

HOW TO DECIDE THE NUMBER OF CIRCULATING SPARE PARTS TO BE KEPT AT SHEDS AND RAILWAY WORKSHOPS

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1. INTRODUCTION

Under the jurisdiction of Kokura Workshop of Japanese National Railways, there are totally twenty-nine locomotive and car sheds, as regards the repair of circulating spare parts.

Each shed, which is in charge of maintenance and operation of rolling stock assigned, has sufficient number of spare parts, so that any parts of locomotives or cars in service, when found defective, can be quickly replaced by those in good condition to maintain the working efficiency of the rolling stock as high as possible. Spare parts that are stored at sheds for this purpose are called "shed spare."

However, as sheds have no persons nor facilities to repair, all defective parts are sent to the workshop, where they are concentratedly repaired. Therefore, the workshop also has a certain number of spare parts so that it can supply to sheds same kinds and quantities of parts in good condition immediately when those defective arrived from sheds. Those spare parts kept at the workshop are named "workshop spare."

Freight trains are regularly operated certain times every week between the workshop and each of sheds in order to transport exclusively those defective or completed parts, and they are called "delivery train."

This treatise is to show the methods of distributing spare parts from the workshop impartially to each of sheds, considering their different factors, and of determining the most economical quantities of workshop spares.

2. CIRCULATION OF SPARE PARTS

As regards the movement of circulating spare parts, we can consider a cycle shown in Fig. 1, where capital letters indicate respec-

tively: S , storehouse of a shed; T , transporting process of defective parts by delivery trains from a shed to the workshop; U , transporting process of repaired parts by delivery trains from the workshop to a shed; X , repairing process at the workshop; Y , repair-waiting process of defective parts at the workshop; Z , dispatch-waiting process of repaired parts at the workshop; and s, t, u, x, y and z mean the numbers of the parts existing at a random instant in processes S, T, U, X, Y and Z respectively.

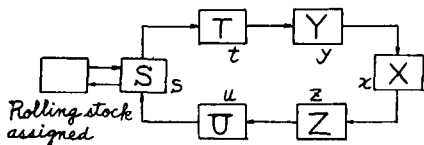


Fig. 1. Circulation of spare parts

When some part of rolling stock in service has got defective, it is exchanged by one of the spares prepared in the storehouse S , so that the rolling stock may work continuously. The defective parts removed from the rolling stock is sent by a delivery train and arrives at the workshop after passing the transporting process T . When the workshop receives some defective parts from sheds, it dispatches, in return, the same number of repaired ones, picking them out of those in dispatch-waiting process Z . When there is no corresponding repaired parts in process Z , this action is taken immediately after necessary number of repaired spares have come in the dispatch-waiting process Z . Therefore, there is no possibility that repaired parts which are greater in number than those defective sent from sheds are delivered from the workshop.

Thus, letting the number of shed spares be symbolized by n , we get

$$s + t + u \leq n \quad (1)$$

We call the state where there is no repaired parts in the dispatch-waiting process Z "extraordinary state," and otherwise "ordinary state." In an extraordinary state, the sign of equality in formula (1) must be taken off, and it, therefore, becomes

$$s + t + u < n \quad (2)$$

In this case, the same number of parts as the difference between the right and left sides of (2) are existing in the repair-waiting process Y and or the repairing process X . Let the number of those parts be indicated by q , then the following equality (3) is always true.

$$s + t + u + q = n \quad (3)$$

When we consider the number of shed spares, one of the most

important facts is that, although n pieces of spares are provided to a shed, the effective number is only s which is almost always smaller than n , and other t , u and q pieces of parts that are outside the storehouse S cannot be available as effective spare parts for immediate use.

When we set an equality

$$t+u+q=v \quad (4)$$

and regard that V means going and returning transporting process, hereinafter referred to "transporting process," taking dispatch delay into consideration in an extraordinary state, and v means number of parts to be involved in the transporting process V at an instant, transporting process of defective parts T and transporting process of repaired parts U in Fig.

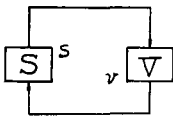


Fig. 2. Circulation on shed side

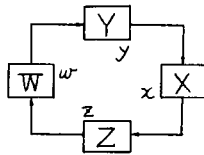


Fig. 3. Circulation on workshop side

1 are connected with each other, making a new "transporting process" V . Fig. 1, therefore, is divided into circulation on shed side and one on workshop side, which are shown by Fig. 2 and 3, both being very simple. Then

the model of the problem becomes more tractable as a result.

3. THE PROBABILITY OF OUT OF STOCK FOR SHED SPARES

In determining the quantities of shed spares, we have to take notice about the fact that each shed has the following three different factors: (1) Number of the rolling stock assigned. (2) Distance from the workshop. (3) Frequency of delivery trains operated to and from the workshop.

In order to know the way to distribute the most reasonable number of spares to each of sheds impartially, taking into consideration the above mentioned three factors as well as the effect of delayed return of repaired parts from the workshop, we study at first the probability of out of stock for shed spares.

Let d be number of days required for a delivery train to make a round trip between the workshop and a shed, ξ be number of days from when a part gets defective till when a delivery train, on which the defective part is loaded, departs from the shed, ζ be number of bays of delay in dispatching a part from the workshop in an extraordinary state, a be frequency of delivery train operation per week, and γ be number

of days from when a part gets defective till when it comes back to the shed, being repaired. ξ , ζ and η are random variables, while d and a are constants for each shed.

Then, referring to Fig. 4, we get

$$\eta = d + \xi + \zeta \quad (5)$$

A defective part is exchanged by a repaired parts kept in the storehouse of the shed S , and is put into the transporting process V . The defective part comes back to the shed being repaired

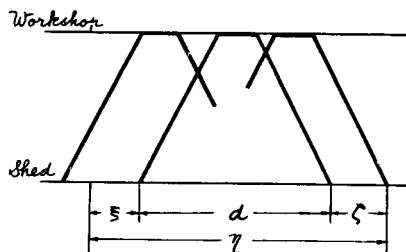


Fig. 4. Diagram of delivery trains

η days after it got defective. Since it may be assumed within the degree of actual cases that there is no limit to the loading capacity of a delivery train for parts, transporting process V can be regarded as a repairing factory having capacity to repair a defective part in η days and to deal with infinite number of parts simultaneously. The number of delivery train operation a is a constant ($a=1, 2$ or 3), which is particular at each shed. As there are seven days in a week, intervals between operations are $6/a$ days for $a-1$ times and $6/a+1$ days for once, e. g., in case of a shed whose a is 3, delivery trains are operated twice with interval of two days and once with that of three days. It is supposed that happenining of defective parts is at random in reference to time. Therefore, in case where defective parts happen once a week in average, it can be considered that on the average defective parts happens $1/7$ time every day. $f(\xi)$ that stands for the probability density function of ξ can be derived from the fact mentioned above as follows;

$$\left. \begin{aligned} f(\xi) &= a/7 \text{ when } 0 < \xi \leq 6/a \\ &= 1/7 \text{ when } 6/a < \xi \leq 6/a + 1 \\ &= 0 \text{ when } 6/a + 1 < \xi \end{aligned} \right\} \quad (6)$$

$\bar{\xi}$ which is the mean value of ξ is, therefore,

$$\begin{aligned} \bar{\xi} &= \int_0^{\infty} \xi f(\xi) d\xi = \int_0^{6/a} \frac{a}{7} \xi d\xi + \int_{6/a}^{6/a+1} \frac{1}{7} \xi d\xi \\ &= (a+48)/14a = 3.43/a + 0.07 \end{aligned} \quad (7)$$

Let the average of η be indicated by $\bar{\eta}$, the average of ζ by $\bar{\zeta}$ and the reciprocal of $\bar{\eta}$ by μ and we have

$$1/\mu = \bar{\eta} = d + \bar{\xi} + \bar{\zeta} = d + 3.43/a + 0.07 + \bar{\zeta} \quad (8)$$

It was observed at our workshop that the number of defective parts happenings comes under Poisson distribution, and the result of χ^2 test also confirmed the fact. Therefore, if we define λ as the average number of happenings of defective parts per day, the transporting process V with Poissonian input of parameter λ is regarded as a group of infinite channels, each of which has the average holding time symbolized by $1/\mu = \eta$. In such a case, $P_\tau(v)$ which means the probability that v pieces of parts exist in transporting process V at a random instant τ has already been proved to be, in the form of its converged value $P(v)$, as follows;

$$\lim_{\tau \rightarrow \infty} P_\tau(v) = P(v) = e^{-\rho} \frac{\rho^v}{v!} \quad (9)$$

where

$$\rho = \lambda / \mu \quad (10)$$

Since the number of spares provided to a shed is n , P_s which signifies the probability of out of stock for shed spares is represented by

$$P_s = \sum_{v=n+1}^{\infty} P(v) = \sum_{v=n+1}^{\infty} e^{-\rho} \frac{\rho^v}{v!} \quad (11)$$

and ρ in (11) is obtained from (8) and (10) as follows;

$$\rho = \lambda / \mu = \lambda(d + 3.43/a + 0.07 + \zeta) \quad (12)$$

4. THE PROPER QUANTITIES OF SHED SPARES

Formula (12) gives us a simple but very important suggestion. It is noticed from (12) that the larger λ , d and ζ are, and the smaller a is, then the larger ρ becomes. Therefore, if we decide n , the number of shed spares, by taking into consideration P_s of each shed calculated by (11) and (12), we succeed to supply the most proper quantities of spares impartially to each of many sheds having complicated characters. The larger λ is, the longer the distance from the workshop is and the smaller a is, the larger n becomes. Further, the larger the number of days of delay in dispatching repaired parts from the workshop is, the larger n becomes, too.

Now considering an inequality

$$P_s = \sum_{v=n+1}^{\infty} e^{-\rho} \frac{\rho^v}{v!} < \frac{1}{100} \alpha \quad (13)$$

let the value of the smallest n satisfying (13) be denoted by n_o , and we know n_o is the proper number of shed spares from the view point of making the probability of out of stock for shed spares less than α per cent.

Table 1 represents for each shed the values of d , number of days required for a delivery train to make a round trip between the workshop

Table 1

Shed no.	Name of shed	a	d	$\bar{\xi}$	$\bar{\zeta}$	$\bar{\eta}$
1	Miyoshi	2	13.47	1.79	1.25	16.51
2	Hiroshima	"	10.68	"	"	13.72
3	Ogoori	"	9.31	"	"	12.35
4	Nishikaratsu, and other one	3	9.41	1.21	"	11.87
5	Asa, and other one	2	8.67	1.79	"	11.71
6	Oita, and other two	3	8.57	1.21	"	11.03
7	Iwakuni	2	7.92	1.79	"	10.96
8	Kumamoto, and other five	3	7.87	1.21	"	10.33
9	Wakamatsu	"	7.62	"	"	10.08
10	Nagasaki, and other one	"	7.26	"	"	9.72
11	Moji, and other four	"	6.40	"	"	8.86
12	Nishiyahata	"	5.85	"	"	8.31
13	Tosu	"	5.62	"	"	8.08
14	Nogata	"	4.90	"	"	7.36
15	Mojiko	"	4.47	"	"	6.93

and the shed; a , frequency of delivery train operation per week; $\bar{\xi}$ number of days of averaged delay in returning repaired parts. The relation between λ and n_0 for each shed can be gained by inserting the values of α , a , d and $\bar{\xi}$ into (12) and (13). Fig. 5 shows the case where $\alpha=1$ per cent, in which λ is the average number of happenings of defective parts per year, and shed nos. correspond to those in Table 1.

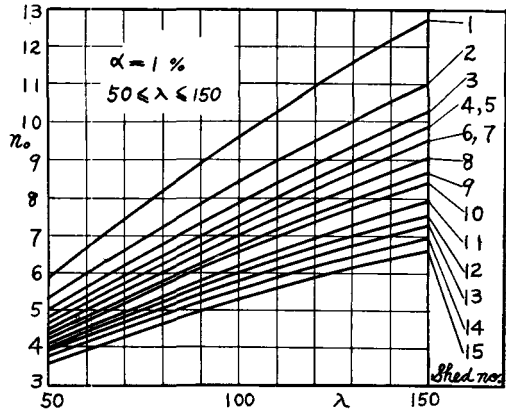


Fig. 5. The relation between n_0 and λ . Shed nos. correspond to those in Table 1.

5. THE PROBABILITY OF OUT OF STOCK FOR WORKSHOP SPARES

In order to make a shed use its provided spares most efficiently and avoid the worst case where necessary parts are not available owing to the lack of spares, it is desirable that, as soon as some defective parts reach the workshop, the same number of repaired ones start for the shed without delay. The greater the number of workshop spares is, the more this requirement is satisfied, while on the contrary, the more spares we have, the larger the cost of having them becomes.

As the first step to find the proper number of workshop spares that minimizes the sum of the losing cost which occurs when there is no repaired parts in the dispatch-waiting process Z and the cost of having workshop spares, we study the probability of out of stock in process Z , adopting the theory of cyclic queues which has already been investigated.

Here the following signs indicate respectively: h , the total number of parts of one kind including both those on rolling stock and spares kept by all of the sheds under the jurisdiction of the workshop; k , the number of channels of repairing process X at the workshop; f , the number of workshop spares; m , the total number of parts kept at all of the sheds and the workshop; W , the shed cycle involving the transporting process V ; w , the number of the parts existing in W at a random instant.

$$\text{We get} \quad m = h + f \quad (14)$$

$$0 \leq w \leq h, \quad 0 \leq x \leq k \quad (15)$$

$$m = x + y + z + w. \quad (16)$$

The number of repairing channels at the workshop is not larger than that of workshop spares, so that we are interested only in the case where

$$f \geq k \quad (17)$$

$$\text{As} \quad m = h + f \geq h + k \quad (18)$$

we adopt equalities defined by

$$m = h + k + l, \quad f = k + l \quad (l \geq 0) \quad (19)$$

Since (15), (16) and (17) show that the case where $y = z = 0$ never happens, we have only to seek the values of, or relations among w , x , y and z in the possible three cases that are shown below.

(i) The case where $y \neq 0$ $z \neq 0$

$y \neq 0$ means the repairing channels are all occupied.

Therefore, $x = k$
 $z \neq 0$ means the ordinary state where there is no dispatch delay,
 then $w = k$

$$\therefore y + z = m - (w + x) = m - (h + k) = l$$

As $y, z \geq 0$, we get

$$0 \leq y, z \leq l$$

(ii) The case where $y \neq 0, z = 0$

By the same reason as stated in (i), we have

$$x = k$$

$$y + w + z = m - x = m - k$$

$$\therefore y + w = m - k = h + l$$

$$\therefore w = h + l - y$$

As $0 \leq w \leq h$, we get

$$l \leq y \leq h + l$$

(iii) The case where $y = 0, z \neq 0$

By the same reason as stated in (i), we have

$$w = h$$

$$x + y + z = m - w = m - h$$

$$\therefore x + z = m - h = k + l$$

$$\therefore x = k + l - z$$

As $0 \leq x \leq k$, we get

$$l \leq z \leq k + l$$

We obtain from above-mentioned (i), (ii) and (iii) the ranges in which the combined values of y and z vary, and moreover the fact that the combined values of x and w are known if combined values of y and z are fixed within the said range. It has also been revealed that $P_t(y, z)$ which signifies the probability of finding y and z pieces of parts at a random instant t in process Y and Z respectively is always nought excepting in cases where combined values of y and z are as indicated by* in Table 2.

$y \backslash z$	0	1	$l-1$	l	$l+k$
0	0	0	0	*	*
1	0					*	0	...	0
...
...
$l-1$	0	*							...
l	*	0							...
...
...
$l+k$	*	0	0

Table 2. Combined value of y and z can exist at the places indicated by*

The number of happenings of defective parts, as stated before, proves to have Poisson distribution. Because the defective parts which arrive at the workshop are the summation of those that have got defective at sheds, it can mathematically be deduced that the distribution of the former is just the same as that of the latter, and this fact was also recognized by actual data at our workshop.

Let the average number of arrival be denoted by λ_0 and the reciprocal of the average repairing time at one of the channels of the repairing process X by μ_0 , supposing the repairing time is exponentially distributed. Thus we are able to apply the theory of cyclic queues to the problem of circulating spare parts represented in Fig. 3. But, in order to make the numerical computation easier, we make an approximation of making h infinite, considering the total number of parts which belong to all sheds is fairly large. In this case $P_t(y, z)$, the probability of state, is shown in terms of $P(y, z)$, its converged value in the stationary state, as follows ;

$$P(l+i, 0) = (\lambda_0/k\mu_0)^{l+i} P(0, l) \quad (20)$$

$$i=1, 2, \dots$$

$$P(0, l+j) = (\mu_0/\lambda_0)^j [k!/(k-j)!] P(0, l) \quad (21)$$

$$j=1, 2, \dots, k-1, k$$

$$P(i, l-i) = (\lambda_0/k\mu_0)^i P(0, l) \quad (22)$$

$$i=0, 1, \dots, l-1, l$$

where $P(0, l)$ can be calculated making use of the fact that the total of probabilities is 1.

Putting β as

$$\beta = \lambda_0/k\mu_0 \quad (23)$$

we will decide $P(0, l)$ first.

$$\begin{aligned} 1 &= \sum_{i=0}^k P(l+i, 0) + \sum_{j=1}^k P(0, l+j) + \sum_{i=0}^{l-1} p(i, l-i) \\ &= P(0, l) \left[\sum_{i=0}^{\infty} \beta^i + \sum_{j=1}^k (1/k\beta)^j k!/(k-j)! \right] \\ &= P(0, l) [K(k, 1/\beta) + 1/(1-\beta)] \end{aligned} \quad (24)$$

where $K(k, 1/\beta) = \sum_{j=1}^k (1/k\beta)^j k!/(k-j)!$

$$= \frac{1}{\beta} + \frac{k-1}{k} \left(\frac{1}{\beta}\right)^2 + \frac{k-1}{k} \cdot \frac{k-2}{k} \left(\frac{1}{\beta}\right)^3 + \dots + \frac{k-1}{k} \cdot \frac{k-2}{k} \dots \frac{1}{k} \left(\frac{1}{\beta}\right)^k \quad (25)$$

From (24) we have

$$P(0, l) = 1/[K(k, 1/\beta) + 1/(1-\beta)] \quad (26)$$

A state where want of parts in the dispatch-waiting process Z is as much as i pieces means the state where $y=l+i$ and $z=0$, and its probability $P(l+i, 0)$ is given by (20). Therefore, the probability of occurrence of extraordinary state where there is no repaired parts can be calculated by (20), (25) and (26).

6. ESTIMATION OF TRAFFIC DENSITY

When we study the proper number of workshop spares, we have to know the value of traffic density $\beta = \lambda_0/k\mu_0$ which includes the average repairing time $1/\mu_0$, the number of repairing channels k and the average number of arrival λ_0 . However, it is impossible to measure the exact value of $1/\mu_0$ for a great many kinds of circulating spare parts. Hence we apply the actually observed distribution of the number of repair-waiting parts y to overcome this difficulty. For the sake of convenience, we put δ as

$$\delta = 1/\beta = k\mu_0/\lambda_0 \quad (27)$$

$P(y)$ which stands for the probability of finding y pieces of parts in the repair-waiting process Y is derived from (20), (21) and (22) as, in the case where $y=0$

$$\begin{aligned} P(0) &= \sum_{j=0}^k P(0, l+j) = P(0, l) + \sum_{j=1}^k (1/k\beta)^j k!/(k-j)! P(0, l) \\ &= [1 + K(k, 1/\beta)] P(0, l) = [1 + K(k, \delta)] P(0, l) \end{aligned} \quad (28)$$

and in the case where $y \geq 1$

$$P(y) = (\lambda_0/k\mu_0)^y P(0, l) = \beta^y P(0, l) = \delta^{-y} P(0, l) \quad (29)$$

That means

$$\begin{cases} P(y) = [1 + K(k, \delta)] P(0, l) & \text{when } y=0 \\ = \delta^{-y} P(0, l) & \text{when } y \geq 1 \end{cases} \quad (30)$$

From (30) we get the theoretical value of $P(y)$ and the distribution of y as well. \bar{y} , the mean value of y becomes, therefore,

$$\begin{aligned} \bar{y} &= \sum_{y=0}^{\infty} y P(y) = \sum_{y=0}^{\infty} y \beta^y P(0, l) = P(0, l) \beta / (1-\beta)^2 \\ &= P(0, l) \delta / (\delta-1)^2 = \delta / (\delta-1) [\delta + (\delta-1) K(k, \delta)] \end{aligned} \quad (31)$$

In the case where $k=1$, for instance, the equality $K(k, \delta) = K(1, \delta) = \delta$ comes out from (25), so that

$$\bar{y} = \delta / (\delta - 1) [\delta + (\delta - 1)\delta] = 1 / \delta(\delta - 1) \quad (32)$$

Therefore,
$$\delta = \frac{1 + \sqrt{1 + 4/\bar{y}}}{2} \quad (33)$$

Especially, when $\bar{y} \gg 1$, it becomes

$$\delta \doteq 1 + 1/\bar{y} \quad (34)$$

The relations between \bar{y} and δ calculated by (31) for the cases where $1 \leq k \leq 10$ and $1 \leq \delta \leq 4$ are represented in Fig. 6.

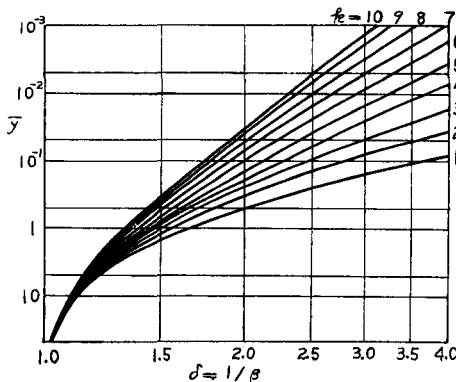


Fig. 6. The relation between $\delta = 1/\beta$ and \bar{y} .

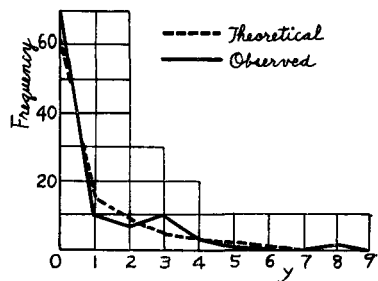


Fig. 7. An example of the theoretical and observed distributions of repair-waiting parts

Fig. 7 shows an example which puts together the theoretical and observed distributions of y . As χ^2 test displayed no significance between them, the consideration described above may be taken for reasonable.

7. THE MOST ECONOMICAL NUMBER OF WORKSHOP SPARES

Let "one transition" mean the interval between arrivals of defective parts, between completions of repairing, or between completion of repairing and arrival of defective parts. When there are no repaired parts in the dispatch-waiting process Z , the workshop has to take special action in order to comply the request of sheds. Therefore, if we regard the losing cost in this case as the expenditure symbolized by b which is one needed to repair a parts, I_1 that signifies the losing cost per one transition becomes

$$I_1 = \sum_{i=0}^{\infty} b i P(1+i, 0) = b \sum_{i=0}^{\infty} i \beta^{1+i} P(0, 1)$$

$$\begin{aligned}
 &= b\beta^t P(0, l) \sum_{t=0}^{\infty} i\beta^t = b\beta^{t+1} P(0, l) / (1-\beta)^2 \\
 &= b\beta^{f-k+1} P(0, l) / (1-\beta)^2
 \end{aligned} \quad (35)$$

Then, letting c be the cost to purchase a parts, r be the rate of interest and I_2 be the interest to have f pieces of parts, we have

$$I_2 = crf_i / 2\lambda_0 \quad (36)$$

Therefore, I which stands for the loss function or the total losing cost of having f pieces of workshop spares is deduced as,

$$I = I_1 + I_2 = crf / 2\lambda_0 + b\beta^{f-k+1} P(0, l) / (1-\beta)^2 \quad (37)$$

On partially differentiating (37) by f and putting $\partial I / \partial f = 0$, we obtain

$$cr / 2\lambda_0 + b\beta^{1-k} P(0, l) \beta^f \log_e \beta / (1-\beta)^2 = 0$$

$$\text{Thus, } \beta^f = -cr(1-\beta) [1 + (1-\beta)K(k, 1/\beta)] / 2\lambda_0 b\beta^{1-k} \log_e \beta \quad (38)$$

Here, introducing a function defined by

$$N(k, 1/\beta) = -2\beta^{1-k} \log_e \beta / r(1-\beta) [1 + (1-\beta)K(k, 1/\beta)] \quad (39)$$

we get

$$\beta^f = (c/b) / \lambda_0 N(k, 1/\beta) \quad (40)$$

After some manipulation

$$f = [\log \lambda_0 - \log(c/b) + \log N(k, \delta)] / \log \delta \quad (41)$$

In the case where $k=1$, for example, $N(1, \delta)$ is obtained from (39) as

$$N(1, \delta) = 2 \log_e \delta / r(\delta-1) \quad (42)$$

If we mean the most economical number of workshop spares by f_1 in this case, it is expressed as

$$\begin{aligned}
 f_1 = & [\log \lambda_0 - \log(c/b) \\
 & + \log(2 \log_e \delta / r(\delta-1))] / \log \delta \quad (43)
 \end{aligned}$$

The relations between f_1 and, c/b and δ , are illustrated in Fig. 8 where r is 0.07 per year.

Formula (43) shows that f_1 of one kind of parts, for instance, whose $\lambda_0 = 3,000$ per year, $\delta = 1.1$ and $c/b = 32$ proves to be 80 by tracing points AB CDE in turn in Fig. 8, where CD must run parallel to the

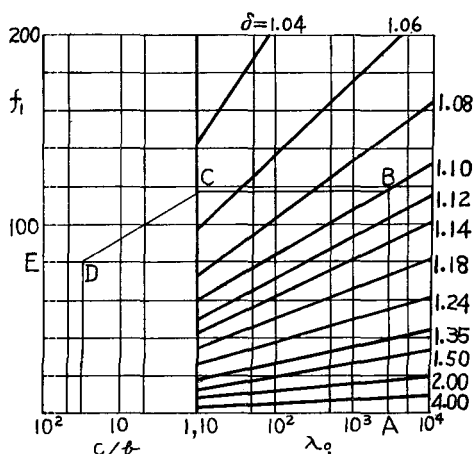


Fig. 8. The way to find the most economical number of workshop spares in the case where $k=1$

line exhibiting $\delta=1.1$.

Next, in the case where $k>1$, making use of (39) and (42), we can change the term of $N(k, \delta)$ as

$$N(k, \delta) = \frac{2 \log_e \delta}{r(\delta-1)} \cdot \frac{\delta^{k+1}}{[\delta + (\delta-1)K(k, \delta)]} = N(1, \delta) \frac{\delta^{k+1}}{\delta + (\delta-1)K(k, \delta)} \quad (44)$$

Upon inserting (44) into (41), we get

$$\begin{aligned} f_k &= \left[\log \lambda_0 - \log(c/b) + \log N(1, \delta) + \log \frac{\delta^{k+1}}{\delta + (\delta-1)K(k, \delta)} \right] / \log \delta \\ &= f_1 + (1/\log \delta) \log [\delta^{k+1} / \{\delta + (\delta-1)K(k, \delta)\}] \end{aligned} \quad (45)$$

where f_k signifies the most economical number of workshop spares in the case where the number of repairing channels is k . Now we take a function $g(k, \delta)$ defined by

$$g(k, \delta) = (1/\log \delta) \log [\delta^{k+1} / \{\delta + (\delta-1)K(k, \delta)\}] \quad (46)$$

and (45) is expressed as

$$f_k = f_1 + g(k, \delta) \quad (47)$$

It is realized from (47) that the most economical number in the case where $k>1$ is given by adding $g(k, \delta)$ to f_1 which can be obtained from Fig. 8 assuming $k=1$. That means $g(k, \delta)$ is, so to speak, a correction term when k is larger than 1. The values of $g(k, \delta)$ for $1 \leq k \leq 10$ and $1 \leq \delta \leq 4$ have been calculated.

It might as well be added in conclusion that the methods of deciding the number of shed and workshop spares described in this treatise have been put to practical use at Kokura Workshop of Japanese National Railways and those methods are also applicable in other workshops employed in the same job.

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