Journal of the
Operations Research
Society of Japan

VOLUME 2 *March 1960* NUMBER 4

A PRE-COMPUTATIONAL REPORT ON JOB SHOP SIMULATION RESEARCH

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(October 29, 1959)

ABSTRACT. This report has been written for the purpose of organizing and recording a collection of thoughts, observations, and notions which have resulted in a simulation model for a generalized job shop production process, an experimental outline for studying the functioning of such systems, and a computer program for the IBM 709 at the Western Data Processing Center, UCLA. The study is not pointed toward any specific production system. Conversely, it is intended to determine, through experimentation, the relative importance of the effects of the system variables, to develop and test hypotheses, and to evaluate alternative queue disciplines under a variety of conditions and goals. The model, computer operations, and experimental outline in their present states are discussed in that order. The results of computations and analysis will be reported separately at a later date.

It is further anticipated that the paper may elicit some suggestions for extensions and/or revisions of the plans at a time when they will be most useful.

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* This work was supported by the office of Naval Research under Task NR 047—003 (Management Sciences Research Project, University of California, Los Angeles). Reproduction in whole or in part is permitted for any purpose of the United States Government. The authors express appreciation to James R. Jackson and Kenneth R. MacCrimmon for their interest and suggestions.

I. INTRODUCTION

This paper reports the current status of simulation research in job shop type production systems being conducted by the Management Sciences Research Project. This report is a sequel to an earlier paper [9] which described the initial simulation program and computations on the National Bureau of Standards Western Automatic Computer (SWAC). The installation of an IBM 709 at the Western Data Processing Center, UCLA, in October, 1958, had considerable effect on the structure of the model. The 709, designed for both scientific computation and data processing, has impressive computing speed, large internal storage, and an efficient input-output system capable of handling a large volume of data. As a result of these improvements in computing facilities it became practical to consider many extensions and revisions of the original model. It is felt that the model, in its present state, retains the generality which characterizes it as an experimental model for research into the basic properties of job shop type systems, rather than a model for simulating the operations of a specific firm. It is further sensed that the important variables of job shop systems are included as variables in the model; whether this is truly the case can be determined, of course, only from extensive experimentation. The relaxation of assumptions and introduction of additional variables resulting in a more complex model may become an important part of the research as the study proceeds. In any event, the authors are of the opinion that a great deal needs to be learned about the basic functioning of the system and the relative effects of certain obvious key variables on general properties of the system's output. The current version of the model has been designed with these basic goals in mind, together with the idea of flexibility for initiating more refined studies which the initial experiments may indicate as important.

It seems to be customary in the introduction of any research report to present a short history of the project since its inception, as well as to point out some rationale for the particular approach being used. In this case, the authors prefer to suggest two references for the interested reader; [20] for the history and [7] for the rational.

It seems appropriate to attempt to classify the research reported here in reference to other simulation studies of job shop type production systems reported by Cornell University [2], [3], General Electric Company [17] and the International Business Machine Corporation [1]. The Cornell research simulator reflects a model similar in structure to the one described here. The authors are not at the present time aware of the experiments that are planned with the program. The General Electric study involves a model designed to reflect the characteristics of an actual system and the experiments were planned primarily to investigate the effectiveness of various queue disciplines under a variety of operation conditions*. The IBM model is a very simple experimental model of a generalized job shop production process. The experiments were designed to ascertain the effects of various parameters and queue disciplines. The IBM study is the most closely related to the one reported here; possessing the same basic experimental philosophy but differing mainly in the complexity of the model and the number and choice of variables for experimentation.

The report consists of three sections and an appendix. The first section is a description of the simulation model. This description includes discussions of the system, the assumptions, the variables, the measures of effectiveness, and the equations which, define and relate the principal terms used throughout the paper. The second section deals with computer operations in some detail. Discussions concerning input data, initialization of the shop, simulation of operations and output data, are intended to provide a clear understanding of the structure of the computer model with no reference to the details or techniques of programming. The final section is devoted to an outline and discussion of experimental plans as they exist just prior to the initial computations. The appendix is a flow chart intended to provide readers interested in the specifics of the computer program with a means for following the routine with little difficulty.

II. THE MODEL

A. THE SYSTEM:

1. Input. The input to system is composed of jobs to be processed

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* See, for example [17], abstract and P. 22.

by the production facilities. Each job is described by a *routing* and a set of operation *processing times*. The job routing specifies the machine centers required for processing the job and the sequence in which they must be used. The set of processing times are identified with the required operations and reflect the total time necessary to perform each operation on the job. Jobs arrive or enter the processing facilities in a continuing statistically specified time pattern. A *due data* may or may not be associated with each job.

2. Processing facilities. The processing facilities (shop) consist af a collection of *machine centers* each of which contains one or more interchangeable *machines* for processing incoming jobs.

3. Output. The output from the processing facilities is made up of completed jobs. A *completion time* is associated with each job.

B. ASSUMPTIONS OF THE MODEL:

1. The arrival of jobs in the shop is statistical in nature. The governing probability distribution remains unchanged during each experimental run, e. g., changes in current shop load conditions do not result in changes in the arrival pattern. The arrivals occur at discrete time intervals.

2. The routing for each job is fixed and is known when the job arrives in the shop. The set of processing times for each job is known when the job arrives in the shop.

3. There is a single queue of jobs for each machine center.

4. There is a priority associated with every job in a queue. Whenever there are idle machines and jobs in queue, jobs are assigned to these machines in order of job priority, i. e., machines are not allowed to idle in anticipation of higher priority jobs.

5. No job is processed on more than one machine at one time. No machine may process more than one job at one time. The total processing time requirement for an operation on any job must be satisfied during a single continuous time interval. This precludes the use of techniques such as job lot "splitting."

6. The processing facilities are fixed. Labor and capital are assumed available to maintain the facilities.

7. No transportation time must be allowed for during the processing.

8. Subcontracting and overtime are not allowed.

9. Machine breakdowns, fabrication errors, and similar disturbances, do not occur in the system.

10. Setup time is implicitly included in the processing time for each operation.

The limitation imposed by the assumptions are considered desirable for initial experimentation because of the complication of the problem in the present restricted form. Any or all of these assumptions may be removed after more has been learned about the basic system.

C. VARIABLES OF MODEL:

The eight factors listed below were selected as the variables of the system for experimantation. Each factor is discussed in some detail below.

Shop load parameters:

1. Mean arrival rate of jobs in the shop. (λ)
2. Mean processing rates at the machine centers. (μ_j), $j=1, 2, \cdots\cdots, N$
3. Shop size (number of machine centers and number of machine in each) N, η_j; $j=1, 2, \cdots\cdots, N$

Operational characteristics of the system:

4. Distribution of arrivals of jobs in the shop.
5. Processing time distributions at the machine centers.
6. Job routing generation procedure.
7. Job los size variation vs. operation complexity variation.

Queue discipline:

8. Policy for resolving conflicts for assignment of jobs in queues.

Factors 1—3 are termed the shop load parameters because they serve to determine the steady state average machine utilizations for each machine center according to the formula:

Average machine utilization (center j)$=\rho_j=\dfrac{K_j\lambda}{\eta_j\mu_j}$, where λ, η_j, and μ_j are the load parameters and K_j is the average number of operations at machine center j per job arrival to the shop. In this model the job routings are based on empirical data. Hence the average K_j are fixed and not listed as a load parameter.

The shop load parameters serve to determine the average utilization vectors which consist of the average utilization at the machine center. Changes in the values of these parameters only affect the capacity of and/or the load upon the system. Thus, these parameters determine the utilization of the system.

The distribution of job arrivals in the shop is the frequency distribution which governs the time pattern of the entry of jobs into the production system. The form of this frequency distribution is one of the variables of the model, and because it affects the subsequent inter-arrival patterns throughout the system, it is termed an operational characteristic of the system.

The frequency distributions of the processing times at the machine centers are considered as a single variable of the model (although they may be changed singly and, in that sense, viewed as N variables). These distributions also influence the entire system of operations and are termed an operational characteristic of the system.

The job routing generation procedure refers to the manner in which samples of job routings are generated from empirical data. Changes in this variable (procedure) correspond to different assumptions about the inter-dependence of successives states in the job routings. This operational characteristic is discussed in detail in Section III on Computer Operations.

The job lot size variation vs. operation complexity variation is introduced in the model as a measure of the "correlation" among the processing times on individual jobs, The processing time distributions from which operation times are assigned are based on empirical data. Each operation time is determined jointly from a job lot size and operation complexity. Lot size remains unchanged throughout the job's operations while the operation complexity varies randomly. An increase (decrease) in the operation complexity variation relative to the lot size variation tends to decrease (increase) the correlation among assigned processing times for individual jobs. The relative variation in these two factors is the final operational characteristic variable of the model.

The queue discipline is a variable of the model. It is considered primarily as a decision variable rather than a descriptive one is the experimentation and is listed separately for this reason.

D. DEFINITIVE EQUATIONS:

The following equations relate some of the terms used in the study. The last equations pertains only to the due data version of the model.

1. Job Arrival Time+Total Processing Time+Total Waiting Time in Queues=Job Completion Time.
2. Job Flow Time=Job Completion Time=Job Arrival Time= Total Processing Time+Total Waiting Time in Queues.
3. Job Tardiness=Job Completion Time−Job Due Data.

E. MEASURES OF EFFECTIVENESS:

At this point, it seems appropriate to re-emphasize the fact that the model described here was designed for experimentation to lead to better understanding of the basic properties and functions of job-shop type production processes; it was *not* designed for simulating or optimizing the performance of any particular system. This distinction becomes important when one considers measures of effectiveness for the model. It seems clear that a measure of effectivness for a model of any specific production system (military production perhaps excluded) should be an economic measure involving properly defined costs, revenues, and functional relationships based on study of the system. For a general experimental model of a job-shop type process on the other hand, it is implied that one abstracts certain basic properties of the system that are clearly related to any reasonable economic measure of effectiveness and studies the functioning of the system with respect to these basic properties, through a systematic analysis involving a variety of load conditions, operational characteristics, and queue disciplines. Although the study of system properties does not correspond to the optimization of a quantitative function, measures of these properties may, nevertheless, be considered the measures of effectiveness for the experimental model.

The model outlined in this section allows for two possibilities in taht each job may or may not have a final due date associated with it. In the discussion which follows we shall refer to these as the due date version (DV) and the non-due date version (NDV) of the model, respectivery.

The basic properties of the system selected as measures of effectiveness for the NDV of the model are:

1. Job *flow time* distribution. Before analysis, the flow times may be adjusted in some manner such as dividing each job flow time by its total required processing time.
2. *Congestion in the shop.* The congestion will be measured in terms of queue-length distributions.

For the DV of the model, one additional property is included as a measure of effectiveness of the operations.

3. Job *tardiness* distribution. The tardiness for any job is simply the difference between its actual completion time in the simulation and its prespecified due date.

III. COMPUTER OPERATIONS

In this section we discuss the computer operations which are used for each experiment. The four major parts of the computer program; input, initialization, simulation of shop operations, and output are considered in that order. The intent here is to provide only an explanatory outline of the routine without discussing the details of the programming techniques.

A. INPUT DATA.

1. Specification of selected shop load parameters, operational characteristics, and queue discipline (external input).

The initial step, prior to the first computer operations, consists of;

A. assigning numerical values for the three shop load parameters: mean shop arrival rate, mean service rates at machine conters, and shop size,

B. selecting distribution functions and making the necessary tables available for internal storage in the computer to describe the operational characteristics of the system: shop arrival interval distribution, machine center service time distributions, job routing probability distribution, and lot size variation vs. operation complexity variation,

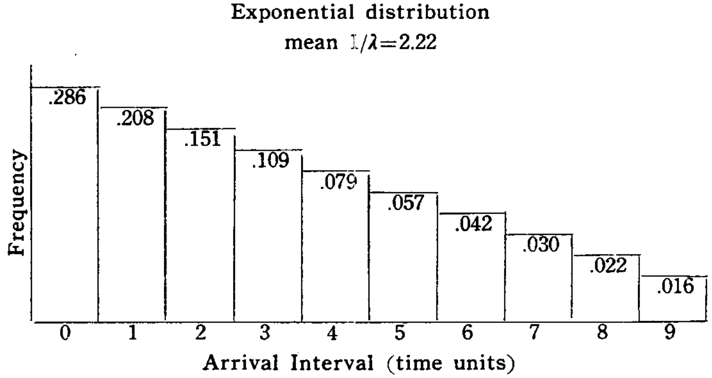
C. specification of the queue discipline to be used in assigning jobs from waiting lines at the machine centers.

The either items of input information are selected externally based upon the shop conditions desired for the experiment. The term

" external input " will be used throughout the remainder of the paper to describe this portion of the input for the simulation. The question of how the numerical values for external input will be selected is reserved for section 3 when the experimental plan is discussed.

2. Determination of job arrival times (internal input)

The arrival interval frequency distribution and the mean arrival interval are specified as part of the external input. For each desired combination of arrival interval frequency distribution form and mean, a table must be computed to specify the cut-off values of the frequency cells. Digits from 0—1000 are used for this purpose. The tables are stored in the computer for reference in this portion of the routine. Figure 1 illustrates the arrival interval data in graphical and stored tabular form approximating an exponential arrival interval frequency distribution.



Arrival Intarval	*Cut-off Poines*
0	0—286
1	286—494
2	494—645
3	645—754
4	754—833
5	833—890
6	890—932
7	932—962
8	962—984
9	984—1000

Figure 1

Arrival Interval Data-Example of Graphical and Tabular Forms

Note that arrival intervals are considered only as discrete synthetic time units ranging from 0—9. The entire simulation is based upon discrete advances of a synthetic "time clock." The range 0—9 for arrival intervals was selected as a compromise between accuracy and economy of operations.

The arrival time for each job is derived from the tabular form of the data as follows:

1. A random number between 0—1000 is generated.*
2. The generated number is identified with an arrival interval cell in the table and the corresponding arrival interval is recorded. This is the arrival time for the first job.
3. Steps 1 and 2 are repeated and a second arrival interval obtained.
4. The second arrival interval is added to the first arrival time. This is the arrival time for the second job.
5. The process is continued until the number of arrival times in the sequence corresponds to the specified maximum number of jobs to be used for the experiment. The complete sequence of arrival times is then stored for use in the simulation.

3. Generation of job sample. (internal input)

The next two phases of the computer operations "generate" a sample of a specified number of jobs to be used in the experiment. The complete specification of each job includes (1) a *routing* which gives the machine centers at which the job will require processing, as well as the order in which the work centers are to be used, a *processing time* for each of the required operations. These are discussed separately below. It should be mentioned that assumptions 2 and 10 of the model are directly related to the job sample generation.

a. Job routing. An empirical study of manufacturing outlines from a number of firms engaged in job shop production provided the basis for the job routing data in the simulation. A sample of 1,000 actual manufacturing outlines was used to derive job routing probability tables as described below. This procedure was selected because many samples of

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* A congruent multiplicative method of the following form recommended by Martin Greenberger [4] is used throughout this simulation

$$U_t = kU_{t-1} \quad k = 2^{18} + 3\,;\ (\text{Modulo } 2^{35})$$

$$U_0 = 200,\ 336,\ 163.\ 251_5$$

jobs are to be used in the experimentation. Use of a routing probability table based on a sample of actual manufacturing outlines has two features: (1) may large samples of routings can be derived from the probability tables in a routine and efficient manner and (2) each of these job samples is related to an empirical sample. The strength of this relationship depends upon how much information is relayed form the empirical routings to the probability tables. Two methods were employed. In the first method the permutational properties of the empirical routings were analyzed by considering pairs of only two operations. This is related to assuming the empirical data as representing Markov processes (i. e., processes for which knowledge of the current state of the job is equivalent to a complete history of the job routing from the point of view of prediction). With this method. termed the *single operation dependence* method, the transition probabilities were obtained by simply counting the number of times a machine center is followed by each other machine center in the empirical data. For the *double operation dependence* method, the transition probabilities came from counting the number of times every permutation of two machine centers was followed by each machine center in the empirical data. This is equivalent to assuming that probabilities of transition depend upon only the previous two processing states of the job. The reason for using the two methods was to build into the experimentation the question: is the degree of inter-dependence in the job routings an important property of the system ?

The single dependence probability table for the four machine center case is given as an example in Figure 2. The upper number in each cell represents the transition probability from the empirical data. The lower number is the cell cutoff value, again based on using numbers from 0—1000, for computer use. The transition probability in a cell is the empirical probability of a job moving from the machine center of the corresponding row to the machine center of the column. The double dependence table has more rows because all combinations of current and previous operations are used to obtain the transition probabilities.

Machine Center	1	2	3	4	Exit
Entry	.543 543	.355 898	.102 1000	— 1000	— 1000
1	.057 57	.439 496	.067 563	.055 618	.382 1000
2	.122 122	.223 345	.130 475	.387 862	.138 1000
3	.183 185	.295 478	.041 519	.302 821	.179 1000
4	.272 272	.172 444	.051 495	.118 613	.387 1000

Single Dependence Method-4 Machine Centers
Figure 2-Job Routing Probabiliey Table

The routing for each job is derived from the appropriate job routing probability table as follows:

1. A random number between 0—1000 is generated.
2. The generated number is identified with a cell in the entry row of the table and the corresponding colum (machine center) is recorded. This is the machine center at which the first operation on the job is o be performed.
3. A second random number is generated. The generated number is now associated with a cell in the row of the table corresponding to the previous operation (or two operations in the double dependence). The column corresponding to this cell is the second machine center in the job routing.
4. The process is continued until either (1) a random number associated with a cell in the exit column is obtained indicating completion of the routing or (2) a routing consisting of twelve operations is obtained (this maximum number of operations was introduced based on the empirical data). In either case, the job routing is complete. The operation sequence is identified with the job number and stored.
5. Step 1—4 are repeated for each job in the sample.

To illustrate the process described above, suppose that the random number sequence (328, 519, 476, 022, 595, 785) were generated. The table is used to obtain the job routing in the following manner:

Previous Machine Center (row)	Random Number	Interval (column)	Machine Center
Entry	328	0—543	1
1	519	496—563	3
3	476	183—478	2
2	022	0—122	1
1	595	563—618	4
4	785	613—1000	Exit

Thus, the job routing is 1, 3, 2, 1, 4.

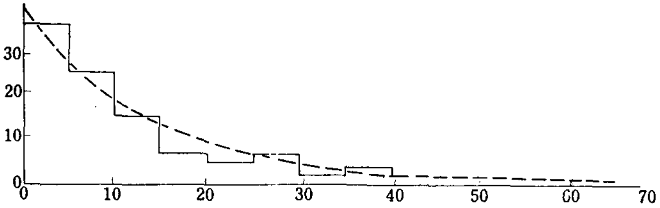
b. Processing Times. Having obtained the routing for each job, it remains only to assign a processing time for each operation which is to be performed. In this model the time required for an operation depends upon three factors: the machine center on which the operation is performed, the lot size of the job, and the complexity of the operation. We shall consider these in turn.

Empirical processing time distributions, taken from a complete history of three months operation of a Los Angeles job shop [14], were used as a basis. The observed distributions were smoothed and truncated for practical reasons, but the indicated distribution forms and means were used. Tabular forms of the selected processing time distributions for each machine center are stored for computer reference. An example of this data, in both graphical and tabular form, is given in Figure 3 for the turret lathe machine center with a truncated exponential distribution fit. The graphical data is condensed into 5 time unit cells for this illustration. in all work, however, single time unit cells are employed. The cell cutoff points in the table are based on the numbers 0—1000.

Machine Center-Turret Lathes

——Observed data

······Truncated exponential fit



Job Processing Time-Machine Hours

Processing Time	1	2	3	4	5	6	7	8	9	10	11	12	13
Cut-Off Point	963	894	829	769	714	662	614	570	529	491	455	422	392
Processing Time	14	15	16	17	18	19	20	21	22	23	24	25	26
Cut-Off Point	363	337	313	290	269	250	232	215	200	185	172	160	148
Processing Time	27	28	29	30	31	32	33	34	35	36	37	38	39
Cut-Off Point	137	127	118	110	102	94	88	81	76	70	65	60	56
Processing Time	40	41	42	43	44	45	46	47	48	49	50	51	52
Cut-Off Point	52	48	45	42	38	36	33	31	28	26	24	23	21
Processing Time	53	54	55	56	57	58	59	60	61	62	63	—	—
Cut-Off Point	20	18	17	16	15	14	13	12	11	10	0	—	—

Figure 3 **Machine Center Processing Time Data-**
Example of Graphical and Tabular Forms

The machine center on which a particular operation is to be performed is the factor which determines which of the processing time tables is used for assigning the processing time. The job lot size and operation complexity are next applied to assign an operation processing time from the appropriate table.

The variation of possible job lot sizes is represented by a range of evenly spaced numbers. A lot size is assigned to each job by merely selecting one of these numbers ar random. The selected number is then used to represent the lot size of the job for each of its operations, i. e., the same job lot size is used for assigning each of its operation processing times.

The variation of possible operation complexity is also represented by a range of evenly spaced numbers. We assume that the operation complexities on a job are independent. Consequently, a new complexity number is selected at random for each operation on the job in assigning the processing times.

The number representing the job lot size (number of prices in job) and the number representing operation complexity (time units per piece) are then multiplied to obtain a number representing the job processing time (time units for the job). This product is translated to an actual processing time by using the distribution of possible products (lot size x complexity), in conjunction with the empirical processing time distribution.

The assignment of processing times to the operations on each job proceeds as follows:

1. The machine center at which the first operation is to be performed is identified from the previously stored job routing.
2. A number is selected at random from the range of numbers representing lot size variation. This number indicates the job lot size and will not be changed until all operation processing times for the job have been assigned.
3. A number is selected at random from the range of numbers representing operation complexity variation. This number indicates the complexity of the operation.
4. The numbers representing lot size and complexity are multiplied. The product represents the job requirement at the machine center.
5. The product of (4) is identified with a processing time by using the cutoff points in the processing time table for the machine center involved. This is the processing time for the first operation.
6. Steps 1 and 3—5 are repreated until each operation on the first job has been treated.
7. Steps 1—6 are repeated for each job.

At this point the input data for the simulation is complete, i. e., a complete specification of the arrival time and processing requirements has been obtained for each job.

B. INITIALIZATIOM OF THE SHOP:

Before proceeding with the simulation, it is necessary to place the shop in some initial condition. There are several evident possibilities.

1. Begin the simulation with an empty shop.

Although this procedure would be comutationally simplest, there are two significant objections to it. The transient phase of the output resulting from the shop seeking its statistical steady state from the zero position would be unnecessarily protracted, and as a result, a relatively large sample would be required to obtain steady state output. Furthermore, the resulting transient effects would be too extreme to represent actual transients occurring in machine shop operations.

2. Attempt to approximate mean steady state queue lengths for each experiment and use these as initial conditions.

This procedure is based upon the idea of reducing transient effects due to experimental parameter changes. First of all, it is not clear how such approximations could be realized for the great majority of planned experimental conditions. In additions, it is felt than the study transients caused by parameter changes should be a part of the experiments because (1) transient effects do occur in real operations and (2) there is a paucity of information available about the extent and time span of the transients except for the simplest theoretical models.

3. Use the final condition of the shop in each experiment as the initial condition for the next run.

The principal objection to this technique is based upon the experimental plan which calls for a rather sharp change in one and only one external input value between any two consecutive experiments. It is felt that the resulting transient would, for the most part, be experimental transients providing no common reference condition for analysis.

4. Use of an initial condition based only on shop load parameters.

The overall experimental plan may be viewed as consisting of one sub-experiment repeated under eight different shop load conditions (three shop load parameters at two levels each). For each repetition of the subexperiment a single initial condition, depending only on the shop load parameters, may be used. This provides a common reference for analysis of transients and can easily be accomplished by computing mean steady state queue lengths from a theoretical waiting line model for each set of shop load parameters. Consequently, this procedure was selected for the experimentation*.

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* One might consider using a single initial condition based on average shop load conditions for the entire experimental plan. There were several reasons for not selecting this procedure.
Within each sub-experiment there will be one run based exactly upon the assumptions used in computing the mean steady state initial conditions for that sub-experiment. Using the initial conditions based on load, the

The theoretical model used for computing mean steady state queue lengths for the initial state of the shop is based on the following conditions. These correspond to the conditions used for one of the experiments for each shop load.

1. Exponential shop arrival interval (Poisson arrival frequency) distribution.
2. Exponential service time distributions for each machine.
3. Independent processing times.
4. Markov type job routing probability table.
5. First come, first served queue discipline. (FCFS)

The mean steady state queue lengths for each machine center were computed from formulas given by Saaty [19]. The results are illustrated for one shop load condition in Figure 4.

		Shop Load Conditions	
Machine Center	*Mean Number of Jobs in Queue*	*Average Utilization*	*No. of Machines*
1. Saws	2	.663	2
2. Lathes	3	.397	7
3. Turret Lathes	2	.335	6
4. Drills	3	.721	3
5. Mills	8	.827	6
6. Bores	2	.572	2
7. Profilers	1	.590	1
8. Grinreds	23	.957	2
TOTAL	44		

Figure 4. Initial Condition Queue Lengths—Example in Tabular Form

The eight initial condition tables are stored in the computer for reference in the initialization process. The initialization process is pro-

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output from this run will provide an estimate of the deviations of the statistical steady state from the expected values for each sub-experiment, i. e., under different load conditions. It is expected that these deviations will vary considerably with load conditions. Further, these estimates will be important information in much of the experimental analysis.

Under high load conditions, the time span of transients seeking a high load steady state from an average load steady state initial condition would be excessive.

If a single average load initial condition were used throughout, the transient effect due to each parameter change would be confounded with the effects due to shop load conditions. With the accepted plan, the transients due to changes in load can be determined from experiments designed for the purpose.

grammed to insure that the total number of jobs in the shop conforms exactly to the mean steady state value. The longest desired queue is also regulated closely. The other queue lengths may vary from the expected values. The initialization is carried out as follows:

1. The machine center with the longest desired initial queue length is identified and the desired queue length recorded.
2. The first job is taken from the stored sample of jobs. If this job requires an operation on the machine center identified in (1), the job is placed in that queue waiting for the corresponding operation. (If there is more than one operation on the machine center, the first is used) If there is no operation calling for the machine center identified in (1), random numbers are used to assign the job to the queue for one of its required operations.
3. Step 2 is repeated until there are almost* the desired number of jobs in the queue for the machine center identified in (1) or until the total number of jobs desired in the shop have been assigned to queues.
4. Jobs remaining after Step 3 are then assigned to the queue for one of their operations using random numbers until the desired total number of jobs have been assigned.

C. SIMULATION OF OPERATIONS:

After the input generation and initialization processes are com-

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* The queue length at which the routine switches is slightly below the desired queue length in anticipation of some random assignments to this queue from the remaining jobs to be assigned. The queue length at which the routine switches is pre-decided and read into the appropriate storage location.

** In order to prevent biased results, the output must consist of the first N arrivals rather than the first N completions. Otherwise the jobs meeting the most resistance would be systematically ignored and replaced by other smooth flowing jobs.

*** The jobs used in the initialization process are not included in the output of the simulaton for the simple reason that they are started at various phases of completion and would provide incomplete data.

**** This terminating condition is necessary to prevent decaying shop load effects from influencing the output.

pleted, the simulation of shop operations begins. The simulation is timed by a synthetic time clock. A simulation cycle consists of (1) recording job arrivals in the shop, (2) movements of jobs with completed operations, and (3) assignments of jobs to idle machines. When each simulation cycle is complete, the time clock advances and the cycle is repeated. The process continues until either the first N jobs to arrive** in the shop after initialization*** are completed (where N is the desired output sample size) or the last**** job in the pre-stored sample arrives in the shop. (This is a signal that the experiment must be terminated unless more jobs are generated by the job generation routine.)

After advancement of the synthetic time clock, the simulation cycle proceeds as follows:

> The current clock time is compared with the shop arrival time for the last job in the job sample. (The pre-generated jobs are stored in order of their arrival times). If the two times are equal the simulation phase is terminated for want of job input. If the clock time is less than the last job arrival time, the simulation cycle continues.

1. ***Recording of job arrivals in the shop.*** The current clock time is compared with the next stored job arrival time. If the clock time is less than the next arrival time, no job arrivals are recorded and the cycle continues. If the two times are equal the corresponding job is removed from the stored job sample and placed in the queue at the proper machine center for its first operation. This involves putting the job identification number and priority* in the proper storage location of the "Queue File." This process is repeated until all jobs arriving at the current clock time are assigned to the proper queues, i. e., until the clock time is less than the next stored arrival time, at which point the simulation cycle proceeds to the next phase.

2. ***Movements of jobs with finished operations.*** This part of the cycle is carried out for each machine in the shop. The current clock time is first compared with the finish time for the operation on the machine.

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* The determination and assignment of priorities to the jobs is handled by choice of an appropriate sub-routine. The sub-routines merely consist of calculating priorities according to the various priority rules for which experimentation is desired.

If the two times are not equal the indication is that either no operation is being performed by the machine or that the operation being performed is not yet finished. In either case no job movement action is taken and the cycle proceeds to the next machine. If the clock time and operation completion time are equal, indicating a change in status from busy to idle, reference is first made to the "Job Loading File" for the pertinent machine, to identify the job number. The job number is used to refer to the "Job Ticket File" to determine the next required machine center for the job. Machine center number 0 represents completion of a job.

If the job is completed the job number is used to determine whether or not it is a job belonging in the desired N job output*. In the latter case the routine proceeds to the next machine. In the former case, the completion of the job is reflected in the output tally which counts the completed job, and the job completion time is recorded on the job ticket.

If the next machine center number is not 0, the job is placed in the queue for the next required machine center.

The job movement part of the cycle is repeated for each machine. The simulation cycle then proceeds.

3. ***Assignments of jobs to idle machines.*** This part of the cycle is also performed for each machine in the shop. First, the current status of the machine is determined as busy or idle.

If the machine is busy no action is taken and the process continues to the next machine. If the machine is idle, the machine center number is used to inspect the "Queue File" to determine whether there are any jobs currently in the queue for that machine center. If there are no jobs in the queue, no assignment can be made and the process continues to the next machine center. If there are jobs in the queue, the job with highest priority is identified, removed from the "Queue File," and placed in the "Job Loading File" along with the machine number to which the assignment is being made. The required processing time is extracted from the "Job Ticket File" and the machine is re-

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* As discussed previously, a job belonging in the desired output is any of the first N arrivals subsequent to initialization.

corded as busy for the corresponding number of future time units. The projected operation completion time is also recorded in the " Job Ticket File " as the ready time for the next required operation.

Each time a job assignment is made to a machine, the job identification number, machine center number, arrival time at the machine center, waiting time in queue, and processing time are determined and placed in the " Chronological Record File." This information is used for the chronological output which provides a complete history of the simulation results and which shall be discussed in the next section.

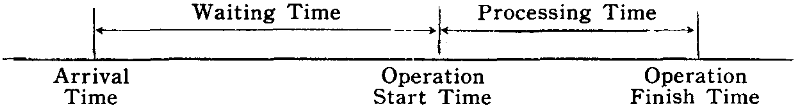
The job assignment part of the cycle is repeated for each machine. The time clock is then advanced one time unit and the simulation cycle is repeated until termination of the simulation.

D. OUTPUT DATA:

1. Chronlogical record of operations and queue lengths.

The chronological record of operation is stored as the simulation proceeds. Each time a job is assigned for a processing operation, the following data is added to the chronological history for tape output:

Job identification number
Machine center identification number
Job arrival time at machine center
Job waiting time in queue at machine center
Job processing time at machine center



Since the progress of any job through the shop consists of a series of cycles like the one illustrated in the schematic above, it is clear that the arrival time, and processing time for each operation are sufficient to provide a complete history of the simulation. This data can be sorted by job number or by machine center number to provide data for individual job and/or machine centers.

In addition to the chronological record of operations, a "snapshot" of the queue length at each machine center is taken at regular time intervals during the simulation. This data is stored for tape output and provides a chronological history of queue lengths and number of jobs in the shop.

2. Gross statistical output.

The gross statistical output provides summary information for each "output" job. The following data are obtained and printed out for each job:

Job identification number
Job arrival time in shop
Job completion time
Total job flow time
Total job processing time
Total job waiting time in queues
Job due date (when applicable)
Job tardiness (when applicable)

IV. EXPERIMENTAL OUTLINE

A. PURPOSES:

The initial experiments are planned with the following purpose in mind:

1. To study the main effects and interaction effects of the variables of the model on certain basic output properties such as mean flow time. This phase of the analysis is designed to answer important questions about the type of system being studied, e. g., what are important variables of job shop-type systems with respect to certain basic output quantities? What types of predictions are possible with respect to the effect on basic output properties of changes in the system parameters and variables? Answers to such questions seem prerequisite to any real understanding of the systems involved and, consequently, to the design of long-run experimentation for obtaining generally useful results. This experimental philosophy is well stated by Kempthorne:

   > "If the research is of the fundamental type, dealing with the formulation of laws and the prediction of effects, we should first determine which factors are important and the degree of the degree of the interdependence of their effects and then isolate some of the factors for detailed study and

return to the general problem when laws have been formulation."*

2. To study the transient effects of changes in variables on certain basic properties such as flow time, congestion. etc. Strictly speaking, this phase of the study chould not be divorced from (1) above since the data for the analysis will derive from the same experiments in the form of a chronological record of operations beginning with the initial conditions. Just what questions this analysis may be of help in solving is difficult to say at this point. Very little is known about the extent and time span of transients under any but the most simplified theoretical conditions. It suffices to note that transient effects due to changes in system values are a part of job shop system operations and, as such, should be systematically introduced and studied in the experiments.
3. To evaluate alternative queue disciplines, relative to different measures of effectiveness, and under a variety of conditions of the variables of the system. For this phase of the study the model will serve as an experimental laboratory for the evaluation of alternative queue disciplines (a decision parameter common to such systems, as differentiated from shop size and arrival rate which may also be decision parameters in some applications).
4. To test certain hypotheses related to job shop systems, which may be useful for research. An example of such an hypothesis is: The assumption of independent waiting times at the work centers does not have any significant effect on job flow time or some other basic output property of the system. This hypothesis can be tested by first computing theoretical results from a model assuming independent waiting times, but otherwise corresponding to the experimental input, and then comparing the theoretical and simulation results by a significance test. Another hypothesis of interest is for a given non-expo-

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* Reprinted with permission from Kempthorne, Oscar, *The Design and Analysis of Fxperiments*, copyright 1952, John Wiley and Sons, Inc.

nential arrival interval distribution of jobs into the shop, the inter-arrival patterns at individual work centers do not differ significantly from the exponential assumption. Other interesting hypotheses will doubtlessly arise in the course of the analysis.

B. INITIAL EXPERIMENTS:

At this time it is not possible to present any such thing as a complete experimental plan. This is true for at least two reasons: First, the plan of experimentation will necessarily develop in large part from the results of the early experiments. Also, a great deal of research remains to be spent on planning methods of analysis. The experiments described below are viewed as the set of initial experiments. It is anticipated that only the experience and results gained from the initial experiments will make it possible to prepare future plans intelligently.

For the initial experiments, each variable of the model will be assigned two different values. The selected values are discussed below.

Shop Load Parameters:

The number of machine centers in the shop was assigned the values 4 and 8. The numerical values for the other shop load parameters are not of interest in themselves. The important point is that the assignment of two values to each of these three load parameters results in $(2)^3=8$ different shop load conditions. The condition of the shop load may be represented by a utilization vector $\rho=(\rho_1, \rho_2, \cdots\cdots, \rho_N)$ where ρ_i for $i=1, 2, \cdots\cdots, N$ is the average steady state machine utilization at machine center i and N is the number of machine centers in the shop. The two values for each of the shop load parameters for the initial experiments were selected in such a way that the eight resulting shop load vectors were:

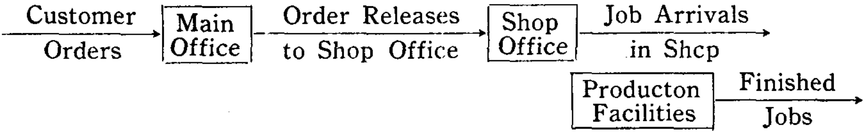
$$\left.\begin{array}{l}(.663,\ .397,\ .335,\ .721\ \ .827,\ .572,\ .590,\ .957)\\(.601,\ .361,\ .303,\ .655\ \ .522,\ .522,\ .542,\ .871)\\(.964,\ .958,\ .969,\ .975\ \ .954,\ .954,\ .961,\ .957)\\(.871,\ .872,\ .877,\ .886\ \ .867,\ .870,\ .882,\ .871)\end{array}\right\}\begin{array}{l}N=8\\ \text{Machine}\\ \text{Centers.}\end{array}$$

$$\left.\begin{array}{l}(.958,\ .821,\ .589,\ .713)\\(.869,\ .746,\ .543,\ .647)\\(.958,\ .953,\ .949,\ .965)\\(.869,\ .865,\ .862,\ .875)\end{array}\right\}N=4\ \text{Machine Centers.}$$

The first vector represents a dispersion of machine utilizations over the eight machine centers such as observed in an empirical study of a local job shop. The maximum utilization was set at a level which experience with previous computations indicated to be a high utilization from the point of view of congestion. The second vector reflects the same empirical dispersion of utilizations, but at a lower level (yet high enough to still present congestion problems of interest). The third and fourth vectores reflect high and low levels, but this time with a well balanced utilization for all machine centers. The last four vectors represent the same selected conditions for the experiments with only four machine centers. To summarize, the shop load parameters were assigned values in such a way as to provide eight shop load conditions. reflecting a considerable variation in loading properties on the system, for the set of initial experiments.

Distribution of arrivals of jobs in the shop:

As mentioned earlier, the arrival pattern of jobs in the shop for each experiment is obtained by drawing a random sample of time intervals between arrivals from a specified frequency distribution. The two distribution function forms* to be used for the initial experiments are the exponential distribution and k=2 Erlang distribution**. These choices are based on empirical data, which indicates that job arrivals in the shop tend to follow a pattern "between" these two theoretical cases. This observation may be rationalized through reference to the concept of an arrival timing channel introduced by Morse [13]. If we conceive the progress of job orders as indicated by the schematic below:



The two phase channel consisting of the main office and the shop office may be viewed as an arrival timing channel between arriving customer orders and job arrivals in the shop. If the timing channel has

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* For the selected distribution forms, the mean arrival intervals, calculated from the specified values of the mean arrival rates, are sufficient to determine the distriobtions.

** An Erlang distribution with variance one half that of the exponential distribution with the same mean.

an exponential holding time distribution, i. e., represents no deliberate delaying behavior in releases of orders, then the distribution of arrival intervals of jobs into the shop will also be exponential. If, on the other hand, the timing channel consists of two exponential phases representing main office and shop office processing respectively, but with extreme deliberate action, in the sense that the main office begins processing a job only after the previous job leaves the shop office, then the job arrival interval distribution would be k=2 Erlang. The empirical data then indicates that the observed situation was somewhere between these two extremes, i. e., some thing of arrivals occured. Statistical tests indicated that this timing was directly related to the current shop load status. The releases were timed to keep the shop load relatively stable. The two distributions for job arrival were selected, therefore, to represent the extreme conditions likely to occur in any such system (for reasonably random customer order patterns).

Processing time distributions at the machine centers:

The two forms* of processing time distributions assigned to the machine centers were also based on empirical data from observation of the machine centers in a shop. The exponential and k=2 Erlang distributions were selected. In the case of processing times, there was some indication that hyper-exponential distributions should be considered. Consequently, if the initial experiments indicate this to be an important consideration, a third value, e. g., a j=2** hyper-exponential distribution may be assigned this variable. One further point should be made clear with regard to the selection of processing time distributions for the initial experiments. For each experiment the same form of distribution is assigned to each machine center. This is an arbitary decision made for convenience. The results of the initial experiments may indicate a need for further experimentation with systematically introduced mixes of distribution forms over the machine centers. As with the need for introducing a third form of distribution, this will depend upon the indicated effect of the processing time distribution variable upon the

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* Again, the mean processing time as calculated from the specified values of the mean processing rates, are sufficient to determine the distribution.

** An hyper-exponetial distribution with variance that of the exponential distribution with then same mean.

measures of effectiveness.

Job routing generation procedure:

As discussed in Section III, the one and two operation dependence methods are used to generate job routing.

These cases were selected partly for simplicity and partly because it was generally agreed that if this variable was to have any important effects, these effects would probably be clear by comparting the Markov process with the first order change from the Markov. This hypothesis is based upon the idea that first order differences are generally the most significant. Again, a significant effect here may indicate a logical extension for additional experiments.

Job lot size variation vs. operation complexity variation $\left(\frac{L}{C}\right)$:

The two values selected for this variable in the initial experiments were the extreme possibilities. It was decided that, in production systems, this variable might well range from the one extreme where lot sizes vary widely and operation complexities are essentially constant (this case represents highly "correlated" processing times for the different operations on a given job) to the other extreme for which lot sizes are essentially constant and operation complexities vary widely (the case of random or non-correlated processing times for the different operations on a given job). In recognition of this possibility, the two specified values for this variable were:

Lot size range=1—10 (integers)

Operation Complexity range=1

$$\frac{L}{C}=10$$

Lot size range=1

Operation Complexity range=1—10 (integers)

$$\frac{L}{C}=\frac{1}{10}$$

Queue discipline:

The two queue disciplines selected for the initial experiments are very simple rules related to the non due data version of the model for which the job flow time distribution is the measure of effectiveness. Job due dates, due date conditioned queue disciplines, and tardiness as a measure of effectiveness will be introduced later.

For a model representing the ultimate in simplicity wherein only two jobs compete at a single machine center for assignment, it is easy to show that the rules listed below lead to the given properties in the resulting flow times:

Rule	*Result*
First-come, first-served	Minimizes the maximum flow time
Assignment of job with smallest required proccessing time at the machine center	Minimizes average flow time (sum of flow times)

These two rules are appealing because of their simplicity. More important than this, however, are two experimental purposes. First, it is an interesting conjecture as to whether or not (or to what extent) the ability of these simple rules to achieve certain interesting flow time properties in the single machine two job model, persists in the complex network model. The use of these rules for the initial experiments should yield enough results to investigate this question. This analysis should also yield valuable data for learning more about the extent to which flow time distributions can be mainpulated by extremely different, yet simple and purposeful, queueing disciplines. The second experimental purpose in choosing these rules for the initial studies is the idea that the results will provide benchmarks for evaluating the performance of other, more complex, rules in future work.

The experimentation will proceed in the following manner:

1. Each of the variables will be assigned one of the selected values as described above. Given this external input data, the computer program will be used to generate a sample of jobs, select the appropriate initial condition for the shop, simulate the operations of the system, and yield the chronological and gross output data for the experiment.

2. One of the variables will then be assigned its other selected value (all other variables maintaining their previous value) and the second experiment will be performed. This process will be continued until an experiment has been performed for each combination of values for the variables. Since there are eight variables at two levels each, this one replication will require $2^8=256$ experiments*

3. Another complete set of experiments will then be performed**. Each of these experiments will differ from the experiment in the first set with the same values for the variables because new numbers will be used to generate the job sample. i. e., arrival intervals routings, and processing times. This procedure will provide a second replication within each cell of the experimental design; the purpose of which is to provide a means for estimating the effect due to sampling from the distributions governing the arrivals, routings, and processing times.

The estimate of sampling effect will provide a means for obtaining measures of other effects relative to a source of variation which would be beyond control in the real systems which the model portrays. It is true, of course, that in any particular job shop system, the uncontrollable variation would consist of several components (e. g., time variation in the statistics of the system or certain variables of the system not subject to control) in addition to the variation due to sampling The latter, however, represents a basic source of uncontrollable variation in all such systems, and, as a consequence, seems to provide a logical base for measuring the relative significance of other effects in a generalized model such as this.

The IBM 709 computing time required for the individual experiments will vary widely due to changes in the variables; particularly shop size and queue discipline. A crude estimate of the average time per experiment has been made as 3 minutes, corresponding to 13 hours per replication of a complete set of 256 experiments. It is clear that, even with the tremendous capabilities of the 709, the scope of the model and the initial experimental plan are such that the computing time requirements are not in any sense trivial.

C. DISCUSSION:

It seems appropriate to conclude this report with a discussion of some of the plans for analysis of the results of the initial experiments and to mention briefly some ideas of interest for future projects.

The first analysis planned are analyses of variance to determine

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[*] The number of replications to be used for each cell has not yet been definitely decided upon.

[**] The possibilities of reducing the number of experiments and/or selecting an efficient order for the variable changes are under consideration.

the effect (and interactions) of the variables on output quantities for the non-due date version of the model. Mean flow time, and mean number of jobs in the shop are examples of cell entries which may be subjected to analyses of variance in order to determine the effects of the systems variables.

Another important phase of the analysis for the non-due date version will be concerned with the study of time graphs from the chronological output of the experiments. An example of such time graphs is mean flow time for jobs completed during a time interval plotted with respect to time. The primary goals of this area of analysis will be concerned with learning more about the time span and extent of transient effects measured from common reference initial conditions.

Sorting of the chronological output by machine center will provide data for analyzing the inter-arrival, waiting time, and queue length distributions at each of the machine centers. Sorting by jobs will provide a detailed history of the progress of each individual job, if such a breakdown is desired. These forms of the data may be particularly useful for testing hypotheses about the internal behavior of the shop and/or for comparing simulation results with results predicted from related theoretical models.

Study of the waiting time distribution or flow time distribution output for the NDV of the model should provide a means for determining a reasonable set of due dates to assign to the jobs for the due date version of the model. The idea here is that the results of the initial experiments with the NDV of the model will give information concerning what the system is capable of (under given load conditions) in terms of the gross distribution of waiting times or flow times. This information on capabilities can then be used to drive due date specifi cations for the jobs in the DDV.

In this way the assigned due dates can be adjusted to the overall capability of the system demonstrated in thə non due date experiments and, at the same time, offer an interesting poblem for evaluating the ability of alternative due date type queue disciplines to condition the completion time of individual jobs relative to their due dates.

The term "due date type queue disciplines" has been mentioned several times. The list below in dicates some of the due date type rules being considered for experimentation. It is to be expected that still other rules will be cosidered as more effort is devoted to the logical development of rules related to various goals.

1. Minimum slack rule : Assign the available job with smallest slack

The slack for each job may be defined in either of two ways depending upon whether an estimate of expected waiting times at future operations is to used:

a. Slack$=D-[t+\Sigma P]$
b. Slack$=D-[t+\Sigma P+\Sigma w]$
where D=job due date
ΣP=future processing requirements
t=time of decision
Σw=expected future waiting times.

2. Slack intensity Rule A: Assign the available job with the smallest value of:

$$\left[\frac{\text{Slack}}{\text{Future Processing Time}}\right]$$

This is a rule which tends to allocate the slack on the jobs, as they proceed through the production process, on the basis of the required processing time remaining.

3. Slack intensity Rule B: Assign the available job with the smallest value of:

$$\left[\frac{\text{Slack}}{\text{Expected Future Waiting Time}}\right]$$

The authors are particularly interested in this rule which tends to allocate slack on the basis of expected future waiting time for the job. The expected future waiting seems to be a measure of the total congestion one may expect the job to experience and, hence, proportional to the desired slack.

Two other rules, namely smallest

$$\left[\frac{\text{Slack}}{\text{Number of Future Operations}}\right]$$

and smallest

$$\left[\frac{\text{Slack}}{\text{Future processing Time+Expected Future Waiting Time}}\right]$$

are closely related to those above.

It is emphasized again that *any meaningful evaluation of these or other proposed rules in the model will depend upon the measures of effectivenss and upon simulation results using the alternative rules.*

Finally, the authors wish to touch on concomitant progress being made on the construction of a job shop model oriented toward studies of the economic aspects of job shop production processes. From the viewpoint of operations researchers, the demand and supply conditions peculiar to production to customer order exemplified in job shop type systems provide a host of highly interesting problems*. The model being constructed enables one to consider various types of decision making faced by job shop firms; overtime operations, sub-contracting, price setting, selection of a particular profile among the alternatives available to them, etc. These decisions will be evaluated on the basis of a broader criterion function such as optimization of profit, properly defined.

APPENDIX

OUTLINE OF THE COMPUTER SIMULATION PROGRAM

The computer simulation consists of the following nine steps:

Step 1 set output tapes ready for writing generated job input data, gross statististical output data, snapshot and chronological output data.

Step 2 Read in an initial random number.

Step 3 Read in a control card which contains desired values for the parameters (8 parameters in all, see section 2 for details).

Step 4 Translate the values specified on the control card read in Step 3, into program control words.

Step 5 Prepare the computer program as instructed by the control words.

Step 6 Generate job inputs (Part I) and write the input data on tape.

Step 7 Initialize the simulated shop (Part II)

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* For a further discussion on this subject, the reader is referred to [10, 11].

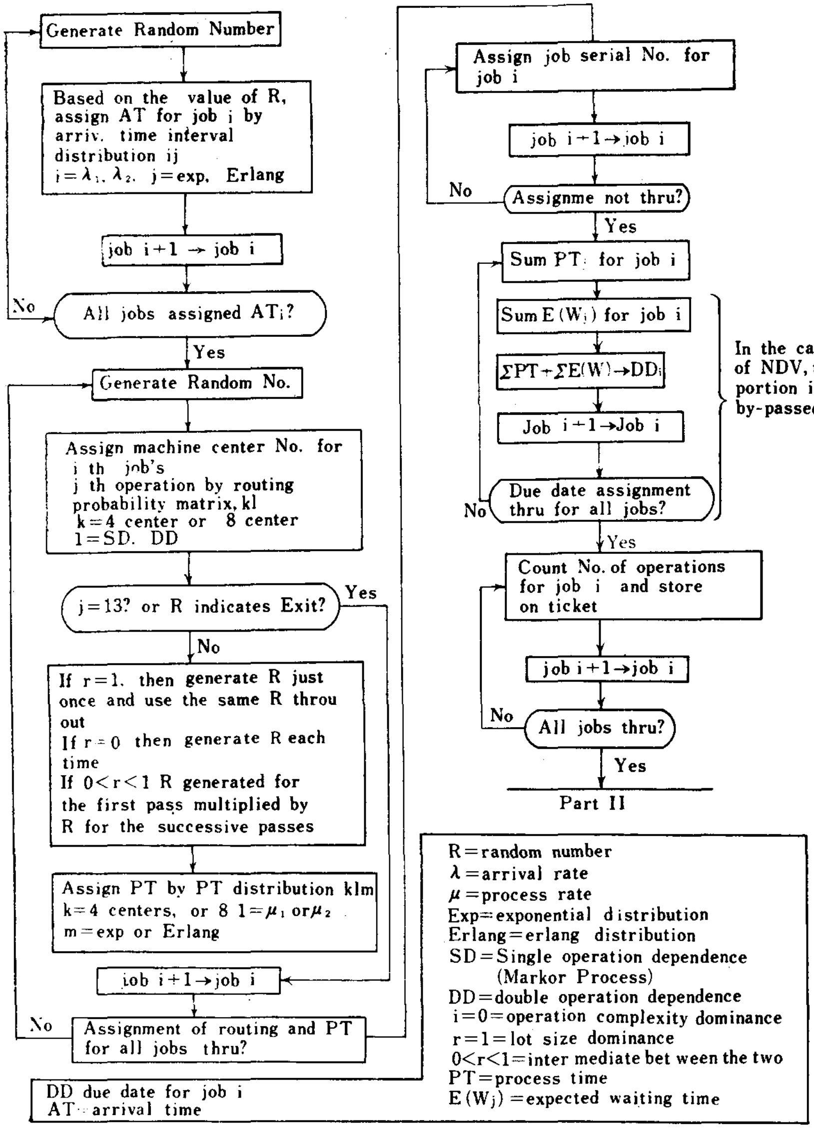
Step 8 Simulate the production processes (Part III)

Step 9 Write the output data (Gross statistical output and snapshot and chronological output) on tapes.

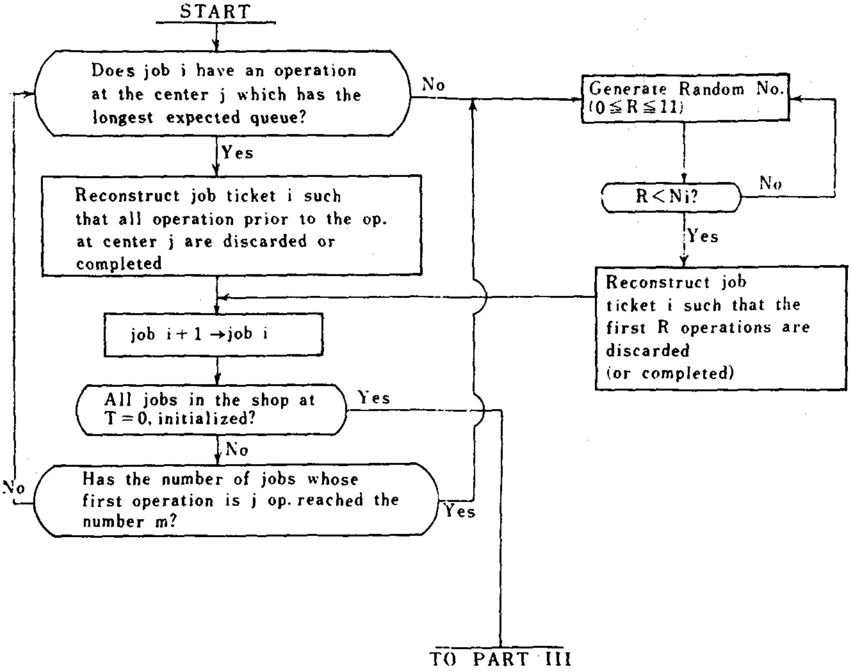
Repeat Steps 2—9 until all control cards run out. Then, a planned experimental run is completed.

Flow charts are given in the following pages only for Parts I, II, and III.

PART I GENERATION OF INPUT DATA



PART II INITIALIZATION

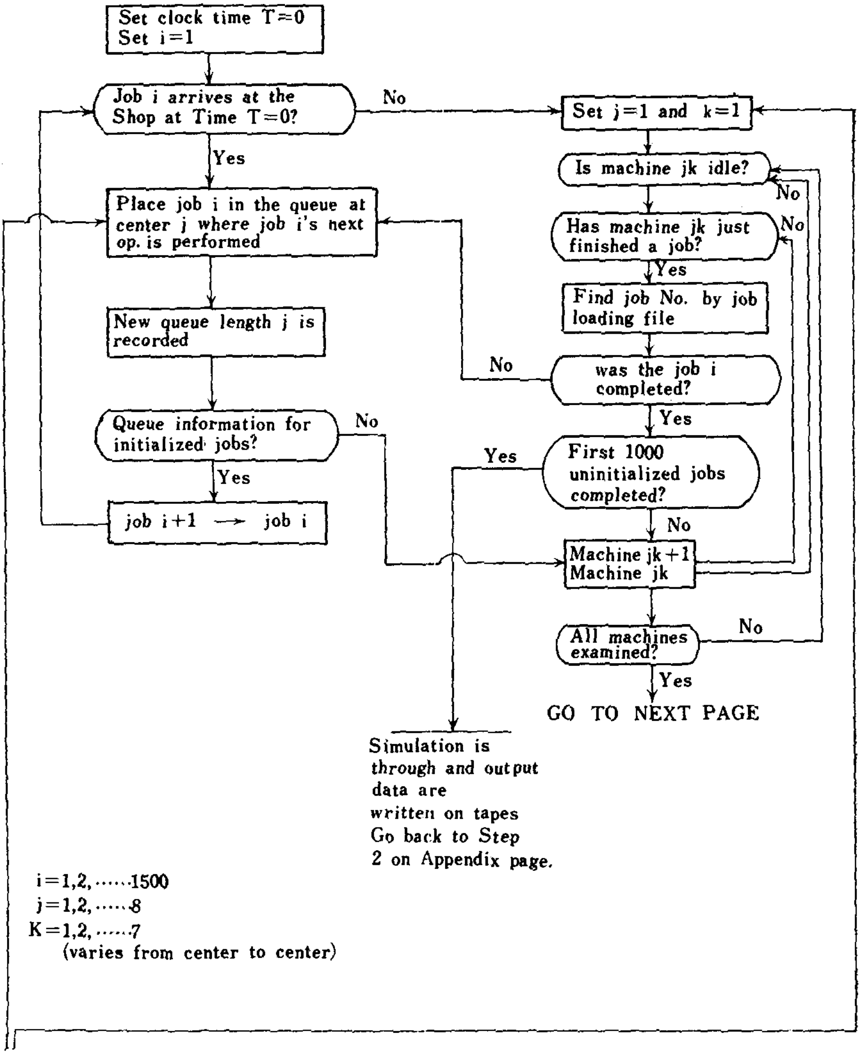


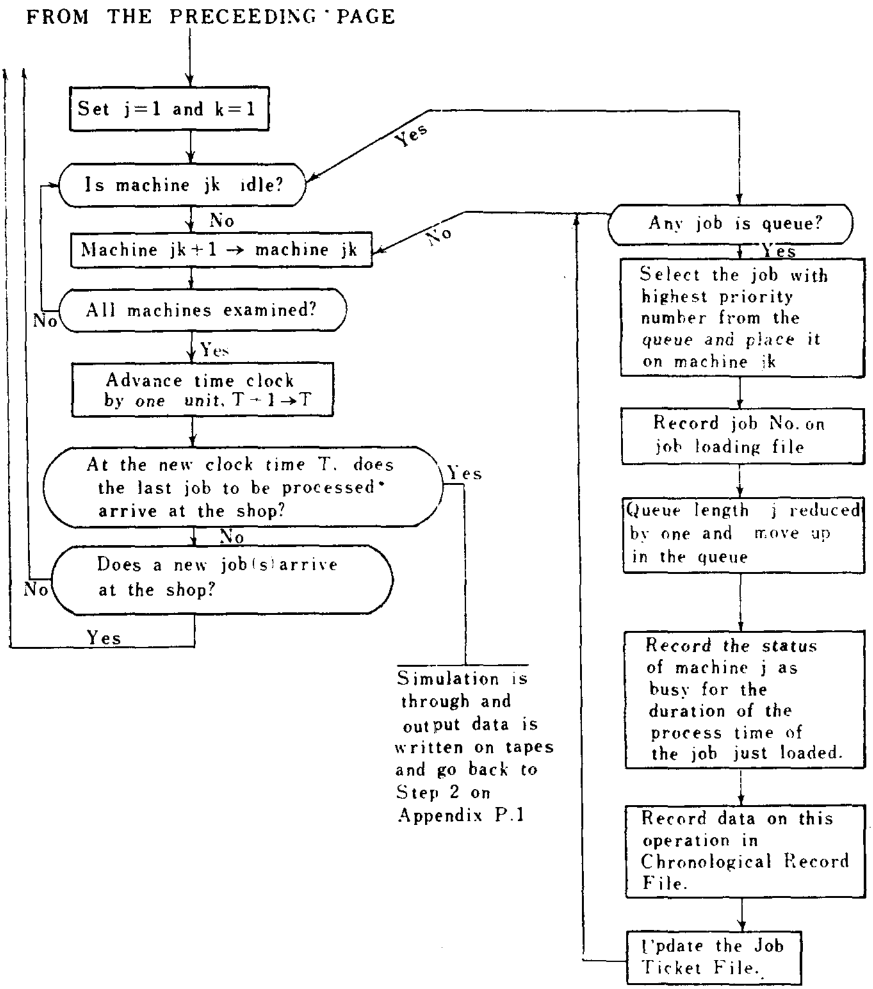
m = the expected number of job lots at the machine center j which has the longest expected queue $-\alpha$ (see footnote on P. 19)

R = Random number (treated as integer) $R_t = KR_{t-1}$ $K = 2^{18}+3$ (Modulo 2^{35})

N = Number of operations for job i
$N = 1, \cdots, 12$ Max. 12

PART III SIMULATION PROPER




FROM THE PRECEEDING PAGE
Set j=1 and k=1
Is machine jk idle?
Yes
No
Machine jk+1 → machine jk
No
All machines examined?
No
Yes
Advance time clock by one unit, T+1→T
At the new clock time T, does the last job to be processed* arrive at the shop?
Yes
No
Does a new job(s) arrive at the shop?
No
Yes
Simulation is through and output data is written on tapes and go back to Step 2 on Appendix P.1
Any job is queue?
Yes
Select the job with highest priority number from the queue and place it on machine jk
Record job No. on job loading file
Queue length j reduced by one and move up in the queue
Record the status of machine j as busy for the duration of the process time of the job just loaded.
Record data on this operation in Chronological Record File.
Update the Job Ticket File.

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