PREDICTION THEORY AND DYNAMIC PROGRAMMING (II)

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The theory of prediction which will be given in this paper is essentially an extension of the previous paper[1].

We discuss some application of the functional equation technique of dynamic programming to the numerical method of this extended prediction theory.

We shall first take notice of results of the previous paper and sketch on the prediction theory extended by L. A. Zade and J. R. Ragazzini[3)4)]. Then we shall discuss the same device as the method of the previous paper.

1. INTRODUCTION

In the previous paper[1], the problem of sep arating a message represented by a sequence a_k described statistically by a given auto—correlation function $R_a(n)$ and cross-correlation function $R_{ba}(n)$ from a signal represented by a sequence b_k with a given auto-correlation function $R_b(n)$ was considered.

We want to choose the $\{A_n\}$ so as to minimize the *rms* of

$$\varepsilon_k = a_{k+s} - \sum_{n=0}^{M} A_n b_{k-n},$$

$$I_M = R_a(0) - 2\sum_{n=0}^{M} A_n R_{ba}(n+s) + \sum_{n=0}^{M}\sum_{n=0}^{M} A_n A_m R_b(m-n). \tag{1}$$

If we write

$$I_M = 1 - E_M,$$

$$E_M = 2\sum A_n \psi_{m+s} - \sum\sum A_n A_m \varphi_{m-n}, \tag{2}$$

where $\dfrac{R_b(k)}{R_a(0)} = \varphi_k$ and $\dfrac{R_{ba}(k)}{R_a(0)} = \psi_k$,

then our problem is reduced to determining the maximum of the unhomogeneous form (2).

To determine the maximum of the unhomogeneous form (2), let us define the iteration formulae

$$f_M(\psi_{0+s},\ \psi_{1+s},\ \cdots\cdots,\ \psi_{M+s})$$
$$=\max_{(A_n)}[2\sum_{n=0}^{M} A_n\psi_{n+s}-\sum_{n=0}^{M}\sum_{n=0}^{M} A_nA_m\varphi_{n-m}]. \qquad (3)$$

It is easy to see that

$$f_0(\psi_{0+s})=\max_{(A_0)}[2\psi_{0+s}A_0-\varphi_0A_{20}]=\frac{\psi^2{}_{0+s}}{\varphi_0}, \qquad (4)$$

and

$$A_0=\frac{\psi_{0+s}}{\varphi_0}.$$

We now wish to derive a recurrence relation between f_M and f_{M-1}. If we fix A_M and the maximizing over the other $\{A_n\}$, we obtain the relation

$$f_M(\psi_{0+s},\ \psi_{1+s},\ \cdots\cdots,\ \psi_{M+s})=\max_{(A_n)}[2\psi_{M+s}A_M-\varphi_0A_M{}^2$$
$$+f_{M-1}(\psi_{1+s}-\varphi_{M-1}A_M,\ \cdots\cdots,\ \psi_{M+s}-\varphi_1A_M)] \qquad (5)$$

We obtain that f_M is a quadratic form in the variables ψ_{i+s},

$$f_M=\sum_{ij=1}^{M} b_{ij}^{(M)}\psi_{i+s}\ \psi_{j+s}. \qquad (6)$$

Then we see that the recurrence relation between f_M and f_{M-1} may be utilize to obtain recurrence relations for the sequences $\{b_{ij}^{(N)}\}$.

2. AN EXTENSION OF WIENER' S THEORY OF PREDICTION

We consider a given time series b_k which is the sum of signal S_k and *a* stationary random disturbance N_k, with the signal being composed of a random component M_k superposed upon a non-random term P_k, i. e.

$$S_k=M_k+P_k,$$
$$b_k=S_k+N_k. \qquad (7)$$

The assumptions made concerning the characteristics of P_k, M_k and N_k are as follows;

(a) P_k is assumed to be representable as a polynominal of a degree k not higer than a specified number.

(b) M_k and N_k are stationary functions of time described respectively by their auto-correlation functions $R_M(n)$ and $R_N(n)$.

(c) M_k and N_k have zero means and are uncorrelated each other.

The problem is to determine the optimum set of numbers A_n in order that a_{k+s} should be represented as closely as possible by

$$\sum_{n=0}^{M} A_n b_{k-n}. \tag{8}$$

Proceeding as in the previous paper, we now choose the A_n so as to let the mean of ε_k equal to zero and to minimize the *rms* of

$$\varepsilon_k = a_{k+s} - \sum_{n=0}^{M} A_n \ b_{k-n}. \tag{9}$$

By hypotheses

$$b_k = P_k + N_k + M_k. \tag{10}$$

Substituting (10) into (9) and making use of the Taylor expansion for $P_{k-\nu}$, it will be found that $\sum A_n b_{k-n}$ may be expressed as

$$\sum A_n b_{k-n} = \mu_0 P_n - \mu_1 P_k' + \frac{\mu_2}{2!} P_k'' + \cdots\cdots + (-1)^n \frac{\mu_n}{n_1'} P_k^{(n)}, \tag{11}$$

where μ_0, μ_1, μ_2 etc, are designated as the moments of A_n, i. e.,

$$\mu_\nu = \sum_{n=0}^{M} n^\nu A_n, \ \nu = 0, \ 1, \ \cdots\cdots, \ n. \tag{12}$$

Since M_k and N_k are stationary (with means zero) it follows that the ensemble means of $\sum A_n b_{k-n}$ and a_{k+s} depend only on the non-random component of the signal, and it is easily found that;

$$\begin{aligned} \mu_0 &= \sum A_n = 1, \\ \mu_1 &= \sum n A_n = \alpha, \\ &\cdots\cdots \\ \mu_\nu &= \sum n^\nu A_n = \alpha^\nu, \end{aligned} \tag{13}$$

which represent the constraints imposed upon A_n when the quantity to be estimated is a_{k+s}.

The problem that remains to be solved is that of minimizing *rms* of

$$\varepsilon_k = \sum A_n (M_{k-n} + N_{k-n}) - M_{k+s} \tag{14}$$

Instead of Eq. (1), we find

$$I_M = R_M(0) - 2\sum A_n R_M(n+s) + \sum\sum A_n A_m \{R_M(n-m) + R_N(n-m)\}. \tag{15}$$

Therefore, our problem is to minimize I_M with respect to the class of A_n satisfying (13). But, it should be noted that the first term in (15) is independent of A_n and hence the maximization of E_M should be considered, as in (2).

$$I_M=1-E_M,$$
$$E_M=2\sum A_n R_M(n+s)+\sum\sum A_n A_m\{R(n+m)+R_n(n-m)\}. \quad (16)$$

The root mean square error approach used here is a approximation developed by N. Wiener and its extension described by L. A. Zadeh and J. R. Ragazzini[3].

3. DYNAMIC PROGRAMMING AND LAGRANGEAN MULTIPLIER

What we do is to combine the functional equation technique with the classical Lagrangean multiplier formalism.

The problem of maximizing E_M with respect to the class of A_n satisfying (17) reduces essentially to an isoperimetric problem in the calculus of variation,

$$G(A_1,A_2,\cdots\cdots,A_M)=[2\sum A_n\phi(n+s)-\sum\sum A_n A_m\{\varphi(n-m)+\phi(n-m)\} \\ +2\lambda_0\sum A_n+2\lambda_1\sum nA_n+\cdots\cdots+2\lambda_\nu\sum n^\nu A_n], \quad (17)$$

where λ_0, λ_1, λ_2, $\cdots\cdots$are the Lagrangean multiplier, and

$$\phi_k=\frac{R_M(k)}{R_M(0)}, \qquad \varphi_k=\frac{R_M(k)}{R_M(0)}.$$

For fixed $\{\lambda\}$, introduce the sequence of functions

$$f_M(\phi_{0+s},\phi_{1+s},\cdots\cdots,\phi_{M+s})=\max_{(A_n)}[G(A_1,A_2,\cdots\cdots,A_M)] \quad (18)$$
$$=\max_{(A_n)}[2\sum A_n\{\phi(n+s)+\lambda_0+\lambda_1 n+\cdots\cdots+\lambda^\nu n^\nu\}-\sum\sum A_n A_m \\ \{\varphi(n-m)+\phi(n-m)\}].$$

Let $A_0(\lambda_0,\lambda_1,\cdots\cdots,\lambda_\nu)$, $A_1(\lambda_0,\lambda_1\cdots\cdots\lambda_\nu)$, $\cdots\cdots$, $A_M(\lambda_0,\lambda_1,\cdots\cdots,\lambda_\nu)$ be a set of values yielding the maximum of $G(A_0, A_1, \cdots\cdots, A_M)$. Then we assert that these values yield the solution of the problem of maximizing (2) subject to the constraints in (13).

We observe that the only difference between these equations(18) and (3) lies in the ϕ_{n+s} which is now increased by $\lambda_0+\lambda_1 n+\cdots\cdots+\lambda_\nu n^\nu$.

4. DETERMINATION OF AN EXPLICIT EXPRESSION FOR A_M

Recalling that f_M is a quadratic form in the variables ϕ_i,

$$f_M(\phi_{0+s},\phi_{1+s},\cdots\cdots,\phi_{M+s})=\sum b_{ij}^{(M)}(\phi_{i+s}+\lambda_0+\cdots\cdots+\lambda_\nu i^\nu) \quad (19) \\ (\phi_{j+s}+\lambda_0+\lambda_1 j+\cdots\cdots+\lambda_\nu j^\nu),$$

and the reurrence relation connecting f_M and f_{M-1} may be utilized to obtain the recurrence relations for the sequence $b_{ij}^{(N)}$,

$$b_{ij}^{(M)}=b_{ij}^{M-i}-\frac{\left(\sum_{k=0}^{M-1}b_{k-i}^{(M-1)}(\varphi_{M-k}+\psi_{M-k})\right)\left(\sum_{l=0}^{M-1}b_{j}^{(M-1)}(\varphi_{M-l}+\psi_{-l})\right)}{-\varphi_0-\psi_0+\sum_{kl=0}^{M-1}b_{Ml}^{(M-1)}(\varphi_{M-k}+\psi_{M-k})(\varphi_{M-l}+\psi_{M-l})}. \quad (20)$$

$$(i,j=1,\ 2,\ \cdots\cdots,\ M-1)$$

$$b_{iM}^{(M)}=-\frac{\sum_{k=0}^{M-1}b_{ki}^{(M-1)}(\varphi_{M-k}+\psi_{M-k})}{-\varphi_0-\psi_0+\sum_{u,l=0}^{M-1}b_{k,l}^{(M-1)}(\varphi_{M-k}+\psi_{M-k})(\psi_{M-l}+\psi_{M-l})}.$$

$$(i=1,\ 2,\ \cdots\cdots,\ M-1)$$

$$b_{MM}^{(M)}=-\frac{1}{-\varphi_0-\psi_0+\sum b_{u,l}^{(M-1)}(\varphi_{M-k}+\psi_{M-k})(\varphi_{M-l}+\psi_{M-l})}. \quad (21)$$

Also, it is readily found that $A_{(M)}$ is given by the following expressions:

$$A_{M}^{(M)}=\frac{(\psi_{M+s}+\lambda_0+\lambda_1 M+\cdots\cdots+\lambda_\nu M^\nu)+\sum_{ij=0}^{M-1}b_{ij}^{(M-1)}(\varphi_{N-j}+\psi_{N-j})(\varphi_{M-l}+\psi_{M-j})}{-\varphi_0-\psi_0+\sum_{ij=0}^{M-1}b_{ij}^{(M-1)}(\psi_{M-j}+\varphi_{M-j})(\psi_{M-j}+\varphi_{M-j})}$$

$$(21)$$

Having obtained the general expression for A_M in the form of (21), there remains the problems of determining the unknown constant ν.

Substituting Eq. (21) into the moment conditions

$$\sum n^r A_M=\alpha^r,\ r=0,\ 1,\ \cdots\cdots r,$$

we get the other linear equations. Solution of this system gives the values of $\{\lambda_\nu\}$ and this completes the process of determining $A_N^{(M)}$

BIBLIOGRAPHY

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2) R. Bellman; Dynamic Progamming, Princeton University Press, Princeton, New Jersey, (1957).

3) L. A. Zadeh and J. R. Bagazzini ; An Extension of Wiener's Theory of Prediction ; Journal of Applied Physics Vol. 21, 645~655 July, (1950)

4) T. Kawata, [Applications of the Theory of the Stochastic Process], Iwanami Koza Gendai Oyosugaku B. 13—d (1958).

OPERATIONS RESEARCH SOCIETY OF JAPAN

SEVENTH NATIONAL (1960 ANNUAL) MEETING

Tokyo, April 23-25, 1960

The Seventh National (and 1960 Annual) Meeting of the OPERATIONS RESEARCH SOCIETY OF JAPAN was held at Waseda University in Tokyo on April 23rd, 24th and 25th, 1960. The sudjects of the individual sessions were:

(on 23rd)

1. Traffic dynamics by Takashi Kishi (Defense Academy)
2. OR and defects by Masaaki Shibuya (Institute of Statistical Mathematics)
3. Design optimization by a threshold passing method by Tatsuki Norimatsu (Electric Laboratory)
4. An application of probability theory to traffic problems by Yoji Kushida (Japanese National Railways)
5. Inversion of matrics including many O elements by Monte Carlo method by Iwaro Takahashi (Production Lab. Waseda University)
6. A multi-channel queuing problem by Toji Makino (Fuji Precision Machine)

SPECIAL LECTURE: Management and OR problems by Takehiko Matsuda (Tokyo Institute of Technology)

(on 24th)

7. Reliability and characteristics of receiving radio tubes by Yoshihiro Saito (NHK Broadcasting Corporation)
8. Optimal conditions in operation of ionized resin films by Yoji Umetani (Tokyo University)
9. Characteristics of dynamic maximin ordering policy by Hiroshi Kasugai and Tadami Kasegai (Waseda University)
10. Prediction Theory and Dynamic Programming by Toshio Odanaka (Metropolitan Technical College)
11. A moving average computer by Kazuo Miyawaki and Kaichi Sogabe (Osaka University)
12. Simulation on railway yard problems by Masayo Kanematsu (Japanese National Railways)
13. Capacity of information channels by Minoru Sakaguchi (University of Electro-communications)

SPECIAL LECTURE: OR and econometrics by Tsunezo Sato (Waseda University)

SPECIAL LECTURE: OR and economic policy by Susumu Kobe (Waseda University)

The factory observation and discussion were carried out by courtesy of the Kawasaki Steel Company at its Chiba Factory on problems of applications of OR to the steel industry on 25th, the last day.

The Annual Business Meeting for 1960 was held at 9 a.m. on April 23rd. At its meeting, the Council elected the staff members, and the plan and budget for 1961 fiscal year were approved.

VISITORS FROM ABROAD

The following experts came to Takyo, Japan, with whom some members held meetings. Those discussions were of great interest, which were reported in the "Keiei-Kagaku" (the Society Japanese Journal.).

Dr. C. J. Craft: 17 June 1959 — Dr. E. L. Arnoff: 3 August 1959

Dr. G. B. Dantzig: 17 November 1959 — Dr. G. Brigham: 25 January 1960

SPECIAL PUBLIC LECTURES AND SEMINARS

THE SOCIETY SPECIAL EVENTS FOR 32ND SESSION OF ISI IN TOKYO, JAPAN

The 32nd session of ISI was held in Tokyo, Japan, May 30th to June 9th, 1960. After that, the official ISI sight-seeing trip to southern Japan was made. On this occasion the Society invited several foreign ISI participants to hold the following special events in Tokyo:

1) SPECIAL PUBLIC LECTURS:

"Scientific Thought and the Refinement of Human Reasoning"
by Sir Ronald A. Fisher (Interpreter: Mr. Kano);

"Statistics and National Planning in India"
by Dr. P. C. Mahalanobis (Interpreter: Dr. Moriguti Sigeiti).

Those lectures were made with the attendance of over eight hundred at the Asahi Press Auditorium on May 28th, and are scheduled to be published in this and next Journal of the Operations Research Society of Japan.

2) SPECIAL SEMINARS:

The subjects of the individual seminars, their chairmen and the guest professors, were:

A. Cox's Renewal Problem by Guest Prof. D. V. Lindley. Chairman: Tatsuo Kawata. Attendants: 10 members.

B. Construction Principles of Simultaneous Equations Models in Econometrics by Guest Prof. H. Wold. Chairman: Koichi Miyasawa. Attendants: 8 members.

C. An Analysis of Water Consumption and Storage Capacity for the Philips Factories in Eindhoven by Guest Prof. H. C. Hamaker. Chairman: Sigeiti Moriguti. Attendants: 9 members.

D. Some Comments on the Least Spuare Estimates in the Gauss-Markoff Model by Guest Prof. C. R. Rao. Chairman : Motosaburo Masuyama. Attendants : 10 members.

Those seminars were held from 13.00 to 20. 30 at Tokyo Kaikan on June 13th, where the Society dinnar party was held at 17. 30 on that day.

PARTICIPATION IN IFORS

The Society dispatched Takehiko Matsuda, Auditor of the Society, to the HQ of IFORS in London for admission to IFORS, in December, 1959.

The following four members were dispatched for participation in the 2nd General Assembly of IFORS held at Aix-en-Province in France, September, 1960 :

T. Kawata (Tokyo Institute of Technology); J. Kondo (Tokyo University)

S. Moriguti (Tokyo University) ; A. Nomoto(Chuo University)

The above items were approved by the Seventh National (1960 Annual) Meeting. For IFORS, qualified members 135 members were figused up in the list of members as of April 1 st, 1960.

AMENDMENTS TO THE SOCIETY CONSTITUTION

The following amendments were approved by the Seventh National Meeting (the Annual General Assembly)

Article 2. The office of the Society shall be located at *the branch office of Kinokuniya Book-Store Co. Ltd., 830 1-chōme Tsunohazu, Shinjuku-ku, Tokyo, Japan.*

Article 7. The membership of the Society shall consist of the following *five* kinds; Honorary Members, Fellows, Regular Members, *Student Members,* and Institutional Members.

Honorary Members shall be those persons who have rendered considerable services to the Society or those who have accumulated significant achievement and experience in any fields related to operations research.

Fellows shall be those persons who have made outstanding contributions research.

Regular Members shall be those individuals who are interested in research or practice in operations research.

Student members shall be college or university students who major in or study operations research.

Institutional Members shall be those juridical persons or other organi zations who support the objectives of the Society and desire to co-operate with it.

Article 8. A person may be approved to acquire each kind of Membership stated above through the following procedures.

1) To an Honorary Member: though the resolution of the General Assembly, based on the recommendation of the Council;
2) To a Fellow: through the approval of the Fellows, based on the recommendation of the Council;
3) To a Regular Member *and a Student member:* through the acceptance by the Council, based on the proposal by two members of the Society;
4) *To a Student Member: After his or her graduation, he or she will be a Regular Member automatically.*
5) To an Institutional Member: through the acceptance by the Council, based on the expression of desire to become an institutional member and on the appointment of the representative.

Article 9. Any Member of the Society enjoys the following privileges (*but the privilege prescribed in the 4th item is not given to the Student Member*):

1) To be informed of the activities of the Society, and to join in its professional meetings;
2) To utilize, according to the rules to be determined, the library which the Society accumulates and takes the custody of;
3) To contribute articles to, and receive every issue of, the journals of the Society;
4) To attend the General Assembly of the Society and participate in voting;
5) To propose his or her desire or opinion about the Society to the Council, and request consideration by the Council.

Article 10. Any Member shall owe the following duties:

1) Fellows and Regular Members shall pay the annual dues of ¥ 1,200 (*Student Member*, ¥ 600), and at the time of their admission to the Society the admission fee of ¥ 300.
2) Institutional Members shall pay the annual dues of ¥ 10,000. The payment may be divided into two instalments.

Article 13. The officers shall be the following procedures:

1) The President and the Vice-presidents shall be elected by the Council from among the Council Members.
2) The Standing Council Members shall be nominated from among the Council Members by the President.
3) The Council Members and the Auditors shall be elected by the Board of Trustees from among the Trustees themselves.
4) The Trustees shall be ellcted by all the Members of the Society at the General Assembly of the Society.
5) *The Student Member shall not be elected as the officers of the Society.*