

CAR FOLLOWING THEORY AND STABILITY LIMIT OF TRAFFIC VOLUME

EIJI KOMETANI AND TSUNA SASAKI

*Department of Civil Engineering, Faculty of Engineering,
Kyoto University*

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ABSTRACT

The authors have developed the fundamental equation of traffic dynamics concerning a traffic flow running down on single lane in one direction. Because of ambiguity of the coefficient " m " relating to the lead car which appears in this fundamental equation, they scrutinized the effects of m on the traffic flow through their car following experiments. In this case, they assume that the minimum value of the safe car space in case of car following practice is expressed as a linear function of speeds of the lead car and that of the following car. In this paper, the value of m obtained by the car following experiment is given and its effect on traffic flow is considered from the standpoint of safety and stability of queue of vehicles, and finally they calculated the traffic volume as stability limit which can prevent multiple collision.

INTRODUCTION

As the traffic volume increases and the passing over becomes harder and harder, the behavior of the following vehicle tends to be fully controlled by the behavior of the lead car. We assume, in such a case, that the following vehicle always follows the lead car keeping the minimum safe car space. We have investigated traffic dynamics concerning the traffic model described here. As the result of car following experiment practice, it was clarified that the existence of the coefficient m in the fundamental equation, hereupon we consider how the character of flow varies due to the existence of the coefficient m .

BASIC EQUATION OF TRAFFIC DYNAMICS

We have assumed in the previous papers that the basic equation of traffic dynamics is given by the following expression,

$$x_k(t-T) - x_{k+1}(t-T) = \alpha v_k(t-T) + \beta v_{k+1}(t) + b_0 \quad (1)$$

where, α , β and b_0 are constants.

For convenience in the analysis we put

$$\alpha = mT, \quad \beta = nT. \quad (2)$$

It should be noted that we put $\alpha = -mT$ in the previous papers.^{3,4)} The reason why we put $\alpha = mT$ in this paper is that the coefficient α was not confined to the negative sign through the successive experiments.

Substituting Eq. (2) into Eq. (1) and differentiating Eq. (1), we get

$$v_k(t-T) - v_{k+1}(t-T) = mT\dot{v}_k(t-T) + nT\dot{v}_{k+1}(t) \quad (3)$$

Laplace transformation is powerful means to solve Eq. (3), if we are careful to the following points: the first, Laplace transformation of $v_k(t)$ and $v_{k+1}(t)$ should be defined as

$$V_k(s) = \int_{0-}^{\infty} v_k(t) e^{-st} dt, \quad V_{k+1}(s) = \int_{0-}^{\infty} v_{k+1}(t) e^{-st} dt,$$

the second, the speed of the lead car, $v_k(t)$ for $0- \leq t < T$ should be given as an initial condition.

In Eq. (3) the first term of both sides are transformed to

$$\int_{0-}^{\infty} v_k(t-T) e^{-st} dt = e^{-sT} V_k(s),$$

and

$$mT \int_{0-}^{\infty} \dot{v}_k(t-T) e^{-st} dt = mT e^{-sT} [s V_k(s) - v_k(0-)],$$

where

$$v_k(t) = 0 \quad \text{for } t < 0-$$

and $v_k(0-)$ is the value of $v_k(t)$ at $t=0-$.

Using the following definitions

$$v_{k+1}(t) = v_{k+1}^*(t) + v_{k+1}^{**}(t) \quad (4)$$

where

$$v_{k+1}^*(t) = 0 \quad \text{for } t < 0- \text{ and } t \geq T$$

$$v_{k+1}^{**}(t) = 0 \quad \text{for } t < T,$$

we obtain as the Laplace transformation of the second term of both sides of Eq. (3)

$$\int_{0-}^{\infty} v_{k+1}(t-T) e^{-st} dt = e^{-sT} \left[\int_{0-}^{\infty} v_{k+1}^*(t) e^{-st} dt + V_{k+1}^{**}(s) \right],$$

and

$$nT \int_{0-}^{\infty} \dot{v}_{k+1}(t) e^{-st} dt = nTs \left[\int_{0-}^T v_{k+1}^*(t) e^{-st} dt + V_{k+1}^{**}(s) \right] - nT v_{k+1}^*(0-),$$

respectively, where $v_{k+1}^*(0-)$ is the value of $v_{k+1}^*(t)$ at $t=0-$. Consequently, Laplace transformation of Eq. (3) is given by

$$V_{k+1}^{**}(s) = \frac{1-mTs}{nTs+e^{-Ts}} e^{-Ts} V_k(s) + \frac{mTe^{-Ts}}{nTs+e^{-Ts}} v_k(0-) + \frac{nT}{nTs+e^{-Ts}} v_{k+1}^*(0-) - \int_{0-}^T v_{k+1}^*(t) e^{-st} dt. \quad (5)$$

If the initial conditions $v_k(0-)$, $v_{k+1}^*(0-)$ and $v_{k+1}^*(t)$ are given, we can find the behavior of a following car by repeated application of Eq. (5). The previous papers are lacking in the consideration the last term of Eq. (5). It is important that $v_{k+1}^*(t)$ expressed by Ep. (5) represents the car following behavior for $t \geq T$ but the behavior for $0- \leq t < T$ is defined by $v_{k+1}^*(t)$ as an initial condition.

CAR FOLLOWING EXPERIMENTS AND THE VALUES OF COEFFICIENTS OF DYNAMIC EQUATION

We performed the previous car following experiments and related observations by several methods such as taking photographs by 8mm cine-camera from top of the road-side building and as measuring the speeds of both vehicles with speed meters mounted on the fifth wheels which were trailed by the experiment vehicles. But those methods could not avoid the error of synchronization between the lead car and the following car, so we made these experiments using trace marking apparatus. This new apparatus is consisted of battery, chronometer, ink-nozzle and a pressure tank containing dye-stuff solution. Each of the lead car and the following car has under the floor one nozzle which injects ink, and you can inject dye-stuff solution from the nozzle to make the position of the vehicle on the road surface continuously while they run on the car following experiment. At the same time, every 0.20 sec. time pulse marks are recorded on the road surface controlled by a common chronometer mounted on the following vehicle, and the time pulse of the lead car is synchronized by the chronometer mounted on the following car connected with a cord. After every car following experiment, the speeds and car space of both vehicles were calculated by measuring the position of time pulse. Effect of non-linearity will appear if the speed changes widely during the car following, thus we made these experiments in two groups

at lower speeds of 20~40 km/h and at higher speeds of 40~80 km/h.

(a) Traffic at lower speeds

We chose a straight stretch of street of two kilometers as the experiment section and the lead car was driven at 20~40 km/h in speed. The road was closed to all vehicles during the experiment because of traffic safety. The driver of the lead car has accelerated and decelerated his vehicle within predetermined range of speed (20~40 km/h) and the driver of the following car was ordered merely to follow the lead car keeping the safe minimum space. After every car following performance we measured the car space and speeds of both vehicles from their loci on the road surface. It was a good idea to have used different coloured dye-stuff solution for the lead car and for the following car respectively to let them mark the paths of vehicles, because it was very convenient and easy to measure necessary data.

Arranging the records of car space and speed, we determined the coefficients α , β , b_0 and T so that equation (1) would be most suitable (to be shown the highest goodness of fit by Chi square test) to these data. The results obtained is shown in Table 1.

Driver	α (SEC)	β (SEC)	T (SEC)	m	n	b_0 (m)
A	0.0420	0.585	0.50	0.084	1.170	4.85
	0.0576	0.519	0.45	0.128	1.154	4.42
B	0.0288	0.427	0.40	0.072	1.068	3.95
	0.0296	0.482	0.40	0.074	1.204	4.12
C	0.0900	0.511	0.50	0.180	1.022	5.07
	0.0792	0.592	0.60	0.132	0.986	5.15
D	0.0858	0.557	0.55	0.156	1.012	4.80
	0.0610	0.592	0.50	0.122	1.184	5.22
E	0.0696	0.748	0.60	0.116	1.246	5.40
	0.0949	0.746	0.65	0.146	1.148	5.18
F	0.0749	0.814	0.70	0.107	1.163	4.62
	0.0598	0.848	0.65	0.092	1.305	4.34
Mean	0.0644	0.618	0.54	0.117	1.138	4.76

Table-1

(b) Traffic at higher speeds

We performed the similar experiments for the speed range of 40~80 km/h. As the results of these car following experiments we obtained the values of coefficients as is shown in Table 2. You can see from

Driver	α (SEC)	β (SEC)	T (SEC)	m	n	b_0 (m)
A	0.211	0.783	0.75	0.301	1.044	5.02
	0.170	0.781	0.75	0.227	1.041	4.83
B	0.126	0.772	0.70	0.180	1.103	4.72
	0.103	0.806	0.75	0.137	1.074	4.50
C	0.183	0.813	0.80	0.229	1.016	5.58
	0.122	0.751	0.70	0.174	1.073	5.24
D	0.194	0.786	0.65	0.298	1.210	4.70
	0.187	0.789	0.60	0.312	1.315	4.31
E	0.161	0.810	0.70	0.230	1.158	3.95
	0.143	0.780	0.75	0.191	1.040	4.28
F	0.114	0.892	0.85	0.134	1.054	4.03
	0.075	0.835	0.80	0.094	1.044	3.75
Mean	0.149	0.800	0.73	0.209	1.098	4.58

Table-2

Table 1 and Table 2 that the values of each coefficient are larger in the case of higher speeds compared to the case of lower speeds.

EFFECT OF THE BASIC COEFFICIENT m

Although the coefficient m does not appear in the papers written by Dr. R. Herman and others,⁽¹⁾⁽²⁾ is not the coefficient m necessary or not? We will consider what meaning will be insisted by the existence of m .

(a) Effects upon the speed-curve in car following

At first, we check the equation of motion for $m=0$ by the speed-curve got from the car following experiment performed by the authors. The goodness of fit examined by Chi square test is shown on Table 3, which tells us that the remarkable goodness of fit is attained if we consider m than neglect it. As is known from Table 3, time delay in case of $m=0$ is almost the same as that in case of $m \neq 0$, but the value of β is larger for $m \neq 0$. Furthermore, as is surmised from Table 3, the value of n in a system where the coefficient m neglected is practically considered same as the value of $n+m$ in a system where the coefficient m is taken into account. Figure 1 shows a part of the speed record in the car following experiment. From these data, we can compute the coefficients of the equation of motion to which the coefficient m is considered: $m=0.301$, $n=1.044$, $T=0.75$ sec. (Driver A in Table 2). If we compute these coefficients for the equation of motion neglecting m , we have $n=$

Driver	m is neglected			m is considered		
	Goodness of Fit	Reaction Time (SEC)	n	Goodness of Fit	Reaction Time (SEC)	
A	25	0.75	1.348	75	0.75	
	50	0.75	1.260	90	0.75	
B	5	0.75	1.322	90	0.70	
	10	0.70	1.201	95	0.75	
C	5	0.80	1.283	75	0.80	
	10	0.70	1.210	50	0.70	
D	5	0.65	1.500	50	0.65	
	5	0.65	1.653	25	0.60	
E	10	0.75	1.415	75	0.70	
	25	0.70	1.184	75	0.75	
F	25	0.80	1.231	50	0.85	
	25	0.80	1.103	75	0.80	

Table-3

1.348, $T=0.75$ sec. In Figure 1 the speed-curve for a case when m is neglected is also plotted. As is illustrated in Figure 1, the effect of m is seen near the transition point between acceleration and deceleration and almost no effect is observed for the other domain.

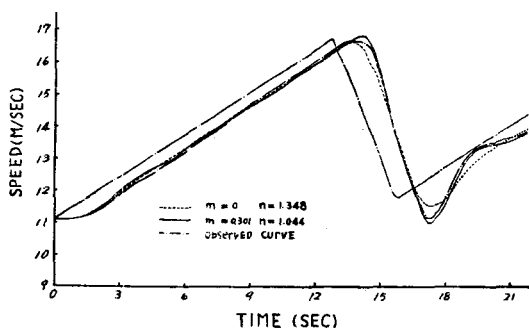


Fig. 1

Next, we performed some experiments to measure brake reaction time in order to answer the question if the reaction time computed from our car following experiment is too small compared to that obtained by other author's experiments.¹⁾ We ordered to the lead car running at the uniform speed to apply sudden full brake, and we computed how long it took until the brakes began to apply to the following vehicle since it recognized the brake application of the lead car. This experiment was also performed with our trace-marking apparatus. As the experimental result is shown in Figure 2, you can find that the reaction time is larger as the speed is higher. As is known from Figure 2, around the speed

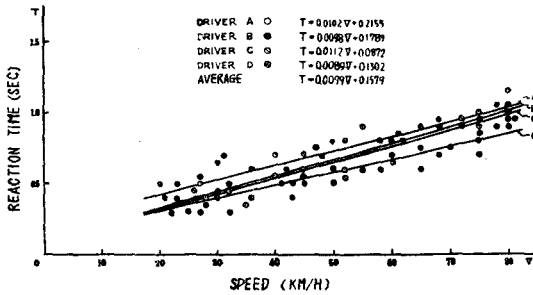


Fig. 2

(1) Indicial response

We now consider the indicial response which tells us clearly the characteristics of the system of cars by a simple mathematical procedure, though it is not a real phenomenon.

At first, we consider a case when all cars are standing still when $t=0$ and the lead car starts to move at $t=0+$ with its initial speed v_0 . In this case, the initial conditions are

$$\begin{aligned} v_1(t) &= v_0 \text{ for } t \geq 0+ \\ v_1(0-) &= 0 \\ v_{k+1}(0-) &= 0 \quad (k=1, 2, 3, \dots) \\ v_{k+1}^*(t) &= 0 \end{aligned}$$

Repeated application of Eq. (5) gives

$$\begin{aligned} V_1(s) &= v_0/s \\ V_2^{**}(s) &= v_0 E(s)/s \\ V_3^{**}(s) &= v_0 E^2(s)/s \\ &\dots\dots\dots \\ V_{k+1}^{**}(s) &= v_0 E^k(s)/s \end{aligned} \quad (6)$$

where,

$$E(s) = (1 - mTs) \exp(-Ts) / [nTs + \exp(-Ts)].$$

Expanding Eq. (6) into polynomials of s , their inverse Laplace transformations are obtained as follows:

$$\frac{v_{k+1}^{**}(t)}{v_0} = \frac{1}{n} \left(\frac{t}{T} - 1 \right) - \frac{1}{2n^2} \left(\frac{t}{T} - 2 \right)^2 + \frac{1}{6n^3} \left(\frac{t}{T} - 3 \right)^3 - \frac{1}{24n^4} \left(\frac{t}{T} - 4 \right)^4 + \dots$$

of 60 km/h the brake reaction time is considered nearly as 0.70~0.80 sec, and thus it is not proper to say that the reaction time obtained in our car following experiment is not necessary too short.

(b) Response of the following cars

$$\begin{aligned}
 & -m \left[\frac{1}{n} U\left(\frac{t}{T}-1\right) - \frac{1}{n^2} \left(\frac{t}{T}-2\right) + \frac{1}{2n^3} \left(\frac{t}{T}-3\right)^2 - \frac{1}{6n^4} \left(\frac{t}{T}-4\right)^3 + \dots \right], \\
 & \frac{v_3^{**}(t)}{v_0} = \frac{1}{2n^2} \left(\frac{t}{T}-2\right)^2 - \frac{1}{3n^3} \left(\frac{t}{T}-3\right)^3 + \frac{1}{8n^4} \left(\frac{t}{T}-4\right)^4 - \frac{1}{30n^5} \left(\frac{t}{T}-5\right)^5 + \dots \\
 & - 2m \left[\frac{1}{n^2} \left(\frac{t}{T}-2\right) - \frac{1}{n^3} \left(\frac{t}{T}-3\right)^2 + \frac{1}{2n^4} \left(\frac{t}{T}-4\right)^3 - \frac{1}{6n^5} \left(\frac{t}{T}-5\right)^4 + \dots \right] \\
 & + m^2 \left[\frac{1}{n^2} U\left(\frac{t}{T}-2\right) - \frac{2}{n^3} \left(\frac{t}{T}-3\right)^2 + \frac{3}{2n^4} \left(\frac{t}{T}-4\right)^3 - \frac{1}{6n^5} \left(\frac{t}{T}-5\right)^4 + \dots \right].
 \end{aligned} \tag{7}$$

Next, we consider the response of following vehicles when the lead car of a queue of cars moving with a uniform speed v_0 stops suddenly at $t=0+$. Initial conditions are

$$\begin{aligned}
 v_1(t) &= 0 \text{ for } t \geq 0+ \\
 v_1(0-) &= v_0 \\
 v_{k+1}(0-) &= v_0 \quad (k=1, 2, 3, \dots) \\
 v_{k+1}^*(t) &= v_0
 \end{aligned}$$

By Eq. (5) we get

$$V_2^{**}(s) = \frac{mTe^{-Ts} + nT}{nTs + e^{-Ts}} v_0 - \frac{v_0}{s} (1 - e^{-Ts}),$$

because

$$\int_{0-}^T v_{k+1}^*(t) e^{-st} dt = v_0 (1 - e^{-Ts})/s$$

Besides, the inverse transformation of the right side of the above equation is $v_0[1 - U(t-T)]$ which vanishes for $t \geq T$. As $V_2^{**}(s)$ is defined only for $t \geq T$, we can write briefly

$$V_2^{**}(s) = (mTe^{-Ts} + nT)v_0 / (nTs + e^{-Ts}) \tag{8}$$

Similarly, we get

$$\begin{aligned}
 V_3^{**}(s) &= E(s) V_2^{**}(s) + D(s) v_0 \\
 &= [1 + E(s)] D(s) v_0
 \end{aligned} \tag{9}$$

$$V_{k+1}^{**}(s) = [1 + E(s) + E^2(s) + \dots + E^{k-1}(s)] D(s) v_0,$$

where

$$D(s) = (mTe^{-Ts} + nT) / (nTs + e^{-Ts}).$$

From Eq. (6) and Eq. (9) we get

$$v_{k+1, start}^{**} + v_{k+1, stop}^{**} = v_0 \tag{10}$$

which is the same relation as that of our previous papers.

How dose the coefficient m affect on the indicial response? The speed of the second car for $T \leq t < 2T$ that is derived from the first equation of Eq. (7) is expressed

$$v_0 \left\{ \frac{1}{n} \left(\frac{t}{T} - 1 \right) - \frac{m}{n} \right\}.$$

The value of this equation will be negative for a moment in the neighborhood of $t=0+$, so far as the sign of m is positive. An impulsive acceleration (or deceleration) of the lead car causes consequently an impulsive acceleration (or deceleration) of the following car because of $v_2^{*+}(T+) = -mv_0/n$.

We can not, however, reject immediately the existence of m in actual traffic. It is because, m being very small relative to n , the inconsistency that the speed of the following car becomes negative is not serious, and in the practical behavior the lead car can not start to move with impulsive acceleration of step function type. In addition, the existence of m has a considerable effect on the stability of traffic flow, to which will be referred in the latter part of this paper.

(2) A case when the lead car is accelerated exponentially

When the lead car of the queue of vehicles, in which all cars are travelling with a uniform speed v_0 , increases its speed exponentially to $v_0 + A$, how do the following cars behave?

In this case, the initial conditions are

$$v_1(t) = v_0 + A(1 - e^{-\lambda t})$$

$$v_k(0-) = v_0 \quad (k=1, 2, 3, \dots)$$

where λ is a constant showing the intensity of acceleration of the lead car. By repeated application of Eq. (5) we have

$$V_1(s) = \frac{v_0}{s} + \frac{\lambda A}{s(s+\lambda)},$$

$$V_2^{**}(s) = \frac{v_0}{s} + \frac{\lambda A}{s(s+\lambda)} E(s),$$

$$V_3^{**}(s) = \frac{v_0}{s} + \frac{\lambda A}{s(s+\lambda)} E^2(s),$$

$$\dots\dots\dots$$

$$V_{k+1}^{**}(s) = \frac{v_0}{s} + \frac{\lambda A}{s(s+\lambda)} E^k(s).$$

In general, movement of the following cars may be represented by the inverse transformation of Eq. (11) in the complicated forms. We show here the only case of the second and the third car:

$$v_2^*(t) = v_0 + AQ_1(t), \quad (12)$$

where

$$\begin{aligned} Q_1(t) = & \frac{t-T}{nT} - \frac{1}{nT\lambda} [1 - e^{-\lambda(t-T)}] - \frac{(t-2T)^2}{2n^2T^2} + \frac{t-2T}{n^2T^2\lambda} - \frac{1}{n^2T^2\lambda^2} [1 - e^{-\lambda(t-2T)}] \\ & + \frac{(t-3T)^3}{6n^3T^3} - \frac{(t-3T)^2}{2n^3T^3\lambda} + \frac{t-3T}{n^3T^3\lambda^2} - \frac{1}{n^3T^3\lambda^3} [1 - e^{-\lambda(t-3T)}] - \dots \\ & - mT \left\{ \frac{1}{nT} [1 - e^{-\lambda(t-T)}] - \frac{t-2T}{n^2T^2} + \frac{1}{n^2T^2\lambda} [1 - e^{-\lambda(t-2T)}] + \frac{(t-3T)^2}{2n^3T^3} \right. \\ & \left. - \frac{t-3T}{n^3T^3\lambda} + \frac{1}{n^3T^3\lambda^2} [1 - e^{-\lambda(t-3T)}] - \dots \right\}. \end{aligned} \quad (13)$$

where

$$\begin{aligned} Q_2(t) = & \frac{(t-2T)^2}{2n^2T^2} - \frac{t-2T}{n^2T^2\lambda} + \frac{1}{n^2T^2\lambda^2} [1 - e^{-\lambda(t-2T)}] - \frac{(t-3T)^3}{6n^3T^3} + \frac{(t-3T)^2}{2n^3T^3\lambda} \\ & - \frac{t-3T}{n^3T^3\lambda^2} + \frac{1}{n^3T^3\lambda^3} [1 - e^{-\lambda(t-3T)}] + \frac{(t-4T)^4}{24n^4T^4} - \frac{(t-4T)^3}{6n^4T^4\lambda} + \frac{(t-4T)^2}{2n^4T^4\lambda^2} \\ & - \frac{t-4T}{n^4T^4\lambda^3} + \frac{1}{n^4T^4\lambda^4} [1 - e^{-\lambda(t-4T)}] - \dots - \frac{2m}{n} \left\{ \frac{t-2T}{nT} \right. \\ & - \frac{1}{nT\lambda} [1 - e^{-\lambda(t-2T)}] - \frac{(t-3T)^2}{2n^2T^2} + \frac{t-3T}{n^2T^2\lambda} - \frac{1}{n^2T^2\lambda^2} [1 - e^{-\lambda(t-3T)}] \\ & + \dots \left. \right\} + \frac{m^2}{n^2} \left\{ 1 - e^{-\lambda(t-2T)} - \frac{t-3T}{nT} + \frac{1}{nT\lambda} [1 - e^{-\lambda(t-3T)}] + \frac{(t-4T)^2}{2n^2T^2} \right. \\ & \left. - \frac{t-4T}{n^2T^2\lambda} + \frac{1}{n^2T^2\lambda^2} [1 - e^{-\lambda(t-4T)}] - \dots \right\}. \end{aligned}$$

From Eq. (12) $v_2^*(t)$ is reduced to

$$v_0 \left\{ \frac{t-T}{nT} - \left(\frac{1}{nT\lambda} + \frac{m}{n} [1 - e^{-\lambda(t-T)}] \right) \right\}$$

for $T \leq t < 2T$. Therefore, the speed of the second car begins to decrease for a moment just as same as the case of indicial response. In actual phenomenon, however, the temporary decrease of speed due to $m > 0$ is negligible since the amount of decrease of speed is less than one percent of original speed.

(c) Effects upon stability

The stability of the system if m is considered differs from those of the system when m is neglected. It is because, as was explained already, the value of n of the latter is equivalent to the value of $n+m$ of the former. Consequently, each system has its own discrimination.

At first, we will consider the stability of the propagation of a disturbance.

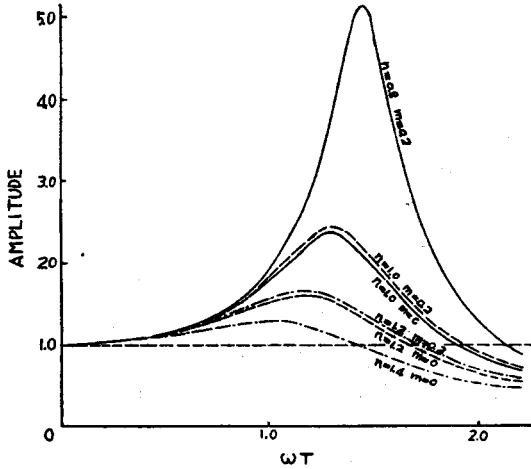


Fig. 3

Figure 3, represents the amplitude of frequency response $|E(j\omega)|$ of each system. Unstable domain of the system for $m=0$ where $E(j\omega) \geq 1$ is satisfied is wider by 10~20 percent than that for $m \neq 0$, provided that the value of n in the system for $m=0$ is equal to $n+m$ in the other system. Besides, the stability condition of the propagation of a disturbance with a frequency

$2\pi/\omega$ is

$$(n^2 - m^2)/n > 2 \sin \omega T / \omega T$$

for the system where m is considered, and

$$n + m > 2 \sin \omega T / \omega T$$

for the system where m is neglected because n in this system is equivalent to in the other system. Therefore, the system for $m > 0$ must have more possibility of being considered unstable than that for $m = 0$.

Traffic flow which must be considered to be stable when m is neglected happens to be regarded as unstable, taking m into consideration.

Next we will consider the stability condition of transient response. As is stated in our previous paper, the condition under which the transient response is stable is given by $n > 2/\pi$ independent of m . However, since the value of n is replaced by $n+m$ when the basic equation for $m=0$ is applied, the stability condition in the system for $m=0$ is attained more easily than that for $m > 0$.

We may come to a similar conclusion on the condition accompanying oscillation in the transient state.

STABILITY LIMIT OF TRAFFIC VOLUME

As the traffic volume increases the passing over becomes harder and harder. If the traffic volume on roads exceeds a certain limit, the rear-end collision will increase promptly and will not be able to prevent it however the driver might try to follow as safe as he can. We call such a limit of traffic volume as the stability limit. In the highway projects we should not assign the future traffic volume exceeding the stability limit for the highway.

On most cases, the stability limit becomes smaller than the traffic capacity on the highway projected. Since the average car space in the following behavior is given by $(n+m)Tv_0+b_0$, we have $v_0/N > (n+m)Tv_0 + b_0$, where N is the traffic volume and v_0 is the average speed of cars. As the stability condition for all frequencies of the disturbance is $(n^2 - m^2)/n > 2$, we have as the stability limit

$$N = 1 / \left[\left(\frac{2}{1-h} \right) T + \frac{b_0}{v_0} \right], \quad (14)$$

where $k = m/n$.

Even if the traffic volume satisfies Eq. (14), we may find the unstable state in some parts of traffic due to the lack of uniformity of traffic. In case of projecting highways, however, we can regard the traffic volume given by Eq. (14) as the traffic limit to keep traffic to be stable. The stability limit of traffic volume is given by Table 4, provided that $b_0 = 5$ meters and $k = 0.2$. In our car following experiments the mean

		$b_0 = 5\text{m}, k = m/n = 0.2$					
$V(\text{KM/H})$	$T(\text{SEC})$	0.75	1.00	1.25	2.50	1.75	2.00
40		1550	1220	1010	860	750	660
60		1660	1290	1050	890	770	680
80		1710	1320	1070	910	780	690
100		1750	1340	1090	920	790	790
120		1780	1360	1100	920	800	700

Table-4

value of k was 0.1051 in case of lower speed and 0.1890 in case of higher speed. In case of projecting expressways, we should adopt $k=1/3$ which is considered to be appropriate from the standpoint of traffic safety, the value of k becoming slightly large as the speed increases.

Figure 4 shows the relation between the number of the rear-end

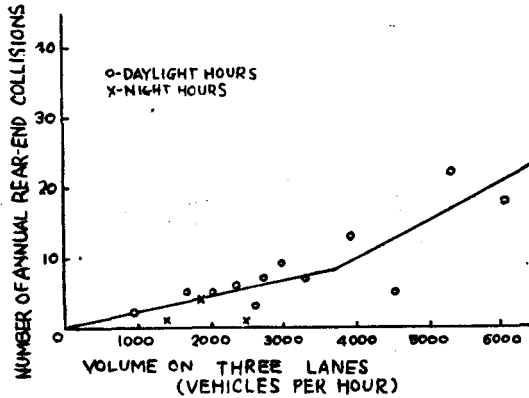


Fig. 4

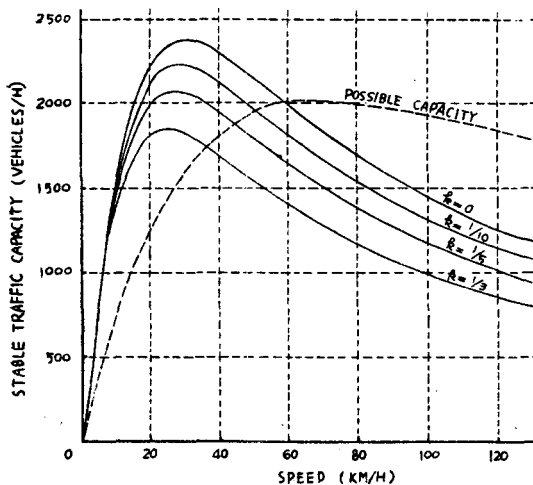


Fig. 5

experiments, we have Figure 5 in which the curves of stability limit for

collision and the traffic volume on three lanes when the rear-end collision occurred. This data is quoted from the paper written by A. F. Malo and others,⁵⁾ and suggests the different character between the domain in which traffic volume is less than about 3,800 vehicles per hour and that more than this volume, as is shown by the broken line.

It is reported in the car following experiments performed in U. S. A. that the reaction time observed is 1.0~2.2 seconds. If we take this value as reliable, the traffic volume greater than 3,800 vehicles per hour seems to exceed the stability limit.

Using the reaction time $T(\text{sec})=0.010$
 $v(\text{km/h})+0.16$ which was obtained through our

$k=0, 1/10, 1/5$, and $1/3$ are shown. As is seen from Figure 5, the stability limit for $v>50$ km/h is smaller than the possible capacity which is used in traffic engineering.

We will consider the express highway from Nagoya to Kobe (Meishin Express Highway) which is at present under construction. It is reported that Meishin Express Highway has two lanes in each direction and its design speed is 120 km/h and its traffic capacity in one direction is estimated 1,710~1,780 vehicles per hour.⁶⁾ This capacity is calculated as a practical capacity in which the effect of mixed traffic is taken into consideration. Considering from the standpoint of stability, we can say that the traffic capacity in one direction on its expressway becomes 1,700($=850 \times 2$) vehicles per hour from the stability limit curve for $k=1/3$ and $v=120$ km/h in Figure 5. In the present case it is very interesting that the stability limit is nearly equal to the practical capacity.

CONCLUSION

In this paper we have got the following conclusion:

- (1) the sign of m was always positive in our car following experiments,
- (2) due to m the speed of following car behaves as if time delay were extended in the neighbourhood of transition point between acceleration and deceleration,
- (3) the value of nT in the system where m neglected corresponds to $(n+m)T$ in the system where m considered,
- (4) neglecting m we shall make an error in diagnosis for traffic flow that we decide it stable even if it is unstable,
- (5) in projecting highways we should not assign the future traffic volume on a projecting highway beyond its stability limit. Because the rear-end collision will occur more frequently in traffic volume greater than stability limit,
- (6) the stability limit may be considered as a practical capacity for traffic at higher speed,
- (7) in projecting roads where the design speed is higher than 50 km/h, we should take the stability limit into consideration,
- (8) in projecting express highways we should adopt the stability limit for $k=1/3$ as a practical capacity which becomes about 1,000 and 850

vehicles per hour on one lane for the design speed of 100 and 120 km/h respectively.

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