J. Op. Res. Japan, Vol. 1, No. 3, 1958.

IS WEATHER FORECASTING PROFITABLE?

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(Recieved December 25, 1957)

A white corrosion product called white rust is produced on the surface of galvanized steel sheets while they are stored. This corrosoin product changes a well polished galvanized surface into a gray and ugly one.

White rust is caused by dews which adhere to the surface of galvanized steel sheets when highly humid air touches them.

By the information from the Weather Forecasting Co. we take the necessary measures to prevent white rust from producing.

The measures taken by us is not always effective, and yet we must pay some money for this information. So we examined the value of the information received from the Weather Forecasting Co. Details are given below.

By rain we mean more than 85 percent humidity. When it rains, dews concentrate on the surface of galvanized steel sheets, causing loss from the white rust on them. To prevent loss, some preparation is needed.

Dews develop about 2 a.m. when the temperature is lowest. Therefore, countermeasures are mostly required during the night. When rain is forecast, some preparation should be done in the evening. We act according to the weather forecast in the evening. We want to know the value of the weather forecast.

THE AMOUNT OF INFORMATION RECEIVED FROM THE WEATHER FORECAST

The amount of information (entropy) received from the weather forecast is as follows :

1. For the person who knows nothing about atmospheric all phenomena, the probability of rain and not rain is $\frac{1}{2}$ (complete beginner), hence, $-\frac{1}{2}\log\frac{1}{2}-\frac{1}{2}\log\frac{1}{2}=1.$

2. For the person who knows the frequency of rain and not rain, assuming that the frequency of rain is $p=20/120=1/6$ (average of four monthes, an amateur), hence

$$-\frac{1}{6}\log\frac{1}{6}-\frac{5}{6}\log\frac{5}{6}=0.6504.$$

3. We know well that a rainy day succeeds a rainy day. And the next day of a fine day is often fine (experienced person). The conditional probability is

$$s=\frac{1}{2} \qquad \text{(rain)}$$

$$r=\frac{9}{10} \qquad \text{(not rain)}$$

Then, determining s, r is as follows:

$$\begin{Bmatrix}\text{also rain}\\ Ps\end{Bmatrix} \qquad \begin{Bmatrix}\text{cease to rain}\\ P(1-s)\end{Bmatrix}$$

$$\begin{matrix}\text{from not rain to rain}\\ (1-P)(1-r)\end{matrix} \qquad \begin{matrix}\text{also not rain}\\ (1-P)r\end{matrix}$$

As the frequency of rain and ceasing to rain is equal,

$$(1-P)(1-r)=P(1-s)$$

$$1-r=\frac{P}{1-P}\cdot(1-s)=\frac{1}{10}$$

from which, the amount of information which such an experienced person receives from the weather forecast is obtained:

$$-\frac{1}{6}\left[\frac{1}{2}\log\frac{1}{2}+\frac{1}{2}\log\frac{1}{2}\right]-\frac{5}{6}\left[\frac{1}{10}\log\frac{1}{10}+\frac{9}{10}\log\frac{9}{10}\right]=0.5574 \text{ bits.}$$

Hence, we can define the grade of knowledge which a complete beginner, amateur, and experienced person have before receiving the weather forecast.

1	......complete beginner
0.6504	......amateur
(background knowledge) (received information)	......experienced person
0.5574	
0	......expert

COMPARING WITH THE VALUE OF THE KNOWLEDGE.

We will compare the usefulness of these background knowledge according to game theory. Let us assume that each kind of cost is estimated as follows:

Cost of the weather forecast ································· ¥ 1,000 per day.
Cost of countermeasures for preventing dews ······ ¥ 7,000 each time.
Loss of failure ··· ¥ 1,000,000 each time.

(I) *Receiving no weather forecast* (Complete beginner.)

		weather		
		rain	not rain	
measures	take	7	7	x
	not take	10,000	0	$1-x$
		y	$1-y$	

Expectant value of loss is as follows:

$$s=7yx+1{,}000y(1-x)+0\cdot(1-y)(1-x)+7(1-y)x=\min$$

$$\frac{\partial s}{\partial y}=0, \qquad x=1$$

The conclusion is that taking measure every day is the best way. In this case the cost amounts to ¥7,000 each time, ¥210,000 a month.

The above is the best that a complete beginner can do.

(II) *Receiving the weather forecast*··············accuracy rate 100 per cent.

		weather	
		rain	not rain
measures	take	8	8
	not take	1001	1

As he knows the mean of the other, average cost is as follows:

$$\frac{1}{6}8+\frac{5}{6}1=2.166$$

per month $2,166\times 30=¥65,000$

profit·········$¥210,000-¥65,000=¥145.000$

(III) *Amateur*

If $y=\frac{1}{6}$, $1-y=\frac{5}{6}$ are known,

$$s=4yx+1000y(1-x)+4(1-y)x=\frac{1000}{6}(1-x)+4x$$

$x=1$ is minimum,

Therefore, the the information which an amateur receives is of no use at all for him.

(IV) *Experienced person*

—rain—

		rain	cease to rain	
measures	take	7	7	x'
	not take	1000	0	$1-x'$

$(s=\frac{1}{2})$

$$\frac{1}{2}7x'+\frac{1}{2}7x'+\frac{1}{2}1000(1-x')+\frac{1}{2}0\cdot(1-x')=\min$$

$x'=1$ is minimum.

—not rain—

		not rain	become rainy	
measures	take	7	7	x
	not take	0	1000	$1-x$

$$r=\frac{9}{10}$$

$$\frac{9}{10}7x+\frac{1}{10}7x+\frac{9}{10}\cdot 0\cdot(1-x)+\frac{1}{10}\cdot 1000(1-x)=\min.$$

$x=1$ is minimum.

Such a degree of background knowledge is of no value for the experienced peason either.

WHAT SHOULD THE ACCURACY RATE OF THE WEATHER FORECAST BE?

Probability of forecasting rain P_1
// of forecasting fine $1-P_1$
// of becoming rain $P=1/6$
// of becoming fine $1-P=5/6$
Accuracy rate of forecasting fine r

(1) Forecasting rain:

		correct (rain)	incorrect (not rain)
measures	take	8	8
	not take	1001	1

As we can always take measures, expectant value is $8P_1$.

(2) Forecasting not rain:

		correct (not rain)	incorrect (rain)
measures	take	8	8
	not take	1	1001

We cannot take measures. Then, $(1-P_1)\,[1\cdot r+1001\cdot(1-r)]$.

The weather forecast is worthwile when the total of both is less than the expectant value of loss of taking measures every day without the weather forecast.

Then, $8P_1+(1-P_1)\,[r+1001(1-r)]<7$

$$P_1<\frac{1000r-994}{1000r-993}$$

Assuming to be $r=\frac{993}{1000}+\varDelta r$,

$$P_1<\frac{\varDelta r-1/1000}{\varDelta r}=1-\frac{1}{1000\varDelta r}.$$

But max $\varDelta r=\frac{7}{1000}$, hence, $P_1<1-\frac{1}{7}=\frac{6}{7}$.

Assuming to be $\varDelta r=\frac{6}{1000}$, $P_1<\frac{5}{6}$............ When P_1 is minimum, it is equal to $\frac{1}{6}$, then, assuming to be $P_1=\frac{1}{6}$ we find

$$\frac{1}{1000\varDelta r}\ 1-\frac{1}{6}=\frac{5}{6}.$$

Therefore,

$$\varDelta r=\frac{6}{5000},$$

from which, min $r=\frac{4971}{5000}=99.42\%$.

If this accuracy rate is satisfied, we may admit some value in weather forecasting.