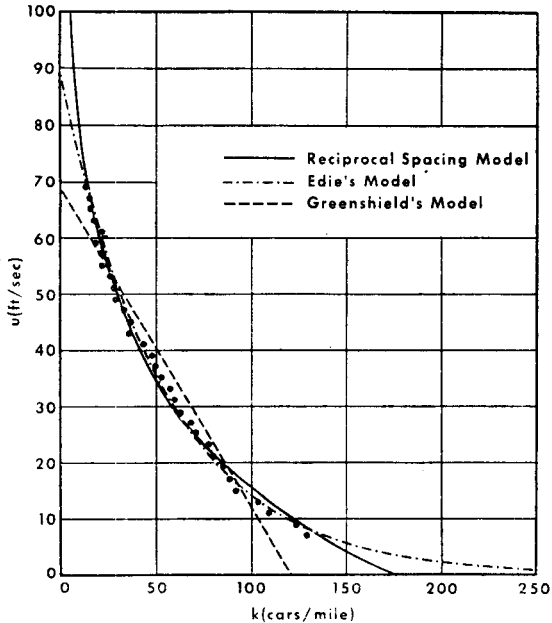


Fig. 2. Data on speed (ft/sec) versus vehicle concentration (cars/mile) plotted together with the three speed-concentration car following relations of Fig. 1.

graphically in Fig. 2 together with the three speed-concentration car following relations of Fig. 1. The data in this figure extends over a considerably larger range than has been previously published. These analytic curves are "least square" fits of the data when the speed u is plotted against $\ln k$ in the case of the reciprocal spacing model; $\ln u$ against k , in the case of the reciprocal spacing speed model; and u against k , in the case of the inverse square spacing model.

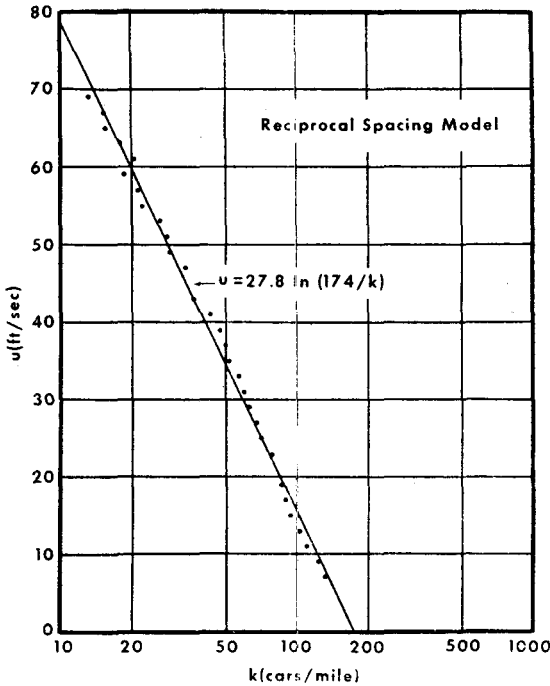


Fig. 3. Speed (ft/sec) versus vehicle concentration (cars/mile). The concentration has been plotted on a logarithmic scale. The solid line is a "least squares" fit of the reciprocal spacing model to the observed data.

How well the data fits the reciprocal spacing and the reciprocal spacing speed models can be seen more easily in Figs. 3 and 4. In Fig. 3 the reciprocal spacing model is compared with the data. Here each speed class has been plotted against the corresponding concentration which has been plotted on a logarithmic scale. The values of the two parameters for this model obtained from a "least squares" fit are

$$a_{1,0} = 18.95 \text{ miles/hr.}, \tag{23}$$

and

$$k_j = 174 \text{ cars/mile.} \tag{24}$$

It is gratifying to note that the numerical value obtained for the sensitivity coefficient of the reciprocal spacing model, $a_{1,0}$, agrees very well

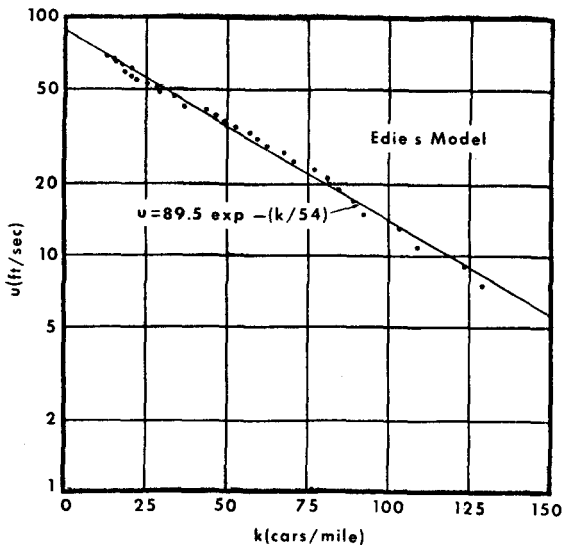


Fig. 4. Speed (ft/sec) versus vehicle concentration (cars/mile). The speed has been plotted on a logarithmic scale. The solid line is a "least squares" fit of the reciprocal-spacing-speed model to the observed data.

with the characteristic speed determined by car following experiments conducted in the Holland Tunnel, namely, 18.2 miles/hr.

Figure 4 has been plotted in order to indicate how the data fits the reciprocal-spacing model. Here each speed class has been plotted on a logarithmic scale against the corresponding concentration. The values of the two parameters for this model, again from a "least squares" fit, are

$$u_f = 89.5 \text{ ft/sec} = 61.0 \text{ miles/hr,} \quad (25)$$

and

$$k_m = 54 \text{ cars/mile.} \quad (26)$$

From a "least square" fit of the data to the inverse square spacing model we obtain for the numerical values of the parameters c and k_j the following:

$$c = 34.5 \text{ ft/sec} = 23.5 \text{ miles/hr.,} \quad (27)$$

and

$$k_j = 120.5 \text{ cars/mile.} \quad (28)$$

It would appear from the reciprocal-spacing-speed model that we

do not obtain experimental values for $a_{2,1}$ and k_m^{-1} which agree closely. However, the model does predict that the characteristic speed, c , is essentially the same as that given by the reciprocal-spacing model. In the former case the characteristic speed is just $1/e^{\text{th}}$ (≈ 0.368) of the "free mean speed" u_f , i. e.,

$$c = 0.368 \times (89.5 \text{ ft/sec}) = 32.9 \text{ ft/sec.} \tag{29}$$

Similarly, we can calculate a semi-empirical value for k_m from the reciprocal spacing model since the concentration at maximum flow is $1/e^{\text{th}}$ of the "jam concentration" k_j , i.e.,

$$k_m = 0.368 \times 174 \text{ cars/mile} = 64 \text{ cars/mile.} \tag{30}$$

Table II summarizes the values of c , k_j , k_m , and $a_{l,m}$ for each of three models that have been discussed above.

TABLE II
Summary of Experimentally Determined Macroscopic Parameters

Model	Sensitivity Coefficient	
Reciprocal Spacing	$a(1, 0) = c = 18.95 \text{ miles/hr}$	
Reciprocal Square Spacing	$a(2, 0) = 2c/k_j = 0.120 \text{ ft}^2/\text{sec}$	
Reciprocal Spacing-Speed	$a(2, 1) = k_m^{-1} = (54 \text{ cars/mile})^{-1}$	
$c(\text{Miles/hr})$	$k_j(\text{cars/mile})$	$k_m(\text{cars/mile})$
18.95	174	64.0
23.5	120.5	60.3
22.8	---	54.0

The original motivation for the reciprocal-spacing-speed model was to attempt to describe low concentration, non-congested traffic. The parameter in this model is the "mean free speed", u_f , i.e., the traffic stream speed as the concentration goes to zero. Edie compared this model with observational data obtained in the Lincoln Tunnel in the concentration range from zero up to a value of 90 cars/mile. The reciprocal spacing model was used for concentrations exceeding this value. Figure 5 shows a comparison of the experimental results with the reciprocal-spacing-speed model in the two ranges of low (0-45 cars/mile) and high concentrations (those exceeding 45 cars/mile). The straight lines are "least square" fits to the data when $\ln u$ is plotted against against k . The reciprocal spacing

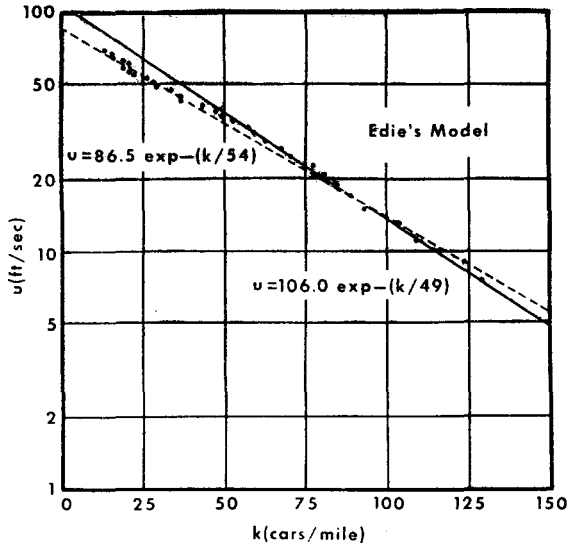


Fig. 5. Speed (ft/sec) versus vehicle concentration (cars/mile). The speed has been plotted on a logarithmic scale. The straight lines are "least square" fits to the observed data. The solid line has been fitted to the data for the concentration range less than 45 cars/mile and the dashed line for the concentration range exceeding 45 cars/mile.

model is similarly compared with the data for these two concentration ranges and is shown in Fig. 6. Again, the straight lines are "least square" fits to the data when the speed u is plotted against $\ln k$. It would appear from Figs. 5 and 6 that it would be difficult to determine which model fits the data more closely than the other in either range of data. Of course, the two-curve fits, be it either model, is a better fit than any one

TABLE III
Correlation Coefficients

Model	Concentration Range		
	0-45	45-130	0-130
Reciprocal Spacing	-0.993	-0.997	-0.996
Reciprocal Square Spacing	-0.993	-0.982	-0.941
Reciprocal Spacing Speed	-0.997	-0.987	-0.968

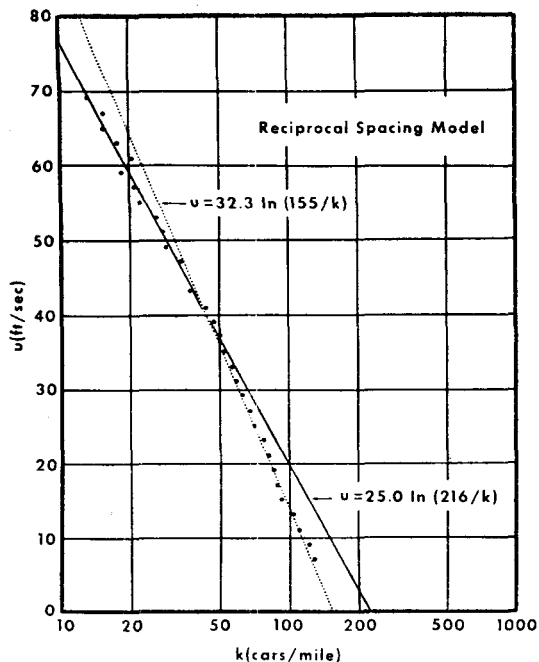


Fig. 6. Speed (ft/sec) versus vehicle concentration (cars/mile). The concentration has been plotted on a logarithmic scale. The straight lines are "least squares" fits to the observed data. The solid line has been fitted to the data for the concentration range less than 45 cars/mile and the dotted line for the concentration range exceeding 45 cars/mile.

curve fitted over the entire range of data. These remarks are further substantiated by Table III where the correlation coefficients are given. These have been calculated from the data for the entire range and for the high and low concentration ranges separately. Even though these difficulties exist it does seem worthwhile to call attention to the apparent discontinuity in the derivative of the speed-concentration curve. It should be pointed out that this discontinuity is of a different kind from what has been mentioned elsewhere. Previously it had been remarked that there was an apparent break in the virtual flow vs. concentration curve near maximum flow where the flow apparently dropped suddenly at a concen-

tration near k_m . This kind of discontinuity suggests that the $u-k$ curve is discontinuous. However, the data shown in Figs. 5 and 6 implies that

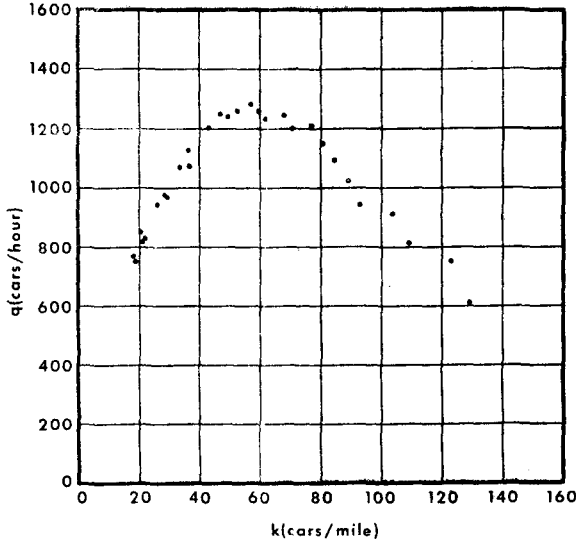
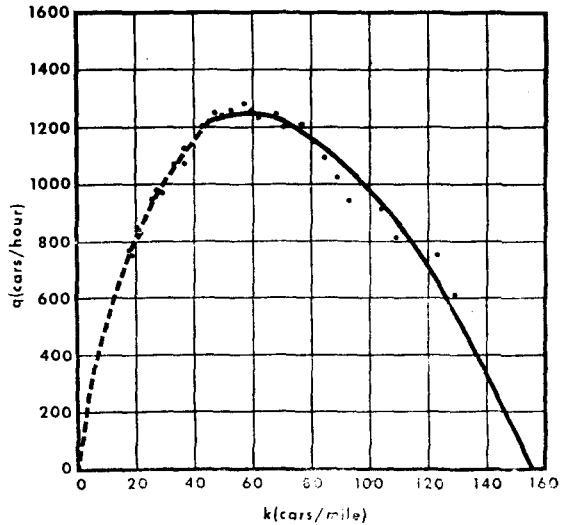


Fig. 7. Virtual flow in Holland Tunnel (cars/hour) versus vehicle concentration (cars/mile). The virtual flow is calculated as the product of the average concentration and the speed of the class in which it lies.

Fig. 8. Virtual flow (cars/hour) versus vehicle concentration (cars/mile). The two curves correspond to the steady state reciprocal-spacing model fitted in the low and high concentration ranges. (See Fig. 6)



the $u-k$ curve is continuous but that its derivative is not.

The experimental virtual flow points are shown in Fig. 7. The virtual flow is that which corresponds to the virtual steady states that have been created by our division of the data into speed classes and is derived from the steady state equation, $q=uk$.

From Fig. 7 it would appear that if there is a real discontinuity in the flow vs. concentration curve near the optimum flow point, it is considerably smaller for the Holland Tunnel than has been suggested for the Lincoln Tunnel⁷. The experimental points in Fig. 8 are shown together with the semi-empirical flow curve obtained from the fit of the reciprocal-spacing model, Eq. (9) in the two regions of high and low concentration. The apparent discontinuity of the speed-concentration curve is reflected in this flow diagram. This again suggests* that single lane traffic has a bimodal character. At least, it would appear that the results from

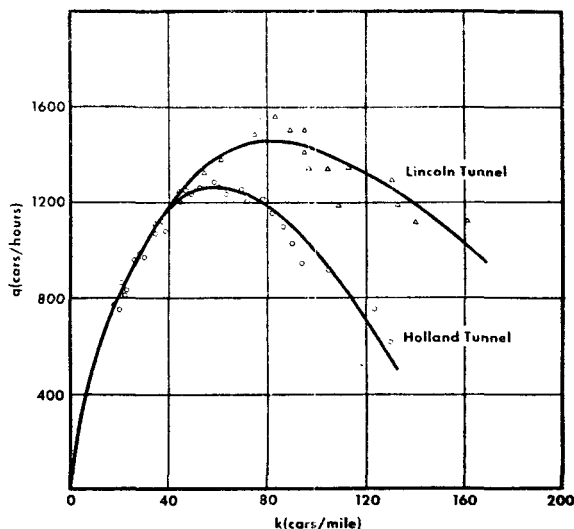


Fig. 9. Flow (cars/mile) versus vehicle concentration (cars/mile) for the Lincoln and Holland Tunnels. The Lincoln Tunnel data are those of Greenberg (reference 4). The two solid curves correspond to "least squares" fits of the reciprocal-spacing model to the data.

* See reference 8.

this large sample are not at variance with this idea.

It is interesting to compare this data from the Holland Tunnel with that from the Lincoln Tunnel reported by Greenberg.⁴ In Fig. 9 we have plotted both sets of data. Both sets of observations have been obtained essentially under the same type of conditions and location with respect to the configuration of the tunnel. It is striking that the data from the Holland Tunnel and the data from the Lincoln Tunnel in the concentration range of 0—45 cars/mile follow essentially the same curve. The main difference appears to be in the breaking point where the interactions are sufficiently strong to significantly change the linearity of the flow when the concentration is further increased. Furthermore, this point for the Holland Tunnel data occurs at approximately the same concentration as the discontinuity occurs in the speed-concentration curve, Fig. 6. The two branches for the higher densities show the marked differences in the traffic behavior in these two tunnels. It would be interesting to see whether the Queen's Mid-Town Tunnel follows this same behavior in the low concentration range. It is known that this tunnel can maintain higher flow levels than either the Lincoln or Holland Tunnels. One might therefore expect that the breaking point would occur at a higher concentration.

IV. ACCELERATION NOISE

In an earlier paper on car following it was suggested that acceleration noise might be used as a means of describing quantitatively the nature of traffic with respect to driver-car-road characteristics. Acceleration noise can be defined as the dispersion in the acceleration distribution function obtained over a given period of time.

Although the results of some experiments related to this subject have been reported, a considerable amount of experimentation needs to be carried out. It is of particular importance to determine the acceleration noise of vehicles as a function of position in a platoon. Some measurements of this kind have been made in the Holland Tunnel.

A control vehicle which maintained constant speed within one mph throughout its transit of the tunnel was injected into the traffic stream and a fixed number of customer vehicles, selected at random, were allowed to follow this control vehicle. The instrumented vehicle then completed the platoon under study. The instrumented vehicle was positioned fifth,

eighth, and tenth in line in the platoon, and the acceleration was recorded. The acceleration noise for these different cases was essentially a constant having the numerical value of approximately $0.035g$. This experiment was duplicated on the test track at the General Motors Technicl Center for platoons containing two, three and four vehicles. The results of these two experiments are shown in Fig. 10 where the acceleration noise has been plotted against vehicle position in the platoon. The curve shown is a visual fit to the data. This indicates that acceleration noise builds up rapidly down the line of vehicles and only requires the interaction of a few vehicles for saturation.

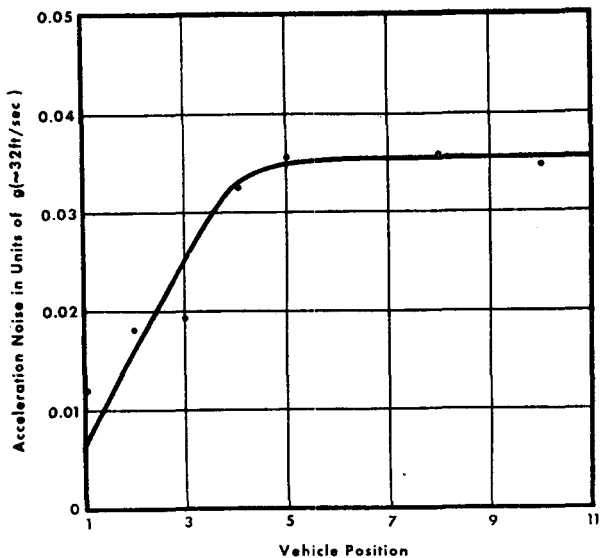


Fig. 10. Acceleration noise defined as the dispersion in the acceleration distribution *versus* vehicle position in a platoon.

In the light of car following experiments and analysis, one might also use the relative speeds between pairs of vehicles as a measure of the noise in a traffic stream particularly since it has been determined that there is a very high correlation between the acceleration and relative speed with respect to the vehicle ahead. In this sense one can define traffic stream noise measured at a point on the roadway as the average of the

absolute values of the relative speeds. We have calculated this parameter as a function of speed classes. It is interesting to note that this speed noise level as defined above is a minimum at approximately the same speed as the characteristic speed, that is to say, it appears that maximum flow occurs when the speed noise is a minimum.⁵ This is in agreement with a suggestion made some time by the Committee on Highway Capacity¹¹ that when the average difference in speed between successive vehicles approaches zero the traffic volume reaches the possible capacity of the facility.

V. CONCLUDING REMARKS

The main discussions in the present paper have been aimed at relating specific car following models to the overall flow characteristics of traffic and to specific car following experiments. While there is no doubt that a strong correlation exists one cannot as yet differentiate unequivocally between the specific models discussed in this paper. It is hoped that with further study aided by improved instrumentation it will be possible to eventually differentiate between the merits of the various microscopic models that have been suggested.

A considerable amount of work still remains to be done concerning the various aspects of car following. For example it would be interesting to determine the influence of forward next nearest neighbors as well as the influence of backward vehicles on the behavior of the driver. In this connection we have been studying car following by means of an instrumented three car platoon on which we expect to report in the near future.

In conclusion we should like to state that we feel it is quite gratifying that it has been possible to determine any connection whatsoever between the micro and the macro worlds of traffic.

ACKNOWLEDGEMENTS

We wish to thank Mr. George Gorday for his invaluable assistance in many areas. Also we acknowledge the computing assistance of Mrs. Joyce Krull and finally wish to thank the Port of New York Authority for their kind cooperation.

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