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TABLE FOR THE CAPACITY OF BINARY COMMUNICATION CHANNELS

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1. INTRODUCTION

First we describe the communication channels we shall treat in this paper. The alphabet in which we send words contains just two letters, which we denote by 0, 1. Thus a sent word of length n is a sequence of n letters, which we shall call an n-sequence, each letter being a 0 or a 1. If we send the word $(x_1, \cdots\cdots, x_n)$, then the received word is $(Y_1, \cdots\cdots, Y_n)$ where $Y_1, \cdots\cdots, Y_2$, are independent chance variables, the probability distribution of Y_i depending only on the value of x_i. If there are just two possible values for Y_i which we can assume are 0, 1, the channel is called a "binary channel. A binary channel is essentially characterized by a 2×2 stochastic matrix

$$\begin{pmatrix} p_1 & 1-p_1 \\ p_2 & 1-p_2 \end{pmatrix}, \quad 0\leqq p_1,\ p_2\leqq 1 \tag{1}$$

where $1-p_1$ anb p_2 are "error probabilities" of obtaining output letter $1-j$ when the input letter was j, $j=0$, 1, respectively. A code (n, N, λ) for the channel is a system

$$\{(u_1, A_1), \cdots\cdots, (u_N, A_N)\}$$

where $u_1, \cdots\cdots, u_N$ are n-sequences, $A_1, \cdots\cdots A_N$, are disjoint set of n-sequences, and, for $j=1, \cdots\cdots, N$,

$$p_r\{(Y_1, \cdots\cdots, Y_n)\in A_j | u_j\}\geqq 1-\lambda.$$

The use of such a code is well-known. If the received word is in

A_j, the receiver assumes that the word u_j was actually sent. Then, no matter which of the words $u_1, \cdots\cdots, u_N$ is sent, the probability that the receiver will be in error is not greater than λ.

The most remarkable and now fundamental theorem of the information theory is the following:

THEOREM. *Let ε be an arbitrary positive number. For sufficiently large n there exists a code of length*

$$N \geqq 2^{n(C-\varepsilon)},$$

where C is a constant determined by the channel characteritic.

The number C is called the *capacity* of the channel. For a binary channel (1) C is given by

$$C = \max_{\substack{0 \leq \xi_1, \xi_2 \leq 1 \\ \xi_1+\xi_2=1}} \sum_{i=1}^{2} \xi_i \left(p_i \log \frac{p_i}{\xi_1 p_1 + \xi_2 p_2} + (1-p_i) \log \frac{1-p_i}{\xi_1(1-p_1)+\xi_2(1-p_2)} \right) \tag{2}$$

A binary channel is called "symmetric" if $p_1+p_2=1$. It is easily found that we have, if symmetric,

$$C = 1 + p_1 \log p_1 + (1-p_1) \log (1-p_1) \tag{3}$$

and the maximizing ξ_1 is equal to $\frac{1}{2}$. The numerical table of the expression (3) was first given by Dolanký-Dolanský [1]. The purpose of the present paper is to give a table of numerical values of (2) for general binary channels (1).

2. THE OPERATIONS-RESEARCH BACKGROUND

Now, besides its interest of somewhat mathematical nature the capacity (2) of binary cammunication channels has now been given a practical importance in the field of operations research. The author feels at this point some neccessity to remark here that Professor K. Kunisawa of Tokyo Institute of Technology has devoted his efforts in these several years to the exploration of applicabilities of the information theory to various operation research problems.

Several case-studies of considerable interest are reported in his book [2], but they are not completely analysed theoretically. The present author proved in a previous paper [4] a theorem stated below which will throw light on the aspects as Prof. Kunisawa viewed.

Let $(p_{ij} | \; i, j=1, \cdots\cdots, n)$ be a stochastic matrix representing the characteristic of a communication channel with the same sizes of both input and output alphabets. Consider a person who observes the output number j. He does not know the input probabilities $\{\xi_i\}$. Hence, of course, he does not know the output probability distribution $\{p(j)\}$, even if he has full knowledge of the channel characteristic (p_{ij}). Given the channel characteristic, he can compute the corresponding output probabilities $p(j)$ if he assumes an input probability distribution $\{\xi_i\}$.

Suppose that if he observes the output number j there occurs a gain for him which is equal to the amount $\log p(j)$ of information available and a cost which is given by a real number X_j. Thus, when the output probabilities are $p(j)$, of which he is ignorant, his expected net gain will be

$$\sum_{j=1}^{n} p(j)(-\log p(j)-X_j). \tag{4}$$

Let the numbers $X_j(j=1, \cdots\cdots, n)$ representing costs be taken such that if an input number i is selected, there results at the output an expected amount of cost $H_i \equiv -\sum_{j=1}^{n} p_{ij}\log p_{ij}$. Or equivalently, $\{X_j\}$ are given by

$$(p_{ij})\begin{bmatrix} X_1 \\ \vdots \\ X_n \end{bmatrix}=\begin{bmatrix} H_1 \\ \vdots \\ H_n \end{bmatrix}. \tag{5}$$

The amount of information transmitted per symbol by the channel equals

$$\sum_{i,j=1}^{n} \xi_i p_{ij} \log\left(p_{ij}\Big/\sum_{i=1}^{n}\xi_i p_{ij}\right) \tag{6}$$

if the input probabilities are $\{\xi_i\}$. Let us call the maximizing probability vector ξ^* of the expression (6) the *matching input-probabilities to the channel*, and let us call the probability vector $\left\{\sum_{i=1}^{n}\xi_i^* p_{ij}\right\}$ the *matching output-probabilities to the channel*. Similarly we shall call the maximizing probabilities $\{p^*(j)\}$ of the expression (4) the *matching probabilities to the costs* $\{X_j\}$. Then we have a theorem [4] as follows:

THEOREM *Let (p_{ij}) be non-singular, and suppose that there exists a probability vector ξ^* such that*

$$\sum_{i=1}^{n}\xi_i^* p_{ij}=e^{-X_j}\left(\sum_{j=1}^{n}e^{-X_j}\right)^{-1} \quad (j=1, \cdots\cdots, n)$$

and $\xi_i^*>0$ $(i=1, \cdots\cdots, n)$, *where* X_j *'s are defined by* (5). *Then the matching probabilities to the costs* $\{X_j\}$ *defined by* (5) *equal the matching output-probabilities to the channel* (p_{ij}). *Moreover the maximum expected net gain when we consider the costs* $\{X_j\}$ *is equal to the capacity*

$$\max_{\xi} \sum_{i,j=1}^{n} \xi_i p_{ij} \log\left(p_{ij} \Big/ \sum_{i=1}^{n} \xi_i p_{ij} \right)$$

of the channel (p_{ij}).

3. THE METHOD AND SOLUTION

The author has given in another paper [3] the solution of our problem (2) as follows. Let $p_1 \neq p_2$, and let ξ_1^*, ξ_2^* be the maximizing probabilities. We solve simultanious equations

$$\begin{cases} p_1X_1+(1-p_1)X_2=H_1 \\ p_2X_1+(1-p_2)X_2=H_2 \end{cases}$$

where $H_i \equiv -p_i \log_2 p_i-(1-p_i)\log_2(1-p_i)(i=1, 2)$ and

$$\begin{cases} p_1\xi_1^*+p_2\xi_2^*=2^{-X_1}(2^{-X_1}+2^{-X_2})^{-1} \\ \xi_1^*+\xi_2^*=1. \end{cases}$$

We obtain from each of these

$$X_1=\frac{(1-p_2)H_1-(1-p_1)H_2}{p_1-p_2}, \qquad X_2=\frac{-p_2H_1+p_1H_2}{p_1-p_2}$$

and

$$\xi_1^*=\left(\frac{2^{-X_1}}{2^{-X_1}+2^{-X_2}}-p_2\right)\Big/(p_1-p_2), \qquad \xi_2^*=\left(\frac{2^{-X_1}}{2^{-X_1}+2^{-X_2}}-(1-p_1)\right)\Big/(p_1-p_2)$$

respectively. We then have $C=\log_2(2^{-X_1}+2^{-X_2})$.

A binary channel can be represented by a point in the unit square as in Fig. 1. By symmetry in the maxim and of (2) it is easily seen that four channels (p_1, p_2), (p_2, p_1) $(1-p_1, 1-p_2)$ and $(1-p_2, 1-p_1)$, each located symmetrically with respect to the diagonals of the square, are equivalent, i.e., have the same values of the capacity. Thus we are only to compute the values in the shaded triangle in

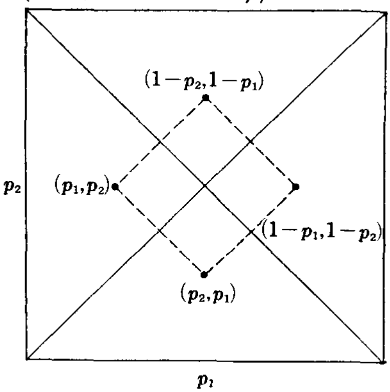

Fig. 1. The equivalence of the four channels

Fig. 1. The five-decimal-place table on page 60-66 gives the values of $\xi_1{}^*$ and C for $p_1=0.01$, 0.02 (0.02) 0.48 and $p_2=0.01$, 0.02 (0.02) 1.00. The computations were performed by the electronic computer HITAC 101.

REFERENCES

[1] Dolanskŷ, L. and Dolanskŷ, M. P. (1952). Table of $\log_2 \frac{1}{p}$, $p \log_2 \frac{1}{p}$ and $p \log_2 \frac{1}{p} + (1-p) \log_2 \frac{1}{1-p}$, *Tech. Rep.* No. 22, *R. L. E., M.I.T.*

[2] Kunisawa, K. (1959). "*Introduction to the Information Theory for OR Workers,*" JUSE Publishing Company, Tokyo. (in Japanese)

[3] Sakaguchi, M. (1959). Notes on statistical applications of information theory IV. *Rep. Stat. Appl. Res., JUSE.*, Vol. 6, 54—57.

[4] Sakaguchi, M. (1961). Some remarks on the capacity of a communication channel, *J. Operations Research Soc. Japan*, Vol. 3, 124—132.

Capacity and the Matching Input-Probability for Binary Channels.

p_1 \ p_2	0.00		0.01		0.02	
	ξ_1*	C	ξ_1*	C	ξ_1*	C
0.02	0.63114	0.01068	0.52806	0.00125		
0.04	.63014	.02150	.55339	.00720	0.52762	0.00253
0.06	.62912	.03247	.56606	.01501	.54246	.00791
0.08	.62809	.04358	.57381	.02376	.55202	.01474
0.10	.62703	.05484	.57901	.03314	.55872	.02252
0.12	.62595	.06625	.58268	.04298	.56366	.03100
0.14	.62485	.07783	.58535	.05321	.56741	.04002
0.16	.62373	.08958	.58730	.06376	.57031	.04951
0.18	.62258	.10150	.58873	.07462	.57257	.05940
0.20	.62141	.11359	.58975	.08576	.57433	.06965
0.22	.62022	.12588	.59045	.09717	.57570	.08025
0.24	.61900	.13835	.59088	.10885	.57673	.09116
0.26	.61775	.15102	.59110	.12078	.57750	.10239
0.28	.61647	.16390	.59114	.13297	.57803	.11392
0.30	.61516	.17699	.59101	.14542	.57837	.12574
0.32	.61382	.19030	.59073	.15813	.57852	.13786
0.34	.61244	.20385	.59033	.17111	.57851	.15028
0.36	.61103	.21764	.58981	.18436	.57837	.16301
0.38	.60958	.23168	.58919	.19790	.57808	.17603
0.40	.60810	.24599	.58846	.21172	.57768	.18937
0.42	.60657	.26057	.58763	.22584	.57716	.20303
0.44	.60500	.27544	.58672	.24028	.57653	.21702
0.46	.60338	.29061	.58571	.25503	.57580	.23135
0.48	.60172	.30610	.58462	.27013	.57496	.24603
0.50	.60000	.32193	.58344	.28558	.57402	.26108
0.52	.59823	.33811	.58217	.30139	.57299	.27651
0.54	.59640	.35466	.58082	.31760	.57185	.29235
0.56	.59450	.37161	.57937	.33421	.57062	.30860
0.58	.59254	.38898	.57784	.35126	.56928	.32531
0.60	.59050	.40679	.57621	.36877	.56785	.34248
0.62	.58839	.42507	.57448	.38676	.56630	.36015
0.64	.58619	.44387	.57265	.40528	.56465	.37835
0.66	.58390	.46321	.57071	.42435	.56287	.39712
0.68	.58150	.48313	.56865	.44402	.56098	.41650
0.70	.57900	.50369	.56647	.46434	.55895	.43654
0.72	.57637	.52494	.56415	.48536	.55677	.45728
0.74	.57361	.54694	.56168	.50714	.55445	.47880
0.76	.57070	.56976	.55905	.52976	.55195	.50117
0.78	.56761	.59349	.55623	.55330	.54927	.52446
0.80	.56434	.61823	.55321	.57787	.54638	.54880
0.82	.56084	.64411	.54996	.60359	.54325	.57430
0.84	.55708	.67129	.54644	.63063	.53985	.60113
0.86	.55302	.69995	.54261	.65917	.53202	.62948
0.88	.54859	.73035	.53840	.68948	.53202	.65960
0.90	.54370	.76285	.53372	.72190	.52744	.69186
0.92	.53824	.79792	.52846	.75693	.52226	.72676
0.94	.53199	.83632	.52240	.79535	.51628	.76505
0.96	.52461	.87932	.51518	.83841	.50913	.80806
0.98	.51530	.92964	.50601	.88891	.50000	.85856

p_2 \ p_1	0.04		0.06		0.08	
	ξ_1*	C	ξ_1*	C	ξ_1*	C
0.02						
0.04						
0.06	0.51593	0.00153				
0.08	.52670	.00522	0.51106	0.00111		
0.10	.53455	.01033	.51931	.00396	0.50837	0.00088
0.12	.54055	.01646	.52572	.00809	.51496	.00323
0.14	.54526	.02339	.53085	.01320	.52029	.00672
0.16	.54903	.03098	.53502	.01913	.52467	.01114
0.18	.55209	.03914	.53846	.02575	.52833	.01636
0.20	.55458	.04779	.54132	.03297	.53140	.02207
0.22	.55662	.05690	.54371	.04073	.53399	.02880
0.24	.55828	.06643	.54569	.04899	.53618	.03588
0.26	.55962	.07634	.54735	.05771	.53802	.04349
0.28	.56069	.08664	.54871	.06686	.53957	.05158
0.30	.56152	.09729	.54983	.07643	.54087	.06012
0.32	.56215	.10830	.55072	.08640	.54193	.06912
0.34	.56258	.11966	.55140	.09677	.54278	.07854
0.36	.56285	.13136	.55191	.10751	.54345	.08838
0.38	.56296	.14342	.55225	.11865	.54395	.09863
0.40	.56293	.15582	.55244	.13016	.54428	.10929
0.42	.56276	.16858	.55248	.14206	.54447	.12037
0.44	.56247	.18170	.55239	.15434	.54451	.13185
0.46	.56205	.19519	.55217	.16703	.54442	.14376
0.48	.56152	.20906	.55182	.18012	.54420	.15609
0.50	.56087	.22333	.55135	.19362	.54385	.16885
0.52	.56011	.23800	.55076	.20755	.54338	18206
0.54	.55924	.25310	.55006	.22193	.54279	.19573
0.56	.55826	.26864	.54924	.23677	.54208	.20988
0.58	.55717	.28465	.54830	.25209	.54125	.22453
0.60	.55597	.30114	.54724	.26791	.54030	.23970
0.62	.55465	.31816	.54606	.28427	.53921	.25541
0.64	.55320	.33572	.54476	.30119	.53808	.27170
0.66	.55164	.35386	.54332	.31872	.53666	.28860
0.68	.54994	.37264	.54175	.33688	.53518	.30615
0.70	.54810	.39208	.54003	.35572	.53354	.32439
0.72	.54611	.41225	.53816	.37531	·53175	.34339
0.74	.54397	.43321	.53612	.39570	.52979	.36320
0.76	.54164	.45504	.53390	.41696	.52764	.38390
0.78	.53912	.47781	.53148	.43919	.52529	.40557
0.80	.53638	.50164	.52884	.46248	.52271	.42832
0.82	.53340	.52665	.52594	.48697	.51988	.45228
0.84	.53013	.55300	.52276	.51282	.51675	.47761
0.86	.52654	.58090	.51924	.54023	.51328	.50450
0.88	.52255	.61060	.51533	.56946	.50940	.53324
0.90	.51809	.64246	.51091	.60087	.50502	.56417
0.92	.51300	.67699	.50588	.63497	.50000	.59782
0.94	.50710	.71496	.50000	.67256		
0.96	.50000	.75771				
0.98						

p_1 \ p_2	0.10		0.12		0.14	
	ξ_1*	C	ξ_1*	C	ξ_1*	C
0.02						
0.04						
0.06						
0.08						
0.10						
0.12	0.50665	0.00074				
0.14	.51208	.00275	0.50546	0.00064		
0.16	.51658	.00579	.51002	.00240	0.50458	0.00057
0.18	.52036	.00872	.51387	.00513	.50847	.00215
0.20	.52355	.01441	.51714	.00868	.51179	.00462
0.22	.52627	.01978	.51995	.01296	.51465	.00788
0.24	.52859	.02577	.52235	.01791	.51711	.01185
0.26	.53056	.03232	.52441	.02347	.51923	.01646
0.28	.53224	.03941	.52617	.02960	.52105	.02168
0.30	.53365	.04699	.52767	.03627	.52262	.02746
0.32	.53484	.05505	.52894	.04344	.52395	.03378
0.34	.53581	.06358	.53000	.05110	.52506	.04061
0.36	.53659	.07255	.53086	.05924	.52599	.04794
0.38	.53719	.08196	.53154	.06784	.52673	.05576
0.40	.53763	.09181	.53206	.07690	.52731	.06406
0.42	.53792	.10209	.53243	.08641	.52773	.07282
0.44	.53806	.11280	.53265	.09637	.52801	.08206
0.46	.53807	.12395	.53272	.10679	.52815	.09176
0.48	.53794	.13554	.53267	.11767	.52814	.10194
0.50	.53769	.14759	.53248	.12901	.52801	.11260
0.52	.53731	.16010	.53217	.14084	.52775	.12375
0.54	.53680	17308	.53173	.15314	.52736	.13540
0.56	.53617	.18655	.53116	.16596	.52684	.14756
0.58	.53542	.20054	.53046	.17929	.52619	.16026
0.60	.53454	.21506	.52964	.19317	.52542	.17350
0.62	.53353	.23013	.52869	.20761	.52451	.18733
0.64	.53239	.24579	.52761	.22266	.52347	.20176
0.66	.53111	.26208	.52638	.23833	.52228	.21683
0.68	.52969	.27902	.52501	.25467	.52095	.23257
0.70	.52812	.29667	.52349	.27173	.51946	.24904
0.72	.52639	.31508	.52180	.28956	.51781	.26629
0.74	.52449	.33432	.51994	.30821	.51597	.28436
0.76	.52239	.35444	.51788	.32777	.51395	.30335
0.78	.52009	.37555	.51561	.34832	.51170	.32334
0.80	.51756	.39775	.51311	.36997	.50922	.34443
0.82	.51476	.42117	.51034	.39283	.50648	.36674
0.84	.51167	.44597	.50727	.41709	.50342	.39045
0.86	.50823	.47234	.50385	.44294	.50000	.41576
0.88	.50437	.50057	.50000	.47064		
0.90	.50000	.53100				
0.92						
0.94						
0.96						
0.98						

p_1 \ p_2	0.16		0.18		0.20	
	ξ_1^*	C	ξ_1^*	C	ξ_1^*	C
0.02						
0.04						
0.06						
0.08						
0.10						
0.12						
0.14						
0.16						
0.18	0.50390	0.00051				
0.20	.50725	.00196	0.50336	0.00047		
0.22	.51014	.00423	.50627	.00181	0.50292	0.00043
0.24	.51264	.00726	.50879	.00393	.50545	.00168
0.26	.51480	.01096	.51098	.00676	.50545	.00367
0.28	.51667	.01530	.51288	.01025	.50958	.00635
0.30	.51828	.02024	.51452	.01437	.51125	.00967
0.32	.51965	.02573	.51593	.01907	.51268	.01360
0.34	.52082	.03197	.51713	.02433	.51390	.01811
0.36	.52258	.03832	.51814	.03012	.51494	.02317
0.38	.52179	.04538	.51896	.03644	.51579	.02876
0.40	.52321	,05293	.51962	.04327	.51648	.03489
0.42	.52367	.06097	.52013	.05061	.51702	.04153
0.44	.52399	.06950	.52049	.05844	.51740	.04868
0.46	.52417	.07851	.52070	.06677	.51764	.05634
0.48	.52422	.08800	.52078	.07559	.51775	.06451
0.50	,52412	.09799	.52072	.08492	.51771	.07319
0.52	.52390	.10848	.52053	.09476	.51755	.08240
0.54	.52355	.11948	.52021	.10512	.51726	.09213
0.56	.52307	.13100	.51976	.11602	.51684	.10240
0.58	.52246	.14307	.51918	.12746	.51628	.11323
0.60	.52172	.15569	.51847	.13947	.51559	.12463
0.62	.52085	.16890	.51763	.15207	.51477	.13663
0.64	.51984	.18273	.51664	.16530	.51381	.14926
0.66	.51869	.19720	.51551	.17917	.51270	.16254
0.68	.51739	.21235	.51424	.19374	.51144	.17652
0.70	.51592	.22823	.51280	.20903	.51001	.19124
0.72	,51430	.24490	.51119	.22512	.50842	.20674
0.74	.51249	.26240	.50940	.24205	.50664	.22310
0.76	.51048	.28082	.50740	.25990	.50466	.24038
0.78	.50826	.30024	.50519	.27875	.50245	.25867
0.80	.50579	.32077	.50273	.29872	.50000	.27807
0.82	.50305	.34253	.50000	.31992		
0.84	.50000	.36569				
0.86						
0.88						
0.90						
0.92						
0.94						
0.96						
0.98						

p_2 \ p_1	0.22		0.24		0.26	
	$\xi_1{}^*$	C	$\xi_1{}^*$	C	$\xi_1{}^*$	C
0.02						
0.04						
0.06						
0.08						
0.10						
0.12						
0.14						
0.16						
0.18						
0.20						
0.22						
0.24	0.50254	0.00041				
0.26	.50476	.00158	0.50222	0.00038		
0.28	.50670	.00347	.50417	.00150	0.50195	0.00037
0.30	.50838	.00602	.50586	.00330	.50364	.00143
0.32	.50983	.00919	.50732	.00574	.50512	.00316
0.34	.51107	.01296	.50858	.00879	.50639	.00551
0.36	.51213	.01731	.50965	.01243	.50747	.00846
0.38	.51300	.02220	.51054	.01664	.50837	.01199
0.40	.51372	.02764	.51127	.02140	.50912	.01608
0.42	.51427	.03360	.51185	.02669	.50970	.02073
0.44	.51468	.04008	.51227	.03252	.51014	.02591
0.46	.51494	.04709	.51255	.03888	.51044	.03163
0.48	.51507	.05461	.51269	.04577	.51059	.03789
0.50	.51506	.06266	.51270	.05319	.51062	.04469
0.52	.51492	.07123	.51258	.06114	.51051	.05203
0.54	.51464	.08034	.51232	.06963	.51026	.05991
0.56	.51424	.08999	.51194	.07868	.50989	.06836
0.58	.51371	.10021	.51142	.08830	.50938	.07738
0.60	.51304	.11101	.51076	.09850	.50874	.08899
0.62	.51223	.12242	.50997	.10932	.50796	.09722
0.64	.51128	.13446	.50904	.12076	.50703	.10808
0.66	.51019	.14715	.50796	.13288	.50596	.11961
0.68	.50894	.16055	.50672	.14569	.50473	.13185
0.70	.50753	.17469	.50532	.15925	.50334	.14483
0.72	.50595	.18962	.50374	.17361	.50176	.15861
0.74	.50418	.20540	.50197	.18882	.50000	.17325
0.76	.50220	.22211	.50000	.20946		
0.78	.50000	.23983				
0.80						
0.82						
0.84						
0.86						
0.88						
0.90						
0.92						
0.94						
0.96						
0.98						

p_2 \ p_1	0.28		0.30		0.32	
	ξ_1*	C	ξ_1*	C	ξ_1*	C
0.30	0.50170	0.00035				
0.32	.50318	.00137	0.50148	0.00034		
0.34	.50446	.00304	.50276	.00133	0.50128	0.00033
0.36	.50554	.00532	.50386	.00294	.50238	.00129
0.38	.50646	.00818	.50478	.00515	.50331	.00286
0.40	.50721	.01162	.50554	.00795	.50407	.00502
0.42	.50781	.01562	.50614	.01131	.50468	.00776
0.44	.50826	.02017	.50660	.01524	.50514	.01106
0.46	.50856	.02526	.50691	.01971	.50547	.01492
0.48	.50873	.03090	.50709	.02474	.50565	.01934
0.50	.50877	.03709	.50713	.03031	.50570	.02431
0.52	.50867	.04382	.50704	.03644	.50562	.02984
0.54	.50844	.05110	.50682	.04313	.50540	.03594
0.56	.50807	.05895	.50647	.05038	.50505	.04261
0.58	.50757	.06738	.50598	.05822	.50457	.04986
0.60	.50694	.07640	.50535	.06666	.50395	.05772
0.62	.50617	.08604	.50458	.07572	.50319	.06619
0.64	.50525	.09632	.50367	.08542	.50228	.07531
0.66	.50419	.10727	.50261	.09579	.50122	.08511
0.68	.50296	.11893	.50139	.10688	.50000	.09562
0.70	.50157	.13134	.50000	.11871		
0.72	.50000	.14455				

p_2 \ p_1	0.34		0.36		0.38	
	ξ_1*	C	ξ_1*	C	ξ_1*	C
0.36	0.50110	0.00032				
0.38	.50203	.00125	0.50093	0.00031		
0.40	.50280	.00279	.50170	.00123	0.50077	0.00030
0.42	.50341	.00491	.50232	.00273	.50139	.00120
0.44	.50388	.00760	.50279	.00482	.50186	.00269
0.46	.50421	.01085	.50312	.00747	.50220	.00474
0.48	.50440	.01467	.50331	.01069	.50240	.00737
0.50	.50445	.01905	.50337	.01447	.50246	.01057
0.52	.50437	.02398	.50330	.01882	.50239	.01433
0.54	.50416	.02949	.50309	.02374	.50218	.01867
0.56	.50382	.03557	.50275	.02924	.50184	.02359
0.58	.50334	.04224	.50227	.03533	.50137	.02910
0.60	.50272	.04952	.50166	.04203	.50076	.03522
0.62	.50196	.05742	.50090	.04935	.50000	.04196
0.64	.50106	.06596	.50000	.05732		
0.66	.50000	.07518				

p_2 \ p_1	0.40		0.42		0.44	
	ξ_1^*	C	ξ_1^*	C	ξ_1^*	C
0.42	0.50062	0.00030				
0.44	.50110	.00118	0.50048	0.00029		
0.46	.50143	.00265	.50081	.00117	0.50034	0.00028
0.48	.50163	.00469	.50101	.00262	.50054	.00116
0.50	.50169	.00730	.50108	.00465	.50064	.00261
0.52	.50162	.01048	.50101	.00725	.50054	.00463
0.54	.50142	.01424	.50081	.01043	.50034	.00723
0.56	.50109	.01858	.50047	.01419	.50000	.01041
0.58	.50061	.02351	.50000	.01855		
0.60	.50000	.02905				

p_2 \ p_1	0.46		0.48	
	ξ_1^*	C	ξ_1^*	C
0.48	0.50020	0.00029		
0.50	.50027	.00116	0.50007	0.00029
0.52	.50020	.00260	0.50000	0.00115
0.54	.50000	.00462		