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REGULATION OF THE CUT BY DYNAMIC PROGRAMMING

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The development of regulation of cut which was the origin of forest management can date back to at least 14 th century. Since then a great many number of method of determining the cut have been developed particularly in Europe and recently in the United States.

The remarkable thing about the methods employed in the actual regulation of the cut is that they are "locked", since no real optimization needs not take place and any volume of annual cut can be produced in one and only one way. Even supposing there are orginally many ways of regulating cut, this comes about through behind-the--scenes optimization. They consider nothing other than biological aspect in an incomplete form, excluding economic problems.

The cause of this non-optimization theory in regulation of the cut comes from assuming that growth rate would not be influenced by cutting to harvest. In this way to treat dynamic problems of regulation of the cut as nothing but special cases of static ones has simply robbed us of the insight that a more direct theory might yield. Actually several forest economists tried to make their theory dynamic by introducing time. However it is not merely the introduction of variables possessing time subscript that makes regulation of the cut dynamic. A problem is fundamentally unchanged if we write y_{t-1} in place of z.

The methods of regulation of the cut can be divided into two classes except for the classification based on volume and area. One is the long-term regulation of the cut and the other short-term one. In actual forest practice the short-term regulation would play a more important role than the long-term one, so that more attention has been paid to it by an increasing number of foresters who have to manage their forests in the dynamic economic environments, although the chief concern of foresters has been directed to the long-term regulation of the cut. It has been the most fundamental problem of forest management to solve these two classes of problems simultaneously. In this paper regulation of the cut which is a key of forest management with the history of hundreds of years will be improved with the aid of dynamic programming.

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TRADITIONAL APPROACH

Among the methods of regulation of the cut put into practice in forestry the most advanced one would be Amortization formula and Horizontal Cut method devised by W. H. Meyer and H. H. Chapman, both of them being professors of Yale University School of Forestry in the United States, because they are only such as having the concept of optimization although their methods are short-term regulations. Amortization formula has the following form :

$$V_n = V_0 (1+g_t)^n - \frac{C}{g_m} \left(\overline{1+g_m}^{\,n} - 1\right) \qquad (1)$$

where

- V_0 : the present volume,
- V_n : the volume n years hence,
- C : the annual cut,
- g_t : the compound interest growth rate of the entire stand, ingrowth included,
- g_m : the compound interest growth rate of the marchantable stand.

Dependable compound interest growth rate will be calculated from the stand projection method with the form in Table 1. Then various values of C is introduced to compute the corresponding V_n 's in order to find the improvement in growing stock in percentage. Finally such cut as will satisfy the needs of forest owner and yet build up the growing stock in a satisfactory manner will be chosen. After the allowable annual cut has been determined, the cruise area should be divided into 10 annual cutting blocks, each of which will produce the required cut when its time of cutting is reached. The following formula is employed to determine the allocation of the annual cut.

$$V_0 = \frac{V_{ac}}{g_t} \ \frac{\overline{1+g_t}^n - 1}{(1+g_t)^n} \left(1 + \frac{g_t}{2}\right) \tag{2}$$

where

V_0 : the total volume on the cruise area,

V_{ac}: the volume on the annual cutting block when cutting time is reached,

g_t : the compound interest growth rate of the stand, ingrowth included.

When V_{ac} is found, then a series of 10 values are computed which relates it to present volume.

Horizontal Cut method differs from the amortization approach in two distinct ways from the computational point of view, In this case the same projection calculation as for the amortization formula as shown in Table 1 is employed. In horizontal cut method simple interest, not compound is implicit in the calculation. But the difference is minimized by calculating from a stand table as of five years hence. It assumes that the cut is taken from the largest trees only and not distributed throughout the entire stands as is the case with the amortization formula.

In this paper an approach will be made on the basis of dynamic programming, using the reasoning of the above-mentioned two methods of regulation of the cut.

DYNAMIC PROGRAMMING APPROACH

Let us assume that we have to regulate the cut in the southern forests at Crossett, Arkansas, which consist of marchantable timber per acre as indicated in Table 1. Timbers to be cut during the next ten years belong to those with DBH (diameter at breast-height, 1.3 meter high) more than 22 inches. And we assume that the state of marchantable timber to be cut and to be left not cut at any particular time may be completely specified by two quantities, the volume of the total growing stock of the forests and the volume of marchantable

Table 1. Prediction of Future Inventory

DBH	No. tree per acre	Volume per tree	Volume per class	10yrs. diameter growth	Mortality	Reduced	10 years hence 1 cls	10 years hence 2 cls.	Tot.	vol. per cls.	culminated sum from bottom
6	15.36				5.9	14.45					
8	9.60				4.7	9.15					
10	8.40			2.26	4.0	8.06					
12	8.26			2.36	3.7	7.59					
14	6.59	140	922.6	2.46	3.5	6.36	6.52	1.05	7.57	1,060	7,817
16	5.08	249	1,264.9	2.55	3.5	4.90	4.90	1.43	6.33	1,576	6,757
18	3.35	322	1,078.7	2.64	3.5	3.23	3.55	1.46	5.01	1,613	5.181
20	1.60	417	667.20	2.73	3.5	1.54	2.20	1.35	3.55	1.480	3,568
22	.61	519	316.59	2.83	3.5	.59	.98	1.03	2.01	1,043	2.088
24	.26	623	161.98	2.92	3.5	.25	.35	.56	.91	567	1.045
26	.08	725	58.00	3.02	3.5	.08	.13	.24	.37	268	478
28	.02	910	18.20	3.10	3.5	.02	.04	.12	.16	146	210
30	.004	1,050	4.20	3.19	3.5	.00	.01	.04	.05	53	64
32								.01	.01	11	
	Total	4.493								7.817	

Note: DBH means a diameter at breast height, 1.3 meter above the surface of the land.

timber left not cut. To simplify the problem for this initial computation, we shall assume that the volume of the cut is unbounded, although the total growing stock at the final stage is given. The purpose of this process will be taken to cut as many marchantable timber as possible over a time period of 10 years, given the final state of the total growing stock. The process is assumed to be discrete, with allocation made every year. At any particular year, t,

$$X_m(t) = \text{the total growing stock of marchantablo timber,} \qquad (3)$$

$X_g(t)$ = the volume of marchantable timber with DBH less than 22 inches which will not be cut.

At any particular time, t, the marchantable timber may be used for either of these three purposes, to be cut as harvest, to be left not cut to increase the volume of marchantable timber with DBH more than 22 inches, or to increase the volume of marchantable timber with DBH less than 22 inches which will increase the volume of harvest because of a fixed rate of the cut to the total growing stock.

Let us write,

$$X_m(t) = z_c(t) + z_m(t) + z_g(t), \qquad (4)$$

where

a. $z_c(t)$: the volume of the cut of marchantable timber with DBH more than 22 inches,

b. $z_m(t)$: the volume of marchantable timber with DBH more than 22 inches but to be left, (5)

c. $z_g(t)$: the volume of marchantable timber with DBH less than 22 inches.

Let us impose the following constraints,

$$\text{a.} \quad d_1 \leqq z_c(t) \leqq a_1 X_m(t), \quad 0 < a_1 < 1, \ d_1 < 0,$$
$$\text{b.} \quad z_g(t) \leqq a_2 X_g(t), \quad a_2 > 1. \qquad (6)$$

The first constraint says that it is not possible to cut less than a fixed volume of marchantable timber and more than a fixed percentage of the total growing stock over any stage, k to $k+1$, while the second asserts that there is not point to increase the volume of marchantable timber with DBH less than 22 inches more than the maximum capacity resulting from timber stand improvement in which timber cut will not be counted as harvest.

Let us now see how the state of the system is affected by the allocation; assuming the linearity,

$$X_m(t+1) = X_m(t) + a_3 X_m(t) - z_c(t), \quad 0 < a_3 < 1,$$
$$X_g(t+1) = X_g(t) + a_4 z_g(t), \quad 0 < a_4 < 1. \tag{7}$$

Finally, let us assume that the volume of merchantable timber cut with DBH more than 22 inches in a state is $z_c(t)$.

It is required to choose the volume of $z_c(t)$, $z_m(t)$ and $z_g(t)$, for $t=0, 1, 2, \cdots, T-1$, so as to maximize the total volume of annual cut over the time period $[0, T]$, given the initial volume,

$$c_1 = X_m(0),$$
$$c_2 = X_g(0), \qquad c_2 < c_1. \tag{8}$$

Let us define, for $T=1, 2, \cdots,$

$f_T(c_1, c_2) =$ the total harvest cut over T stages, starting with initial volume, c_1 and c_2, and using an optimal policy, (9)

and

$$f_T(c_1, c_2) = \max_{\{z\}} L(z), \ L(z) = \sum_{t=0}^{T-1} z_c(t).$$

Then, clearly

$$f_1(c_1, c_2) = \max z_c(0) = a_1 c_1. \tag{10}$$

Using the principle of optimality, we have

$$f_T(c_1, c_2) = \max_{(z)} [z_c + f_{T-1}(c_1 + a_3 c_1 - z_c, \ c_2 + a_4 z_g)], \tag{11}$$

for $T=2, 3, \cdots, T$, where the maximization is over the region in z — space defined by

$$\text{a.} \ z_c, z_m, z_g > 0, \qquad \text{b.} \ z_c + z_m + z_g = c_1, \tag{12}$$
$$\text{c.} \ d_1 \leq z_c \leq a_1 c_1, \qquad \text{d.} \ z_g \leq a_2 c_2.$$

Table 2. Calculation of 10 stage Process

c_1 : 4,493 bd. ft.	a_1 : .0250	d_1 : 100 bd. ft.
c_2 : 3,394 bd. ft.	a_2 : .01875	d_2 : 6,800 bd. ft.
	a_3 : .06325	
	a_4 : 0.1919	

	Information made available by calculation		Acual condition using optimal policy	
	Normalized condition at		Actual condition at	
stage	beginning of stage c_1 · c_2	vertex	beginning of stage	annual cut
1	(1, .875)	(4)	(3,934, 4,493)	112.3
2	(1, .874)	(4)	(4,084.7, 4,664.9)	100.0
3	(1, .872)	(4)	(4,421.1, 4,860.0)	100.0
4	(1, .868)	(4)	(4,403.5, 5,067.4)	100.0
5	(1, .864)	(4)	(4,572.2, 5,287.9)	100.0
6	(1, .859)	(4)	(4,747.2, 5,522.4)	100.0
7	(1, .854)	(4)	(4,929.1, 5,771.7)	100.0
8	(1, .847)	(4)	(5,117.9, 6,036.8)	118.2
9	(1, .843)	(4)	(5,313.9, 6.300.4)	153.9
10	(1, .842)	(4)	(5,517.4, 6,545.0)	159.0
11	(1, .842)	(4)	(5,728.7, 6,800.0)	

Total allocation to the annual cut : 1,143.4 bd. ft.

CALCULATION

The numerical calculation above by the method of successive approximations was made employing the technique depicted in the paper written to me by Richard Bellman and Stuart Dreyfus with the title "On the Computational Solution of Dynamic Programming Process …A Bottleneck process Arising in the Study of Interdependent Industries", which is to be published soon.

From the forestry point of view, dynamic programming applied to regulation of the cut consider the problems simultaneously which almost all existing methods of volume regulation have dealt with. In this paper only two processes were assumed but the number of processes can be increased at least to four with the conventional technique of computation of dynamic programming in case of 10 stage process. Computation of this kind of problem indicated in Table 2 is easy and it will take not so much time. There are other aspects of forest management which were overlooked here. These aspects can be considered simultaneously with the input-output model published in the Journal of Japanese Forestry Society, No. 5, Vol. 40.