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ON RUIN PROBLEM WITH NUMERICAL TABLES

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INTRODUCTION

The main aim of the paper is devoted to certain problems connected with Bernoulli trials in which the probabilities of success and failure are p and q, respectively.

We shall consider the gambler who wins a dollar for each success and loss a dollar for each failure. We shall suppose that the gambler and his adversary own a total of dollars and start with z and $a-z$ dollars, respectively. The game continues until the gambler's capital either is reduced to zero or increases to a, that is, until one of the two players is ruin.

We are interested in

1) the probability of the gambler's ruin ; q_z,
2) the expected duration of the game ; D_z,
3) the expectation of gain of the game ; $E(G)$,
4) the case where the unit is changed from a dollar to half a dollar, etc.

Physical applications and analyses suggest another intuitive interpretation. We imagine that the trials are performed at times $t=1, 2, 3, \cdots\cdots$ and interpret their results in terms of the motion of a variable point or particle on the x-axis. At $t=0$ this particle has the position $x=z$, and at $t=1, 2, 3, \cdots\cdots$ it moves a unit step to the right or left according to whether the corresponding trial results in success or failure. Thus the position of our particle at time n represents the gambler's capital at the conclusion of the n-th trial. The trials terminate when the particle for the first time to reach to either $x=0$ or $x=a$. The limiting positions $x=0$ and $x=a$ are called absorbing barriers ; the gambler's ruin is interpreted as absorption at $x=0$.

The ruin problem which we discussed in above, being treated according to this model, may be called a kind of primitive Monte Carlo method.

This problem has already been taken up by W. Feller and many others and a table for the numerical value has been made, [1] but more systematic tables are needed for industry including random factors as a plan for business stratagy.

In this paper, we discuss a case of the values $+c$ and $-c$ with probabilities p and q respectively and discuss the plan of calculation of q_z, $E(G)$, D_z. And we describe the method used when $a=10$, $c=1$, and attached some graphs and tables.

THEORETICAL CONSIDERATION

1) The probability of the gambler's ruin ; q_z.

We consider the problem stated in the introduction. Let q_z be the probability of the gambler's ultimate ruin, and assume the values $+c$ and $-c$ with probabilities p and q, respectively.

After the first trial the gambler's fortune is either $z-c$ or $z+c$ and therefore we must have

$$\left.\begin{aligned} &q_z = pq_{z+c} + qq_{z-c}, \\ &q_0 = 1, \quad q_a = 0. \end{aligned}\right\} \qquad (1)$$

provided $c<z<a-c$.

We shall derive an explicit expression for q_z by the method of particular solutions.

Suppose first that $p \neq q$. It is easily verified that the difference equation (1) admits the two particular solutions $q_z=1$ and $q_z=(q/p)^{z/c}$. It follows that for arbitrary constants A and B the sequence

$$q_z = A + B(q/p)^{z/c}$$

represents a formal solution of (1). We wish to adjust the constant A and B so that the boundary conditions will be satisfied. This means that A and B must satisfy the two linear equations $A+B=1$ and $A+B(q/p)^{a/c}=0$. Thus

$$q_z = \frac{(q/p)^{a/c} - (q/p)^{z/c}}{(q/p)^{a/c} - 1}, \tag{2}$$

represents a formal solution of the difference equation (1) satisfying the boundary conditions. The formula breaks down if $p=q=1/2$.

It follows that when $p=q=1/2$ all solutions of (1) are of the form $q_z = A + B_Z$ and in order to satisfy the boundary condition, we put

$$q_z = 1 - \frac{z}{a}. \tag{3}$$

2) The expectation of gain ; $E(G)$

Under this system the gambler's ultimate gain of loss is a random variable G which assumes the values $a-z$ and $-z$ with probabilties $1-q_z$ and q_z, respectively.

The expectation of gain is formed to be

$$E(G) = a(1-q_z) - z. \tag{4}$$

Introducing the values q_z from (3), it is found that if $p=q=1/2$, then

$$E(G) = 0. \tag{5}$$

3) The expected duration of the game ; D_z.

The argument which lead to the difference equation and the boundary conditions (1) shows directly that the expected duration D_z satisfies the difference equation with the boundary conditions

$$\left.\begin{aligned} &D_z = pD_{z+c} + qD_{z-c} + 1, \quad 0<z<a, \\ &D_0 = 0, \ D_a = 0. \end{aligned}\right\} \tag{6}$$

If $p \neq q$, then $D_z = \frac{z}{c} \cdot \frac{1}{q-p}$ is a formal solution of (1). It follows that if $p \neq q$ all solutions of (1) are of the form

$$D_z = \frac{z}{c} \cdot \frac{1}{(q-p)} + A + B(q/p)^{z/c}$$

The values of the constants A and B follow again from the boundary conditions (6), according to which we must have $A+B=0$ and $A+B(q/p)^{a/c}=-\frac{a}{c}\frac{1}{q-p}$.

Solving for A and B, we find

$$D_z=\frac{z}{c}\frac{1}{q-p}-\frac{a}{c}\frac{1}{q-p}\frac{1-(q/p)^{z/c}}{1-(q/p)^{a/c}}. \tag{7}$$

Again the formula breaks down if $p=q=1/2$.

It follows that when $p=q=1/2$ all solutions of (6) are of the form $D_z=-z^2+A+Bz$. The required solution D_z which satisfies the boundary conditions is then

$$D_z=z(a-z). \tag{8}$$

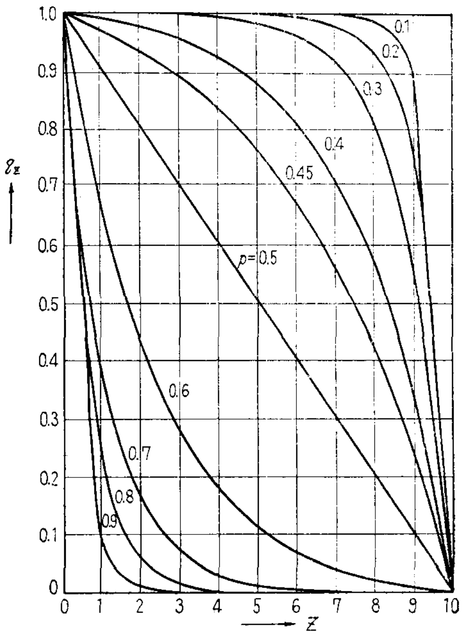

Fig. 1

PLAN OF CALCULATION

From the fact that q_z, $E(G)$, and D_z are functions of the variables a, c, p and z, we first considered the analysis of these functions as the plan of calculation.

1) Consideration for change of p.

If

$$q_{z,p}=\frac{(q/p)^{a/c}-(q/p)^{z/c}}{(q/p)^{a/c}-1},$$

then

$$q_{a-z,p}=1-q_{z,q} \tag{9}$$

Thus $q_{z,p}$ and $q_{z,q}$ are symmetric with each other in regard to the point $(z=q/z, q_z=0.5)$. (Fig. 1).

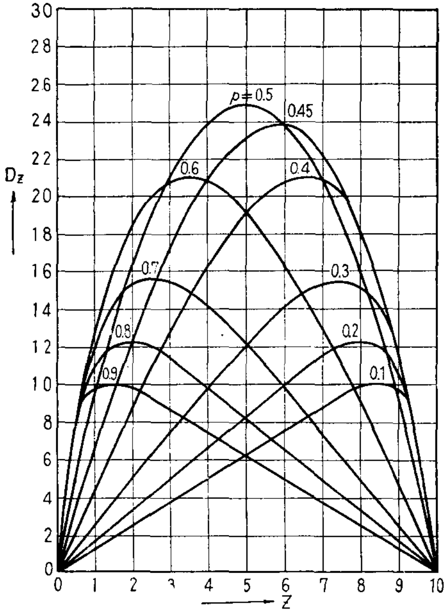

Fig. 2

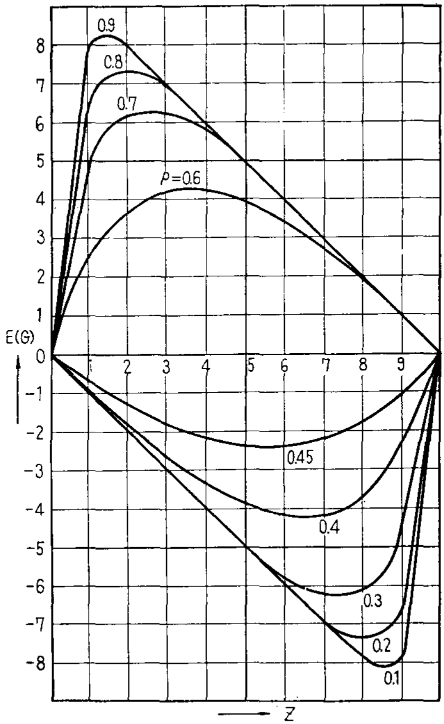

Fig. 3

Also, if we set

$$E_{z\cdot p}=a(1-q_z)-z,$$

then we have

$$E_{a-z,\ p}=-E_{z,q}. \tag{10}$$

Thus $E_{z\cdot p}$ and $E_{z\cdot q}$ are symmetric with each other in regard to points $(z=q/z, E=0)$. (Fig. 2).

Similarly, for

$$D_{z\cdot p}=\frac{z}{c}\,\frac{1}{q-p}-\frac{a}{c}\,\frac{1}{q-p}\,\frac{1-(q/p)^{a/c}}{1-(q/p)^{z/c}},$$

we have

$$D_{a-z,\ p}=D_{z,\ q}. \tag{11}$$

Thus $D_{z,p}$ and $D_{z,\ q}$ are symmetric with each other in regard to line $z=q/z$. (Fig. 3)

2) Consideration for the change of c.

If we set

$$q_z=\frac{\{(q/p)^{1/c}\}^{a}-\{(q/p)^{1/c}\}^{z}}{\{(q/p)^{1/c}\}^{a}-1},$$

and

$$(q/p)^{1/c}=q'/p', \tag{12}$$

then we obtain

$$q_z=\frac{(q'/p')^{a}-(q'/p')^{z}}{(q'/p')^{a}-1}.$$

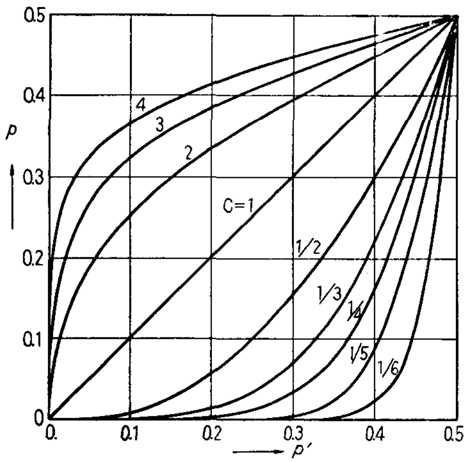

Fig. 4

Hence, q_z which assume values $+c$ and $-c$ with probabilities p and q, respectively is equivalent to the q_z which assume values $+1$ and -1 with probabilities p and q respectively.

From (12), we have

$$\left.\begin{aligned} p'&=\frac{1}{1+(q/p)^{1/c}}, \\ p&=\frac{1}{1+(q'/p')^{c}}. \end{aligned}\right\} \tag{13}$$

Obviously, p_c and $p^{1}/_{c}$ are symmetric with each other in regard to line $p=p'$. (Fig. 4).

Also, since

$$E(G)=a(1-q_z)-z,$$

we can discuss simiiarly $E(G)$ and q_z for the change of c.

we can expect that

$$D_z=\frac{1}{c}\left[\frac{z}{q-p}-\frac{a}{q-p}\,\frac{1-(q'/p')^z}{1-(q/p)^a}\right]. \tag{14}$$

Consequently, D_z is given as $1/c$ of D_z in case of values $+1$ and -1 with probabilities p' and q', respectively.

3) Consideration for the change of a.

Now, when $c=1$ and a changes to c_0a, if z and c change to c_0z and c_0, then q_z, $E(G)$ and D_z are invariant by physical meaning. Really, we have

$$q_z=\frac{\{(q/p)^{a/c}\}^c-\{(q/p)^{z/c}\}^c}{\{(q/p)^{z/c}\}^c-1}=\frac{(q/p)^a-(q/p)^z}{(q/p)^z-1}. \tag{15}$$

Also $E(G)$ and D_z can be proved similarly.

From above consideration, we have following conclusions for the plan of calculation.

1) p may be changed in the range of $[0\sim0.5]$

2) z and c may be changed at the same ratio of change as that of a.

3) If we make numerical table of q_z, $E(G)$ and D_z for p and z in fixed a and c, then we can obtain q_z, $E(G)$ and D_z for any a, c, p and z by this table through suitable transformations.

PREPARATION OF THE TABLES AND DESCRIPTION OF THE METHOD USED

We made the table for following values. (Table 1, Table 2).

$a=10,$

$c=z,$

$p=0.5\sim0.4$ [0.01], $0.4\sim0.1$ [0.1],

$z=1\sim10$ [1].

The method used for the calculation of this numerical tables for any a, p, z, c are following.

First, we determine m as $ma=10$, and transform

$$\begin{pmatrix} a_1 \\ z_1 \\ c_1 \\ p_1 \end{pmatrix} \longrightarrow \begin{pmatrix} ma=10 \\ mz=z' \\ mc=c' \\ p \end{pmatrix}$$

In this transformations, q_z, D_z and $E(G)$ are invariant by equation (15).

Secondary, we transform to p' from p as $c=1$ by equation (13). that is:

$$\begin{pmatrix} 10 \\ z' \\ c' \\ p \end{pmatrix} \longrightarrow \begin{pmatrix} 10 \\ z' \\ c''=1 \\ p' \end{pmatrix}$$

Table I

P/C	1/10	1/9	1/8	1/7	1/6	1/5	1/4	1/3	1/2	1	2	3	4
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.49	0.40	0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50	0.50	0.50
0.48	0.31	0.33	0.35	0.36	0.38	0.41	0.42	0.44	0.46	0.48	0.49	0.49	0.50
0.47	0.23	0.25	0.28	0.30	0.33	0.35	0.38	0.41	0.44	0.47	0.48	0.49	0.49
0.46	0.17	0.19	0.22	0.25	0.28	0.31	0.34	0.38	0.42	0.46	0.48	0.49	0.49
0.45	0.12	0.14	0.17	0.20	0.23	0.27	0.31	0.35	0.40	0.45	0.47	0.48	0.49
0.44	0.08	0.10	0.13	0.16	0.19	0.23	0.28	0.32	0.38	0.44	0.47	0.48	0.48
0.43	0.06	0.07	0.09	0.12	0.16	0.20	0.24	0.30	0.36	0.43	0.46	0.48	0.48
0.42	0.04	0.05	0.07	0.09	0.13	0.17	0.22	0.27	0.34	0.42	0.46	0.47	0.48
0.41	0.03	0.04	0.05	0.07	0.10	0.14	0.19	0.25	0.33	0.41	0.45	0.47	0.48
0.40	0.02	0.03	0.04	0.06	0.08	0.12	0.16	0.23	0.31	0.40	0.45	0.47	0.47
0.39	0.01	0.02	0.03	0.04	0.06	0.10	0.14	0.21	0.29	0.39	0.44	0.46	0.47
0.38	0.01	0.01	0.02	0.03	0.05	0.08	0.12	0.19	0.27	0.38	0.44	0.46	0.47
0.37		0.01	0.01	0.02	0.04	0.07	0.11	0.17	0.26	0.37	0.43	0.46	0.47
0.36		0.01	0.01	0.02	0.03	0.05	0.09	0.15	0.24	0.36	0.43	0.45	0.46
0.35			0.01	0.01	0.02	0.04	0.08	0.14	0.22	0.35	0.42	0.45	0.46
0.34				0.01	0.02	0.04	0.07	0.12	0.21	0.34	0.42	0.44	0.46
0.33				0.01	0.01	0.03	0.06	0.11	0.20	0.33	0.41	0.44	0.46
0.32					0.01	0.02	0.05	0.09	0.18	0.32	0.41	0.44	0.45
0.31					0.01	0.02	0.04	0.08	0.17	0.31	0.40	0.43	0.45
0.30					0.01	0.01	0.03	0.07	0.16	0.30	0.40	0.43	0.45
0.29						0.01	0.03	0.06	0.14	0.29	0.39	0.43	0.44
0.28						0.01	0.02	0.06	0.13	0.28	0.38	0.42	0.44
0.27						0.01	0.02	0.05	0.12	0.27	0.38	0.42	0.44
0.26						0.01	0.02	0.04	0.11	0.26	0.37	0.41	0.43
0.25							0.01	0.04	0.10	0.25	0.37	0.41	0.43
0.24							0.01	0.03	0.09	0.24	0.36	0.41	0.43
0.23							0.01	0.03	0.08	0.23	0.35	0.40	0.43
0.22							0.01	0.02	0.07	0.22	0.35	0.40	0.42
0.21								0.02	0.07	0.21	0.34	0.39	0.42
0.20								0.02	0.06	0.20	0.33	0.39	0.41
0.19								0.01	0.05	0.19	0.33	0.38	0.41
0.18								0.01	0.05	0.18	0.32	0.38	0.40
0.17								0.01	0.04	0.17	0.31	0.37	0.40
0.16								0.01	0.04	0.16	0.30	0.37	0.40
0.15								0.01	0.03	0.15	0.30	0.36	0.39
0.14									0.03	0.14	0.29	0.35	0.39
0.13									0.02	0.13	0.28	0.35	0.38
0.12									0.02	0.12	0.27	0.34	0.38
0.11									0.02	0.11	0.26	0.33	0.37
0.10									0.01	0.10	0.25	0,32	0.37

Tables of $P=\dfrac{1}{1+(q'/p')c}$

$C=1.$ 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 1/9, 1/10, 1, 2, 3, 4,
$P'=0.50\sim0.10$ [0.01].

Table II

p	q	z	qz	$1-qz$	$E(G)$	Dz
0.5	0.5	1	0.9	0.1	0	9
0.5	0.5	2	0.8	0.2	0	16
0.5	0.5	3	0.7	0.3	0	21
0.5	0.5	4	0.6	0.4	0	24
0.5	0.5	5	0.5	0.5	0	25
0.5	0.5	6	0.4	0.6	0	24
0.5	0.5	7	0.3	0.7	0	21
0.5	0.5	8	0.2	0.8	0	16
0.5	0.5	9	0.1	0.9	0	9
0.49	0.51	1	0.917	0.083	−0.2	8.5
0.43	0.51	2	0.831	0.169	−0.3	15.5
0.49	0.51	3	0.741	0.259	−0.4	20.4
0.49	0.51	4	0.647	0.353	−0.5	23.7
0.49	0.51	5	0.550	0.450	−0.5	25.0
0.49	0.51	6	0.449	0.551	−0.5	24.3
0.49	0.51	7	0.343	0.657	−0.4	21.6
0.49	0.51	8	0.233	0.767	−0.3	16.7
0.49	0.51	9	0.119	0.881	−0.2	9.6
0.48	0.52	1	0.932	0.068	−0.3	8.0
0.48	0.52	2	0.858	0.142	−0.6	14.6
0.48	0.52	3	0.779	0.221	−0.8	19.8
0.48	0.52	4	0.692	0.308	−0.9	23.1
0.48	0.52	5	0.599	0.401	−1.0	24.7
0.48	0.52	6	0.497	0.503	−1.0	24.3
0.48	0.52	7	0.388	0.612	−0.9	21.9
0.48	0.52	8	0.268	0.732	−0.7	16.1
0.48	0.52	9	0.140	0.860	−0.4	9.9
0.47	0.53	1	0.945	0.055	−0.5	7.5
0.47	0.53	2	0.883	0.117	−0.8	13.9
0.47	0.53	3	0.813	0.187	−1.1	18.9
0.47	0.53	4	0.735	0.263	−1.4	22.5
0.47	0.53	5	0.646	0.356	−1.5	24.3
0.47	0.53	6	0.546	0.454	−1.5	24.3
0.47	0.53	7	0.433	0.567	−1.3	22.1
0.47	0.53	8	0.305	0.695	−1.1	17.6
0.47	0.53	9	0.162	0.838	−0.6	10.3
0.46	0.54	1	0.956	0.044	−0.6	7.0
0.46	0.54	2	0.905	0.095	−1.1	13.1
0.46	0.54	3	0.844	0.156	−1.4	18.1
0.46	0.54	4	0.774	0.226	−1.7	21.7
0.46	0.54	5	0.690	0.310	−1.9	23.8
0.46	0.54	6	0.478	0.407	−1.9	24.1
0.46	0.54	7	0.478	0.522	−1.8	22.5
0.46	0.54	8	0.343	0.657	−1.4	17.9
0.46	0.54	9	0.185	0.815	−0.9	10.7

p	E	p	qz	$1-qz$	$E(G)$	Dz
0.45	0.55	1	0.965	0.035	−0.7	6.6
0.45	0.55	2	0.923	0.077	−1.2	12.3
0.45	0.55	3	0.872	0.128	−1.7	17.2
0.45	0.55	4	0.809	0.191	−2.1	20.9
0.45	0.55	5	0.732	0.268	−2.3	23.2
0.45	0.55	6	0.638	0.362	−2.4	23.8
0.45	0.55	7	0.523	0.477	−2.2	22.3
0.45	0.55	8	0.382	0.618	−1.8	18.2
0.45	0.55	9	0.210	0.790	−1.1	11.0
0.44	0.56	1	0.973	0.027	−0.7	6.1
0.44	0.56	2	0.939	0.061	−1.4	11.6
0.44	0.56	3	0.895	0.105	−2.0	16.3
0.44	0.56	4	0.840	0.160	−2.4	20.0
0.44	0.56	5	0.770	0.230	−2.7	22.5
0.44	0.56	6	0.680	0.320	−2.8	23.3
0.44	0.56	7	0.566	0.434	−2.7	22.1
0.44	0.56	8	0.420	0.580	−2.2	18.4
0.44	0.56	9	0.235	0.765	−1.4	11.3
0.43	0.57	1	0.979	0.021	−0.8	5.7
0.43	0.57	2	0.952	0.048	−1.5	10.9
0.43	0.57	3	0.916	0.084	−2.2	15.4
0.43	0.57	4	0.867	0.133	−2.7	19.1
0.43	0.57	5	0.804	0.196	−3.0	21.7
0.43	0.57	6	0.719	0.281	−3.2	22.8
0.43	0.57	7	0.607	0.393	−3.1	21.9
0.43	0.57	8	0.458	0.542	−2.6	18.4
0.43	0.57	9	0.261	0.739	−1.6	11.5
0.42	0.58	1	0.984	0.016	−0.8	5.3
0.42	0.58	2	0.963	0.037	−1.6	10.2
0.42	0.58	3	0.933	0.067	−2.3	14.5
0.42	0.58	4	0.891	0.109	−2.9	18.2
0.42	0.58	5	0.834	0.166	−3.3	20.9
0.42	0.58	6	0.755	0.245	−3.6	22.2
0.42	0.58	7	0.646	0.354	−3.5	21.6
0.42	0.58	8	0.495	0.505	−3.0	18.5
0.42	0.58	9	0.287	0.713	−1.9	11.7
0.41	0.59	1	0.988	0.012	−0.9	4.9
0.41	0.59	2	0.971	0.029	−1.7	9.5
0.41	0.59	3	0.947	0.053	−2.5	13.7
0.41	0.59	4	0.911	0.089	−3.1	17.3
0.41	0.59	5	0.861	0.139	−3.6	20.0
0.41	0.59	6	0.787	0.213	−3.9	21.5
0.41	0.59	7	0.682	0.318	−3.8	21.2
0.41	0.59	8	0.531	0.469	−3.3	18.4
0.41	0.59	9	0.313	0.687	−2.1	11.9

Table II

p	q	z	qz	1−qz	E(G)	Dz
0.4	0.6	1	0.991	0.009	−0.9	4.6
0.4	0.6	2	0.978	0.022	−1.8	8.9
0.4	0.6	3	0.958	0.042	−2.6	12.9
0.4	0.6	4	0.928	0.072	−3.3	16.4
0.4	0.6	5	0.884	0.116	−4.8	19.2
0.4	0.6	6	0.817	0.183	−4.2	20.7
0.4	0.6	7	0.716	0.284	−4.2	20.8
0.4	0.6	8	0.565	0.435	−3.7	18.3
0.4	0.6	9	0.339	0.661	−2.4	12.1
0.35	0.65	1	0.998	0.002	−1.0	3.3
0.35	0.65	2	0.995	0.005	−2.0	6.4
0.35	0.65	3	0.989	0.011	−2.9	9.6
0.53	0.65	4	0.978	0.022	−3.8	12.5
0.35	0.65	5	0.956	0.044	−4.6	15.2
0.35	0.65	6	0.918	0.082	−5.2	17.1
0.35	0.65	7	0.846	0.154	−5.5	18.2
0.35	0.65	8	0.711	0.289	−5.1	17.0
0.35	0.65	9	0.462	0.538	−3.6	12.1
0.3	0.7	1	1.000	0.	−1.0	2.5
0.3	0.7	2	0.999	0.001	−2.0	5.0
0.3	0.7	3	0.997	0.003	−3.0	7.4
0.3	0.7	4	0.994	0.006	−4.0	9.8
0.3	0.7	5	0.985	0.015	−4.9	12.1
0.3	0.7	6	0.966	0.034	−5.7	14.1
0.3	0.7	7	0.920	0.080	−6.2	15.5
0.3	0.7	8	0.813	0.187	−6.1	15.3
0.3	0.7	9	0.564	0.436	−4.6	11.6
0.2	0.8	1	1.000	0.	−1.0	1.7
0.2	0.8	2	1.000	0.	−2.0	3.3
0.2	0.8	3	1.000	0.	−3.0	5.0
0.2	0.8	4	1.000	0.	−4.0	6.7
0.2	0.8	5	0.999	0.001	−5.0	8.3
0.2	0.8	6	0.996	0.004	−6.0	9.9
0.2	0.8	7	0.984	0.016	−6.8	11.4
0.2	0.8	8	0.938	0.062	−7.4	12.3
0.2	0.8	9	0.750	0.250	−6.5	10.5
0.1	0.9	1	1.000	0.	−1.0	1.3
0.1	0.9	2	1.000	0.	−2.0	2.5
0.1	0.9	3	1.000	0.	−3.0	3.8
0.1	0.9	4	1.000	0.	−4.0	5.0
0.1	0.9	5	1.000	0.	−5.0	6.3
0.1	0.9	6	1.000	0.	−6.0	7.5
0.1	0.9	7	0.998	0.002	−7.0	8.8
0.1	0.9	8	0.988	0.012	−7.9	9.9
0.1	0.9	9	0.889	0.111	−7.9	9.9

p	q	z	qz	1−qz	E(G)	Dz
0.9	0.1	1	0.111	0.889	7.9	9.9
0.9	0.1	2	0.0123	0.9877	7.9	9.8
0.9	0.1	3	0.0014	0.9986	7.0	8.7
0.9	0.1	4	0.0002	0.9998	6.0	7.5
0.9	0.1	5	0.	1.	5.0	6.2
0.9	0.1	6	0.	1.	4.0	5.0
0.9	0.1	7	0.	1.	3.0	3.7
0.9	0.1	8	0.	1.	2.0	2.5
0.9	0.1	9	0.	1.	1.0	1.3
0.8	0.2	1	0.250	0.75	6.5	10.8
0.8	0.2	2	0.063	0.937	7.4	12.3
0.8	0.2	3	0.016	0.984	6.8	11.4
0.8	0.2	4	0.004	0.996	6.0	9.9
0.8	0.2	5	0.001	0.999	5.0	8.4
0.8	0.2	6	0.	1.	4.0	6.7
0.8	0.2	7	0.	1.	3.0	5.0
0.8	0.2	8	0.	1.	2.0	3.3
0.8	0.2	9	0.	1.	1.0	1.7
0.7	0.3	1	0.428	0.572	4.7	11.8
0.7	0.3	2	0.184	0.816	6.2	15.4
0.7	0.3	3	0.079	0.921	6.2	15.5
0.7	0.3	4	0.034	0.966	5.7	14.0
0.7	0.3	5	0.014	0.986	4.9	12.2
0.7	0.3	6	0.006	0.994	3.9	9.9
0.7	0.3	7	0.002	0.998	3.	7.4
0.7	0.3	8	0.	1.	2.	5.0
0.7	0.3	9	0.	1.	1.	2.5
0.6	0.4	1	0.661	0.339	2.4	12.0
0.6	0.4	2	0.435	0.565	3.7	18.3
0.6	0.4	3	0.284	0.716	4.2	20.8
0.6	0.4	4	0.183	0.817	4.2	20.8
0.6	0.4	5	0.116	0.884	3.8	19.2
0.6	0.4	6	0.074	0.926	3.3	16.4
0.6	0.4	7	0.042	0.958	2.6	12.9
0.6	0.4	8	0.022	0.978	1.8	9.9
0.6	0.4	9	0.009	0.991	0.9	4.6

Tables of *qz*, *E*(*G*), *Dz*.

p=0.5∼0.4 [0.01]

0.35, 0.3, 0,2, 0.1.

0.9, 0.8, 0.7, 0.6.

X=1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

Using the table, we find out q_z, $E(G)$ and D_z for $a'=10$, z', $c''=1$ and p'. And by equation (2), (4), and (7) respectively, we have

$$q_z = q_z', \tag{16}$$

$$E_z(G) = mE_z'(G), \tag{17}$$

$$D_z = mD_z', \tag{18}$$

Thus, we have q_z, D_z and $E(G)$ for a, p, z and c.
If, $0.5 \leq p < 1$, then by equation

$$q_{z,q} = 1 - q_{a-z,p} \tag{9}$$

$$E_{z,q} = -E_{a-z,p} \tag{10}$$

$$D_{z,q} = D_{a-z,p} \tag{11}$$

we can obtain the case of $0 > p > 0.5$.

Ex 1. Obtain q_z, $E(G)$ and D_z for $p=0.45$, $a=100$, $z=90$. $c=1$ Using the table 1 and following transformation

$$\begin{pmatrix} a=100 \\ z=\ 90 \\ c=\ \ 1 \\ p=0.45 \end{pmatrix} \longrightarrow \begin{pmatrix} 10 \\ 9 \\ 1/10 \\ 0.45 \end{pmatrix} \longrightarrow \begin{pmatrix} 10 \\ 9 \\ 1 \\ 0.12 \end{pmatrix}$$

and also, using the table II and the equation (16), (17), (18), we have

$$q_z \fallingdotseq 0.89, \quad E(G) = -79 \times 10, \quad D_z = 99 \times 10.$$

Ex 2. Obtain q_z, $E(G)$ and D_z for $p=0.65$, $a=100$, $z=90$ and $c=5$. We first obtain q_z, $E(G)$ and D_z for $p=0.45$, $a=100$, $z=10$ and $c=5$.

By the transformation

$$\begin{pmatrix} a=100 \\ z=\ 10 \\ c=\ \ 5 \\ p=0.45 \end{pmatrix} \longrightarrow \begin{pmatrix} a=10 \\ z=\ 1 \\ c={}^1/_2 \\ p=0.45 \end{pmatrix} \longrightarrow \begin{pmatrix} 10 \\ 1 \\ 1 \\ 0.40 \end{pmatrix}$$

We have

$$q_z=0.991, \quad E(G)=-0.9\times 10, \quad D_z=4.6\times 10.$$

Using the equation (9), (10) and (11), we have

$$q_{90,\ 0\cdot 65}=1-q_{10,\ 0\cdot 45}=0.009,$$

$$D_{90,\ 9\cdot 65}=D_{10,\ 0\cdot 45}=46,$$

$$E_{90,\ 0\cdot 65}=-E_{10,0\cdot 45}=9.$$

CONCLUSION

In this paper, we attached simple tables but are needed more accurate table as Operations Research.

We assumed the value $+c$ and $-c$ with probailities p and q respectively, but in practice we have the case of the values $+c$ and $-d\,(c \neq d)$. This point is being investigated.

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