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STOCHASTIC PROGRAMMING PROBLEM WITH FIXED CHARGE RECOURSE

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Abstract In this paper, we introduce a class of stochastic programming problem with fixed charge recourse in which a fixed cost is imposed if the value of the continuous recourse variable is strictly positive. The algorithm of a branch-and-cut method to solve the problem is developed by using the property of the expected recourse function. Then, the problem is applied to a power generating system. The numerical experiments show that the proposed algorithm is quite efficient. The mathematical programming model defined in this paper is quite useful for a variety of design and operational problems.

Keywords: Stochastic optimization, stochastic programming with fixed charge recourse, branch-and-cut, generation planning

1. Introduction

Mathematical programming has been applied to many problems in various fields. However for many actual problems, the assumption that the parameters involved in the problem are deterministic known data is often unjustified. These data contain uncertainty and are thus represented as random variables, since they represent information about the future. Decision-making under conditions of uncertainty involves potential risk. Stochastic programming (Birge [3], Birge and Louveaux [4], Kall and Wallace [7]) deals with optimization under uncertainty. A stochastic programming problem with recourse is referred to as a two-stage stochastic problem. In the first stage, a decision has to be made without complete information on random factors. After the value of random variables are known, recourse action can be taken in the second stage. For the continuous stochastic programming problem with recourse, an L-shaped method (Van Slyke and Wets [22]) is well-known. The L-shaped method was used to solve stochastic programs having discrete decisions in the first stage (Laporte and Louveaux[13]). This method was applied to solve a stochastic concentrator location problem (Shiina [18, 19]).

For a multistage stochastic programming with recourse, Louveaux [14] introduced the concept of block-separable recourse. By utilizing this property, the problem is transformed into a two-stage stochastic program with recourse. A typical problem which has the property of block separable recourse is the multistage electric power capacity expansion problem. Shiina and Birge [20] proposed an L-shaped algorithm to solve the problem by reformulating the problem into one which has integer variables only in the first stage decisions.

In this paper, we consider a stochastic integer programming problem which does not possess the property of block separable recourse. If integer variables are involved in a second stage problem, optimality cuts based on the Benders [2] decomposition do not provide facets of the epigraph of recourse function. It is difficult to approximate the recourse function which is in general nonconvex and discontinuous, since the function is defined as the value function of the second stage integer programming problem. Carøe and Tind [5] generalized the L-shaped method for a mixed integer first and second stage variable. But from a practical standpoint, the method for this implementation is not known because it is necessary to add nonlinear and discontinuous cuts to the master problem.

For stochastic programs with simple integer recourse, Louveaux and van der Vlerk [15] investigated the property of the problem. Klein Haneveld, Stougie, and van der Vlerk [11, 12] proposed an algorithm to construct a convex envelope of the recourse function. Ahmed, Tawarmalani, and Sahinidis[1] developed a finite algorithm based on the branching of the first stage integer variables.

However, variables involved in the stochastic program with simple integer recourse are restricted to having a nonnegative integer value. Such restriction of variables to pure integers makes application of the problem difficult. Therefore, we consider a practical stochastic programming model which is applicable to various real problems. In this paper, we define a stochastic program with fixed charge recourse in which a fixed cost is imposed if the value of the continuous recourse variable is strictly positive. This mathematical programming model is quite useful for a variety of design and operational problems which arise in diverse contexts, such as investment planning, capacity expansion, network design and facility location.

In Section 2, the basic model of the stochastic programming problem with recourse and the L-shaped method are shown. Then, we define the stochastic program with fixed charge recourse, which is a natural extension of the continuous simple recourse. In Section 3, we investigate the property of the recourse function. The algorithm of a branch-and-cut method to solve the problem is shown in Section 4. In Section 5, we develop a heuristic algorithm using a dynamic slope scaling procedure. The electric power industry is undergoing restructuring and deregulation, and it is necessary for electric power utilities or power generators to incorporate uncertainty into power generation planning. Hence, the development of an effective algorithm to solve the stochastic programming problem is required. Shiina and Birge [21] developed an algorithm which solves the short-term scheduling of power plants. In Section 6, we present numerical results for a power generating system obtained from our solution approach.

2. Stochastic Programming with Fixed Charge Recourse

2.1. Basic concepts

We first form the basic two-stage stochastic linear programming problem with recourse as (SPR).

$$
\left|\begin{array}{rl}
\text{(SPR): min} & c^\top x + \mathcal{Q}(x) \\
\text{subject to} & Ax = b \\
& x \geq 0 \\
\text{where} & \mathcal{Q}(x) = E_{\tilde{\xi}}[Q(x, \tilde{\xi})] \\
& Q(x, \xi) = \min\{q^\top y(\xi) \mid Wy(\xi) \geq \xi - Tx, y(\xi) \geq 0\}, \xi \in \Xi
\end{array}\right.
$$

In the formulation of (SPR), c is a known n_1-vector, b a known m_1-vector, $q(> 0)$ a known n_2-vector, and A and W are known matrices of size $m_1 \times n_1$ and $m_2 \times n_2$, respectively. The first stage decisions are represented by the n_1-vector x. We assume the m_2-random vector $\tilde{\xi}$ is defined on a known probability space. Let Ξ be the support of $\tilde{\xi}$, i.e. the smallest closed set such that $P(\Xi) = 1$.

Given a first stage decision x, the realization of random vector ξ of $\tilde{\xi}$ is observed. The second stage data ξ become known. Then, the second stage decision $y(\xi)$ must be taken so as to satisfy the constraints $Wy(\xi) \geq \xi - Tx$ and $y(\xi) \geq 0$. The second stage decision $y(\xi)$ is assumed to cause a penalty of q. The objective function contains a deterministic term $c^\top x$ and the expectation of the second stage objective. The symbol $E_{\tilde{\xi}}$ represents the mathematical expectation with respect to $\tilde{\xi}$, and the function $Q(x,\xi)$ is called the recourse function in state ξ. The value of the recourse function is given by solving a second stage linear programming problem.

It is assumed that the random vector $\tilde{\xi}$ has a discrete distribution with finite support $\Xi = \{\xi^1, \dots, \xi^S\}$ with $\mathrm{Prob}(\tilde{\xi} = \xi^s) = p^s, s = 1, \dots, S$. A particular realization ξ of the random vector $\tilde{\xi}$ is called a scenario. Given the finite discrete distribution, the problem (SPR) is restated as (DEP), the deterministic equivalent problem for (SPR).

$$\left|\begin{array}{rl} \text{(DEP): min} & c^\top x + \sum_{s=1}^{S} p^s Q(x,\xi^s) \\ \text{subject to} & Ax = b \\ & x \geq 0 \\ \text{where} & Q(x,\xi^s) = \min\{q^\top y(\xi^s) \mid Wy(\xi^s) \geq \xi^s - Tx, y(\xi^s) \geq 0\}, s = 1, \dots, S \end{array}\right.$$

To solve (DEP), an L-shaped method (Van Slyke and Wets [22]) has been used. This approach is based on Benders [2] decomposition. The expected recourse function is piecewise linear and convex, but it is not given explicitly in advance. In the algorithm of the L-shaped method, we solve the following problem (MASTER). The new variable θ denotes the upper bound for the expected recourse function such that $\theta \geq \sum_{s=1}^{S} p^s Q(x,\xi^s)$.

$$\left|\begin{array}{rl} \text{(MASTER): min} & c^\top x + \theta \\ \text{subject to} & Ax = b \\ & x \geq 0 \\ & \theta \geq 0 \end{array}\right.$$

Let x^*, θ^* be the optimal solution of (MASTER), then the following second stage problem is solved for $s = 1, \dots, S$.

$$\begin{aligned} Q(x^*,\xi^s) &= \min\{q^\top y(\xi^s) \mid Wy(\xi^s) \geq \xi^s - Tx^*, y(\xi^s) \geq 0\} && (2.1) \\ &= \max\{(\xi^s - Tx^*)^\top \pi(\xi^s) \mid \pi(\xi^s)^\top W \leq q^\top, \pi(\xi^s) \geq 0\} && (2.2) \end{aligned}$$

If minimization problem (2.1) is infeasible for some scenario ξ^s, the optimal objective value of maximization problem (2.2) is infinite above or problem (2.2) is infeasible. Leaving out the latter case, we have a dual solution $\bar{\pi}(\xi^s) \geq 0$ which satisfies the following inequalities.

$$(\xi^s - Tx^*)^\top \bar{\pi}(\xi^s) > 0 \quad \text{and} \quad \bar{\pi}(\xi^s)^\top W \leq 0 \tag{2.3}$$

To cut off solution x^*, the feasibility cut (2.4) is added to the formulation of (MASTER).

$$(\xi^s - Tx)^\top \bar{\pi}(\xi^s) \leq 0 \tag{2.4}$$

If minimization problem (2.1) is feasible for $\forall \xi \in \Xi$ and $\theta^* < \sum_{s=1}^{S} p^s Q(x^*,\xi^s)$, let $\pi^*(\xi^s)$ be the solution of problem (2.2). In this case, the optimality cut (2.5) is added as an outer approximation of $\sum_{s=1}^{S} p^s Q(x,\xi^s)$.

$$\theta \geq \sum_{s=1}^{S} p^s (\xi^s - Tx)^\top \pi^*(\xi^s) \tag{2.5}$$

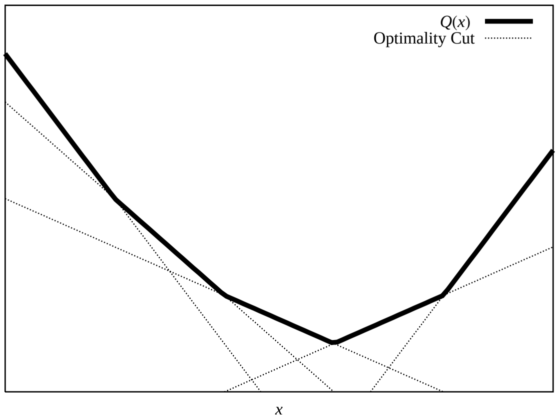

Figure 1: Expected recourse function and optimality cuts

The recourse function is given by an outer linearization using a set of optimality cuts as shown in Fig 1. In the case of $n_2 = 2 \times m_2$ and $W = (I, -I)$, the problem (SPR) is said to have a simple recourse. In Section 3, we consider the problem with $m_2 = n_2$ and $W = I$.

2.2. Definition of the problem

In this section, we define the stochastic program with fixed charge recourse as (SPFCR), in which a positive fixed cost is imposed if the value of the continuous recourse variable is strictly positive. The problem (SPFCR) requires a fixed charge to be incurred, as compared to the problem with simple continuous recourse.

$$
\begin{array}{|rl}
\text{(SPFCR): min} & c^\top x + \mathcal{Q}(x) \\
\text{subject to} & Ax = b \\
 & x \geq 0 \\
\text{where} & \mathcal{Q}(x) = \sum_{s=1}^{S} p^s Q(x, \xi^s) \\
 & Q(x, \xi^s) = \min\{q^\top y(\xi^s) + f^\top z(\xi^s) \mid \; y(\xi^s) \geq \xi^s - Tx, \\
 & \qquad\qquad\qquad\qquad\qquad\qquad\qquad 0 \leq y(\xi^s) \leq Mz(\xi^s), \\
 & \qquad\qquad\qquad\qquad\qquad\qquad\qquad z(\xi^s) \in \{0,1\}^{n_2}\}, s = 1, \ldots, S
\end{array}
$$

In the formulation of (SPFCR), c is a known n_1-vector, b a known m_1-vector, $q(> 0)$ a known n_2-vector, $f(> 0)$ a known n_2-vector, and A, T, known matrices of size $m_1 \times n_1$ and $n_2 \times n_2$, respectively. The problem (SPFCR) can be viewed as a natural extension of the problem (DEP) with $m_2 = n_2$, $W = I$. The first stage decisions are represented by the n_1-vector x. The second stage decisions are $n_2 (= m_2)$-vector $y(\xi) \geq 0$ and $z(\xi)$, where $z(\xi)$ is restricted to n_2-binary vector. If the value of the i-th recourse variable $y_i(\xi^s)$ is positive, the value of $z_i(\xi^s)$ must be one. Hence a fixed cost f_i is imposed on the recourse cost when $y_i(\xi^s) > 0$.

Let $\tilde{\xi}_i$ and Ξ_i be the i-th component of the random vector $\tilde{\xi}$ and the support of $\tilde{\xi}_i$, respectively. We make the following assumptions.

Assumption 2.1 *The random variables $\tilde{\xi}_i, i = 1, \ldots, n_2$ are independent and follow a discrete distribution. A probability p_i^s is associated with each outcome ξ_i^s, $s = 1, \ldots, |\Xi_i|$ of $\tilde{\xi}_i$. The random variable $\tilde{\xi}_i$ takes only positive values and is bounded as $0 < \xi_i^s < \infty, s = 1, \ldots, |\Xi_i|, i = 1, \ldots, n_2$.*

Assumption 2.2 *The first stage feasible set $\{x | Ax = b, x \geq 0\}$ is non-empty and compact.*

Then, the support of $\tilde{\xi}$ is described as $\Xi = \Xi_1 \times \cdots \times \Xi_{n_2}$. And the positive constant M can be taken so as to satisfy $M \geq \max\{\xi_i^s, s = 1, \ldots, |\Xi_i|, i = 1, \ldots, n_2\}$. From Assumptions 1 and 2, the feasible solutions $y(\xi^s)$ and $z(\xi^s)$ exist for all first stage feasible solution x and scenario ξ^s. So (SPFCR) has a relatively complete recourse.

Then we define the new variables $\chi = Tx$, where χ is called a tender to be bid against random outcomes. The problem (SPFCR) can be transformed into (SPFCRT) as follows.

$$\begin{array}{rl}
\text{(SPFCRT): min} & c^\top x + \Psi(\chi) \\
\text{subject to} & Ax = b \\
& x \geq 0 \\
& \chi = Tx \\
\text{where} & \Psi(\chi) = \sum_{s=1}^{S} p^s \psi(\chi, \xi^s) \\
& \psi(\chi, \xi^s) = \min\{q^\top y(\xi^s) + f^\top z(\xi^s) \mid \; y(\xi^s) \geq \xi^s - \chi, \\
& \qquad 0 \leq y(\xi^s) \leq Mz(\xi^s), \\
& \qquad z(\xi^s) \in \{0,1\}^{n_2}\}, s = 1, \ldots, S
\end{array}$$

The new recourse function $\psi(\chi, \xi)$ of $\chi = (\chi_1, \ldots, \chi_{n_2})^\top$ is separable in $\chi_i, i = 1, \ldots, n_2$.

$$\psi(\chi, \xi) = \sum_{i=1}^{n_2} \psi_i(\chi_i, \xi_i) \tag{2.6}$$

$$\begin{aligned}
\psi_i(\chi_i, \xi_i) = \min\{q_i y_i(\xi_i) + f_i z_i(\xi_i) \;\; | \;\; & y_i(\xi_i) \geq \xi_i - \chi_i, \\
& 0 \leq y_i(\xi_i) \leq M z_i(\xi_i), \\
& z_i(\xi_i) \in \{0,1\}\}
\end{aligned} \tag{2.7}$$

It is shown that the expected recourse function $\Psi(\chi)$ is also separable in $\chi_i, i = 1, \ldots, n_2$ as (2.8), where $\Psi_i(\chi_i) = \sum_{s=1}^{|\Xi_i|} p_i^s \psi_i(\chi_i, \xi_i^s)$ denotes the expectation of the i-th recourse function (2.7).

$$\begin{aligned}
\Psi(\chi) &= \sum_{s=1}^{S} p^s \psi(\chi, \xi^s) \\
&= \sum_{s_1=1}^{|\Xi_1|} \cdots \sum_{s_{n_2}=1}^{|\Xi_{n_2}|} p_1^{s_1} \cdots p_{n_2}^{s_{n_2}} \sum_{i=1}^{n_2} \psi_i(\chi_i, \xi_i^{s_i}) \\
&= \sum_{i=1}^{n_2} (\sum_{s_1=1}^{|\Xi_1|} \cdots \sum_{s_{n_2}=1}^{|\Xi_{n_2}|} p_i^{s_i} \prod_{\substack{j=1 \\ j \neq i}}^{n_2} p_j^{s_j}) \psi_i(\chi_i, \xi_i^{s_i}) \\
&= \sum_{i=1}^{n_2} \sum_{s_i=1}^{|\Xi_i|} p_i^{s_i} \psi_i(\chi_i, \xi_i^{s_i}) \\
&= \sum_{i=1}^{n_2} \Psi_i(\chi_i)
\end{aligned} \tag{2.8}$$

Example 1 Let $x = (x_1, x_2)^\top$, $\tilde{\xi} = (\tilde{\xi}_1, \tilde{\xi}_2)^\top$, and set the parameters as

$$q_i = 2, f_i = 4, i = 1, 2 \text{ and } T = \begin{pmatrix} 1.2 & 0.4 \\ 0.5 & 1.0 \end{pmatrix}.$$

Suppose $\tilde{\xi}_1, \tilde{\xi}_2$ follow the discrete uniform distribution with $\Xi_i = \{1, 2, 3, 4\}, i = 1, 2$ and $p_i^s = 1/4,\ i = 1, 2, s = 1, \ldots, 4$. Two distinct expected recourse functions $\mathcal{Q}(x)$ and $\Psi(\chi)$ are illustrated in Figure 2. The separability of $\Psi(\chi)$ in tender variables $\chi_i, i = 1, 2$ is easily seen.

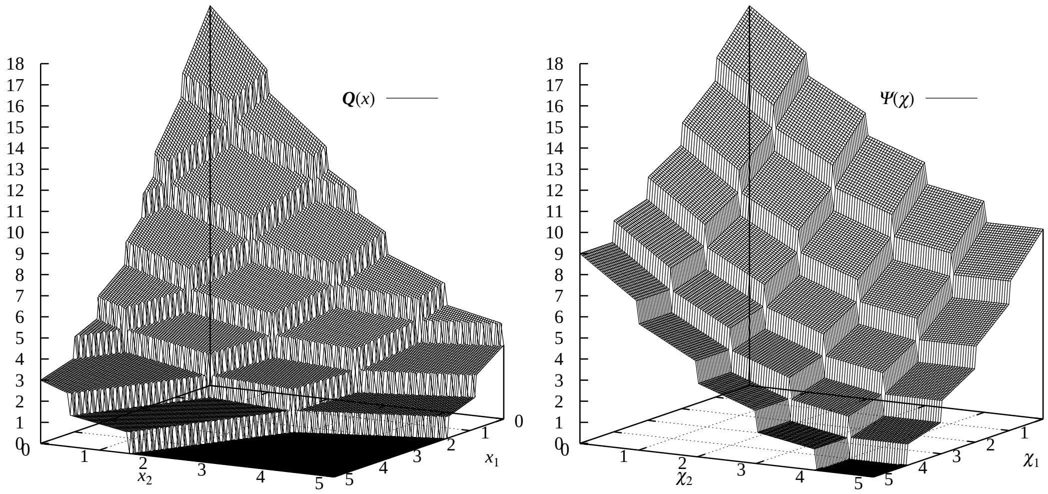

Figure 2: Expected recourse function $\mathcal{Q}(x)$ and $\Psi(\chi)$

3. Property of the Recourse Function

The optimal solution $(y_i^*(\xi_i), z_i^*(\xi_i))$ of the problem (2.7) to define the i-th recourse function $\psi_i(\chi_i, \xi_i)$ is described as (3.1).

$$(y_i^*(\xi_i), z_i^*(\xi_i)) = \begin{cases} (\xi_i - \chi_i, 1) & \text{if } \xi_i > \chi_i \\ (0, 0) & \text{otherwise} \end{cases} \tag{3.1}$$

Therefore, $\Psi_i(\chi_i)$ is calculated as $\Psi_i(\chi_i) = \sum_{s=1}^{|\Xi_i|} p_i^s \psi_i(\chi, \xi^s) = \sum_{s=1}^{|\Xi_i|} p_i^s (q_i y_i^*(\xi_i^s) + f_i z_i^*(\xi_i^s))$. Without loss of generality, it is assumed that the possible realization of random variable ξ_i is monotonically ordered so that $\xi_i^1 \geq \xi_i^2 \geq \cdots \geq \xi_i^{|\Xi_i|}$. The expected i-th recourse function $\Psi_i(\chi_i)$ is calculated as follows.

$$\Psi_i(\chi_i) = \begin{cases} 0, & \text{if } \xi_i^1 \leq \chi_i \\ p_i^1 \left\{ q_i(\xi_i^1 - \chi_i) + f_i \right\}, & \text{if } \xi_i^2 \leq \chi_i < \xi_i^1 \\ p_i^2 \left\{ q_i(\xi_i^2 - \chi_i) + f_i \right\} + p_i^1 \left\{ q_i(\xi_i^1 - \chi_i) + f_i \right\}, & \text{if } \xi_i^3 \leq \chi_i < \xi_i^2 \\ \vdots & \\ \sum_{k=1}^{j-1} p_i^k \left\{ q_i(\xi_i^k - \chi_i) + f_i \right\}, & \text{if } \xi_i^j \leq \chi_i < \xi_i^{j-1} \\ \sum_{k=1}^{j} p_i^k \left\{ q_i(\xi_i^k - \chi_i) + f_i \right\}, & \text{if } \xi_i^{j+1} \leq \chi_i < \xi_i^j \\ \vdots & \\ \sum_{k=1}^{|\Xi_i|-1} p_i^k \left\{ q_i(\xi_i^k - \chi_i) + f_i \right\}, & \text{if } \xi_i^{|\Xi_i|} \leq \chi_i < \xi_i^{|\Xi_i|-1} \\ \sum_{k=1}^{|\Xi_i|} p_i^k \left\{ q_i(\xi_i^k \chi_i) + f_i \right\}, & \text{if } 0 \leq \chi_i < \xi_i^{|\Xi_i|} \end{cases} \tag{3.2}$$

Proposition 3.1 *The expected i-th recourse function $\Psi_i(\chi_i)$ is discontinuous at the points $\chi_i = \xi_i^s, s = 1, \ldots, |\Xi_i|$, and $\Psi_i(\chi_i)$ is lower semicontinuous (l.s.c.).*

Proof. To prove the discontinuity of $\Psi_i(\chi_i)$, we show the right-hand limit differs from the left-hand limit at the point $\chi_i = \xi_i^j$.

$$\begin{aligned}
\text{right-hand limit} &= \lim_{\chi_i \to \xi_i^j + 0} \Psi_i(\chi_i) \\
&= \lim_{\chi_i \to \xi_i^j + 0} \sum_{k=1}^{j-1} p_i^k \left\{ q_i(\xi_i^k - \chi_i) + f_i \right\} \\
&= \sum_{k=1}^{j-1} p_i^k \left\{ q_i(\xi_i^k - \xi_i^j) + f_i \right\} \\
\text{left-hand limit} &= \lim_{\chi_i \to \xi_i^j - 0} \Psi_i(\chi_i) \\
&= \lim_{\chi_i \to \xi_i^j - 0} \sum_{k=1}^{j} p_i^k \left\{ q_i(\xi_i^k - \chi_i) + f_i \right\} \\
&= \sum_{k=1}^{j} p_i^k \left\{ q_i(\xi_i^k - \xi_i^j) + f_i \right\} \\
&= \sum_{k=1}^{j-1} p_i^k \left\{ q_i(\xi_i^k - \xi_i^j) + f_i \right\} + p_i^j f_i
\end{aligned}$$

It can be easily seen that the function $\Psi_i(\chi_i)$ is linear continuous except for at the points $\chi_i = \xi^s, s = 1, \ldots, |\Xi_i|$. Next, we prove that the function $\Psi_i(\chi_i)$ is lower semicontinuous at $\chi_i = \xi_i^s$.

For any $\varepsilon > 0$, set $\overline{\delta}$ as (3.3). If $\Psi_i(\xi_i^s) - \varepsilon \geq 0$, there exists $\overline{\chi}_i$ which satisfies $\xi_i^s < \overline{\chi}_i \leq \xi_i^1$ and $\Psi_i(\overline{\chi}_i) \geq \Psi_i(\xi_i^s) - \varepsilon$. Let $\gamma > 0$ be an arbitrary positive constant.

$$\overline{\delta} = \begin{cases} \overline{\chi}_i - \xi_i^s & \text{if } \Psi_i(\xi_i^s) - \varepsilon \geq 0. \\ \gamma(>0) & \text{otherwise} \end{cases} \tag{3.3}$$

For $\chi_i < \xi_i^s + \overline{\delta}$, it follows that $\Psi_i(\chi_i) - \Psi_i(\xi_i^s) > -\varepsilon$ if $\Psi_i(\xi_i^s) - \varepsilon \geq 0$. Otherwise, if $\Psi_i(\xi_i^s) < \varepsilon$, $\Psi_i(\chi_i) - \Psi_i(\xi_i^s) > \Psi_i(\chi_i) - \varepsilon \geq -\varepsilon$. From Assumption 1, there exists a positive $\zeta > 0$ which satisfies $0 < \xi_i^s - \zeta$. Setting $\delta = \min\{\xi_i^s - \zeta, \overline{\delta}\}$ yields that for any $\varepsilon > 0$, there exists a $\delta > 0$ such that $|\chi_i - \xi_i^s| < \delta$ implies $\Psi_i(\chi_i) - \Psi_i(\xi_i^s) > -\varepsilon$, which completes the proof. □

From Proposition 1, it is evident that the objective function of (SPFCRT) is lower semicontinuous. Therefore, problem (SPFCRT) has an optimal solution since the first stage feasible set is compact and nonempty from Assumption 2.

Then we consider the lower bound of the expected recourse function, taking the convex envelope of the function $\Psi_i(\chi_i)$. To obtain the lower bound, we exploit the algorithm to compute the convex hull of a given set of n points in a plane. Computing the convex hull is one of the most important research problems in computational geometry (Preparata-Shamos [17]). Graham [6] proposed a method of computing the convex hull with $O(n \log n)$ time. Graham proved that computing the extreme points of the convex hull requires $O(n)$ time after sorting points in an increasing order of the angle they and some interior point make with horizontal axis. This algorithm proceeds computing the angle formed by the two previously scanned points and the new points. It determines whether the point is involved in the set of the extreme points of the convex hull by computing the signed area of the three points. In the case of calculating the convex envelope of the expected recourse function, only comparisons of the slope are required, since the function is monotonically nonincreasing. The algorithm is shown as follows.

Algorithm to calculate lower bound of $\Psi_i(\chi_i)$

Step 0 Let $\xi_i^{|\Xi_i|+1} = 0$, and the list of points to scan $\{\xi_i^1, \ldots, \xi_i^{|\Xi_i|}, \xi_i^{|\Xi_i|+1}(=0)\}$. Sort $\xi_i^s, s = 1, \ldots, |\Xi_i|$ in non-increasing order so as to satisfy $\xi_i^1 \geq \ldots \geq \xi_i^{|\Xi_i|} > \xi_i^{|\Xi_i|+1}(=0)$ by substituting indices if required. Set the start point to scan ξ_i^1 and $k = 1$.

Step 1 Let $SUCC(\xi_i^k)$ denote the successor of ξ_i^k in the list. If $k \geq 2$, let $PRED(\xi_i^k)$ be the predecessor of ξ_i^k. If $SUCC(\xi_i^k) = \xi_i^{|\Xi_i|+1}(=0)$, then stop.

Step 2 If $\dfrac{\Psi_i(SUCC(SUCC(\xi_i^k))) - \Psi_i(SUCC(\xi_i^k))}{SUCC(SUCC(\xi_i^k)) - SUCC(\xi_i^k)} \geq \dfrac{\Psi_i(SUCC(\xi_i^k)) - \Psi_i(\xi_i^k)}{SUCC(\xi_i^k) - \xi_i^k}$ for three points, go to Step 3. Otherwise, go to Step 4.

Step 3 The following three points ξ_i^k, $SUCC(\xi_i^k)$, $SUCC(SUCC(\xi_i^k))$ are involved temporarily in the list of extreme points. Set the point to scan $SUCC(\xi_i^k)$ and $k = k + 1$, then go to Step 1.

Step 4 The point $SUCC(\xi^k)$ is not involved in the list of extreme points, remove $SUCC(\xi^k)$ from the list of points to scan. If $k \geq 2$, set the point to scan $PRED(\xi_i^k)$ and $k := k - 1$, then go to Step 1.

The time complexity of the loop from Step 1 to Step 4 is $O(|\Xi_i|)$ since each point is scanned as $SUCC(\xi_i^k)$ only once in Step 2. If the algorithm terminates, $k+1$ is the number of points involved in the set of extreme points. The point $(\xi_i^{|\Xi_i|+1}, \Psi_i(\xi_i^{|\Xi_i|+1}))$ is always involved in the list of extreme points because $\xi_i^{|\Xi_i|+1}$ is scanned as $SUCC(SUCC(\xi_i^k))$ in the second to last iteration.

Example 2 The recourse function $\Psi_1(\chi_1)$ of Example 1 and its lower bound are illustrated in Figure 3. The function is discontinuous at $\chi_1 = 1, 2, 3, 4$.

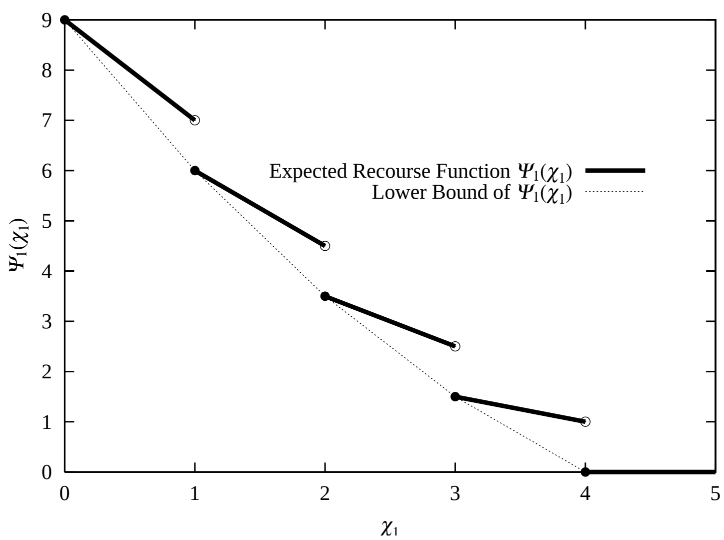

Figure 3: Expected recourse function and its lower bound

4. Branch-and-Cut Method

In this section, we develop a branch-and-cut method to solve (SPFCRT). The algorithm to obtain the lower bound for the expected recourse function $\Psi_i(\chi_i)$ is provided in the previous section. Let $cl_i(j), j = 1, \ldots, l(i)$ be the indices for the scenarios involved in the set of extreme points of the convex envelope of $\Psi_i(\chi_i)$. The number of extreme points involved in the set is denoted as $l(i)$. We define the smallest upper bound for $\Psi_i(\chi_i)$ as θ_i. The following $(l(i) - 1)$ valid inequalities of (4.1) provide the lower bound for $\Psi_i(\chi)$.

$$\theta_i \geq \frac{\Psi_i(\xi_i^{cl_i(j+1)}) - \Psi_i(\xi_i^{cl_i(j)})}{\xi_i^{cl_i(j+1)} - \xi_i^{cl_i(j)}}(\chi_i - \xi_i^{cl_i(j)}) + \Psi_i(\xi_i^{cl_i(j)}), j = 1, \ldots, l(i) - 1 \qquad (4.1)$$

The validity of (4.1) is evident because the inequality (4.1) passes two points $\left(\xi_i^{cl_i(j)}, \Psi_i(\xi_i^{cl_i(j)})\right)$ and $\left(\xi_i^{cl_i(j+1)}, \Psi_i(\xi_i^{cl_i(j+1)})\right)$. First, we solve the following problem (M_0) in which all inequalities of (4.1) are added.

$$\begin{array}{|ll}
(\mathrm{M}_0)\text{:min} & c^\top x + \sum_{i=1}^{n_2} \theta_i \\
\text{subject to} & Ax = b \\
& Tx = \chi \\
& x \geq 0 \\
& \theta_i \geq 0, i = 1, \ldots, n_2 \\
& \theta_i \geq \frac{\Psi_i(\xi_i^{cl_i(j+1)}) - \Psi_i(\xi_i^{cl_i(j)}))}{\xi_i^{cl_i(j+1)} - \xi_i^{cl_i(j)}}(\chi_i - \xi_i^{cl_i(j)}) + \Psi_i(\xi_i^{cl_i(j)}), j = 1, \ldots, l(i) - 1, i = 1, \ldots, n_2
\end{array}$$

After solving the problem (M_0), the optimal solution $(x^*, \chi^*, \theta_1^*, \ldots, \theta_{n_2}^*)$ is obtained. If the relation $\theta_{i_1}^* < \Psi_{i_1}(\chi_{i_1}^*)$ and $\xi_{i_1}^{j_1+1} \leq \chi_{i_1} < \xi_{i_1}^{j_1}$ hold for some random variable i_1 and scenario j_1, the problem (M_0) is split into the following two problems (M_1) and (M_2).

$$\begin{array}{|ll}
(\mathrm{M}_1)\text{:min} & c^\top x + \sum_{i=1}^{n_2} \theta_i \\
\text{subject to} & Ax = b \\
& Tx = \chi \\
& x \geq 0 \\
& \theta_i \geq 0, i = 1, \ldots, n_2 \\
& \theta_i \geq \frac{\Psi_i(\xi_i^{cl_i(j+1)}) - \Psi_i(\xi_i^{cl_i(j)})}{\xi_i^{cl_i(j+1)} - \xi_i^{cl_i(j)}}(\chi_i - \xi_i^{cl_i(j)}) + \Psi_i(\xi_i^{cl_i(j)}), j = 1, \ldots, l(i) - 1, i = 1, \ldots, n_2 \\
& \theta_{i_1} \geq \frac{\Psi_{i_1}(\xi_{i_1}^{j_1+1}) - \{\Psi_{i_1}(\xi_{i_1}^{j_1}) + p_{i_1}^{j_1} f_{i_1}\}}{\xi_{i_1}^{j_1+1} - \xi_{i_1}^{j_1}}(\chi_{i_1} - \xi_{i_1}^{j_1}) + \Psi_{i_1}(\xi_{i_1}^{j_1}) + p_{i_1}^{j_1} f_{i_1} \\
& \chi_{i_1} \leq \xi_{i_1}^{j_1}
\end{array}$$

$$\begin{array}{|ll}
(\mathrm{M}_2)\text{:min} & c^\top x + \sum_{i=1}^{n_2} \theta_i \\
\text{subject to} & Ax = b \\
& Tx = \chi \\
& x \geq 0 \\
& \theta_i \geq 0, i = 1, \ldots, n_2 \\
& \theta_i \geq \frac{\Psi_i(\xi_i^{cl_i(j+1)}) - \Psi_i(\xi_i^{cl_i(j)})}{\xi_i^{cl_i(j+1)} - \xi_i^{cl_i(j)}}(\chi_i - \xi_i^{cl_i(j)}) + \Psi_i(\xi_i^{cl_i(j)}), j = 1, \ldots, l(i) - 1, i = 1, \ldots, n_2 \\
& \chi_{i_1} \geq \xi_{i_1}^{j_1}
\end{array}$$

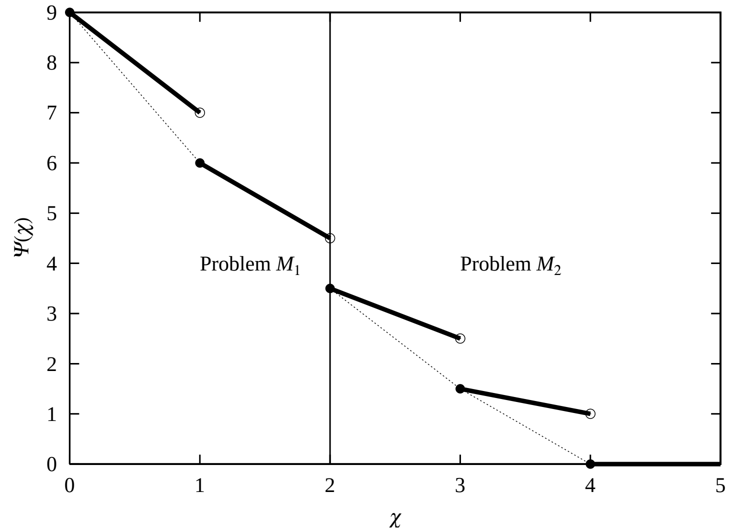

Figure 4: Branching into two subproblems (M_1) and (M_2)

The linear constraints $\chi_{i_1} \leq \xi_i^{j_1}$ and $\xi_i^{j_1} \leq \chi_{i_1}$ are added to create subproblems (M_1) and (M_2), respectively. These constraints represent branching. At the same time, the optimality cut (4.2) is added to subproblem (M_1).

$$\theta_{i_1} \geq \frac{\Psi_{i_1}(\xi_{i_1}^{j_1+1}) - \{\Psi_{i_1}(\xi_{i_1}^{j_1}) + p_{i_1}^{j_1} f_{i_1}\}}{\xi_{i_1}^{j_1+1} - \xi_{i_1}^{j_1}}(\chi_{i_1} - \xi_{i_1}^{j_1}) + \Psi_{i_1}(\xi_{i_1}^{j_1}) + p_{i_1}^{j_1} f_{i_1} \tag{4.2}$$

Figure 4 shows a decomposition of problem (M_0) into subproblems (M_1) and (M_2). The optimality cut (4.2) provides the maximal lower bound for $\Psi_i(\chi_i)$ if $\chi_{i_1} \in [\xi_{i_1}^{j_1+1}, \xi_{i_1}^{j_1})$, because the right-hand side of (4.2) equals $\Psi_{i_1}(\chi_{i_1})$ as shown in (4.3).

$$\begin{aligned}
& \frac{\Psi_{i_1}(\xi_{i_1}^{j_1+1}) - \{\Psi_{i_1}(\xi_{i_1}^{j_1}) + p_{i_1}^{j_1} f_{i_1}\}}{\xi_{i_1}^{j_1+1} - \xi_{i_1}^{j_1}}(\chi_{i_1} - \xi_{i_1}^{j_1}) + \Psi_{i_1}(\xi_{i_1}^{j_1}) + p_{i_1}^{j_1} f_{i_1} \\
= \; & \frac{\sum_{k=1}^{j_1} p_{i_1}^k \{q_{i_1}(\xi_{i_1}^k - \xi_{i_1}^{j_1+1}) + f_{i_1}\} - \sum_{k=1}^{j_1-1} p_{i_1}^k \{q_{i_1}(\xi_{i_1}^k - \xi_{i_1}^{j_1}) + f_{i_1}\} - p_{i_1}^{j_1} f_{i_1}}{\xi_{i_1}^{j_1+1} - \xi_{i_1}^{j_1}}(\chi_{i_1} - \xi_{i_1}^{j_1}) \\
& + \sum_{k=1}^{j_1-1} p_{i_1}^k \left\{q_{i_1}(\xi_{i_1}^k - \xi_{i_1}^{j_1}) + f_{i_1}\right\} + p_{i_1}^{j_1} f_{i_1} \\
= \; & \sum_{k=1}^{j_1} p_{i_1}^k q_{i_1}(\xi_{i_1}^{j_1} - \chi_{i_1}) + \sum_{k=1}^{j_1-1} p_{i_1}^k \left\{q_{i_1}(\xi_{i_1}^k - \xi_{i_1}^{j_1}) + f_{i_1}\right\} + p_{i_1}^{j_1} f_{i_1} \\
= \; & \sum_{k=1}^{j_1} p_{i_1}^k \{q_{i_1}(\xi_{i_1}^k - \chi_{i_1}) + f_{i_1}\} \\
= \; & \Psi_{i_1}(\chi_{i_1}) \text{ if } \chi_{i_1} \in [\xi_{i_1}^{j_1+1}, \xi_{i_1}^{j_1})
\end{aligned} \tag{4.3}$$

Branch-and-Cut method to solve (SPFCRT)

Step 0	Set $N = 0, w^* = \infty$ and $\mathcal{P} = \{\mathrm{M}_0\}$.
Step 1	If $\mathcal{P} = \phi$, then stop.
Step 2	Choose a problem $\mathrm{M}_k \in \mathcal{P}$. Set $\mathcal{P} = \mathcal{P} \setminus \mathrm{M}_k$.
Step 3	Solve M_k. If M_k is infeasible, go to Step 1. If M_k has an optimal solution, let $(x^k, \chi^k, \theta_1^k, \dots, \theta_{n_2}^k)$ be the optimal solution. Calculate the lower bound $w^k = c^\top x^k + \sum_{i=1}^{n_2} \theta_i^k$. If $w^k \geq w^*$, go to Step 1. Otherwise, if $w^k < w^*$, go to Step 4.
Step 4	If $\theta_i^k \geq \Psi_i(\chi_i^k), i = 1, \dots, n_2$, refine the temporary solution as $(x^*, \chi^*, \theta_1^*, \dots, \theta_{n_2}^*) = (x^k, \chi^k, \theta_1^k, \dots, \theta_{n_2}^k)$. Go to Step 1.
Step 5	If $\theta_{i_1}^* < \Psi_{i_1}(\chi_{i_1}^*)$ and $\xi_{i_1}^{j_1+1} \leq \chi_{i_1} < \xi_i^{j_1}$ for some scenario j_1 of random variable i_1, Divide problem M_k into M_{N+1} and M_{N+2}. Let M_{N+1} and M_{N+2} be the problem which is obtained by adding optimality cut (4.2) plus $\chi_{i_1} \leq \xi_i^{j_1}$ to M_k and the problem which is obtained by adding the constraint $\chi_{i_1} \geq \xi_i^{j_1}$ to M_k. $\mathcal{P} = \mathcal{P} \cup \{\mathrm{M}_{N+1}, \mathrm{M}_{N+2}\}, N = N + 2$ and go to Step 1.

In the algorithm of branch-and-cut, the number of the interval in which the optimality cut (4.2) yields the correct value of the expected recourse function, increases by at least one after branching. Finite convergence comes from the assumption that each element of random vector $\tilde{\xi}$ follows a discrete distribution with finite support.

5. Heuristic Algorithm by Dynamic Slope Scaling Procedure

In this section, we consider a heuristic algorithm to solve (SPFCRT). For the fixed charge network flow problem, Kim and Pardalos [8] developed a new approach, called the dynamic slope scaling procedure (DSSP), which solves successive linear programming problems with recursively updated objective functions. Kim and Pardalos [9, 10] modified DSSP, which repeats the reduction and refinement of the feasible region. In this section, DSSP is used to obtain a feasible solution to the second stage integer programming problem which defines the recourse function. Consider problem (M_0). Let $(x^*, \chi^*, \theta_1^*, \dots, \theta_{n_2}^*)$ be the optimal solution of (M_0). The i-th expected recourse function $\Psi_i(\chi_i)$ is approximated by the variable θ_i which satisfies the inequality (5.1) if $\chi_i^* < \xi_i^1$.

$$\theta_i \geq \frac{\Psi_i(\chi_i^*)}{\chi_i^* - \xi_i^1}(\chi_i - \chi_i^*) + \Psi_i(\chi_i^*) \tag{5.1}$$

The border line of the inequality constraint (5.1) passes $(\chi_i^*, \Psi_i(\chi_i^*))$ and $(\xi_i^1, \Psi_i(\xi_i^1)) = (\xi_i^1, 0)$ as shown in Figure 5. Hence, $(\chi_i, \theta_i) = (\chi_i^*, \Psi_i(\chi_i^*))$ satisfy (5.1) with equality.

Then we solve problem (M_1) which is obtained by adding inequality constraints (5.1), (5.2), (5.3),(5.4) and (5.5) to the formulation of (M_0).

$$\xi_i^{j+1} \leq \chi_i \leq \xi_i^j, \forall i \text{ such that } \xi_i^{j+1} < \chi_i^* < \xi_i^j \text{ and } 1 \leq j \leq |\Xi_i| \tag{5.2}$$

$$\xi_i^2 \leq \chi_i \leq \xi_i^1, \forall i \text{ such that } \chi_i^* = \xi_i^1 \tag{5.3}$$

$$\xi_i^{j+1} \leq \chi_i \leq \xi_i^{j-1}, \forall i \text{ such that } \chi_i^* = \xi_i^j \text{ and } 2 \leq j \leq |\Xi_i| \tag{5.4}$$

$$0 = \xi_i^{|\Xi_i|+1} \leq \chi_i \leq \xi_i^{|\Xi_i|-1}, \forall i \text{ such that } \chi_i^* = \xi_i^{|\Xi_i|+1} = 0 \tag{5.5}$$

The constraint (5.1) is not a valid inequality, however it is regarded as a linear approximation of the expected recourse function as shown in Figure 5. The algorithm of DSSP is shown as follows.

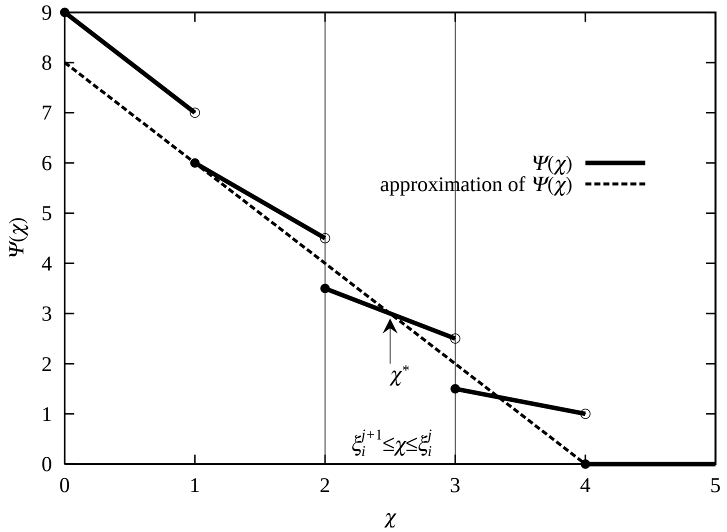

Figure 5: Dynamic slope scaling procedure

Heuristic algorithm by dynamic slope scaling procedure

Step 0	Given positive $\varepsilon > 0$, set $N = 0$.
Step 1	Solve M_N to obtain the optimal solution $(x^N, \chi^N, \theta_1^N, \ldots, \theta_{n_2}^N)$.
Step 2	If $N \geq 1$ and $\sum_{i=1}^{n_1} \lvert x_i^N - x_i^{N-1} \rvert + \sum_{j=1}^{n_2} \lvert \chi_j^N - \chi_j^{N-1} \rvert + \sum_{j=1}^{n_2} \lvert \theta_j^N - \theta_j^{N-1} \rvert < \varepsilon$, then stop.
Step 3	If $N \geq 1$, remove all inequalities of (5.1), (5.2), (5.3), (5.4) and (5.5) added in iteration $N - 1$.
Step 4	Add inequalities (5.1), (5.2), (5.3), (5.4) and (5.5) to the formulation of M_N. Set $N = N + 1$, go to Step 1.

6. Application to Power Generation Problem

We consider the application of the problem (SPFCRT) to the electric power generation problem. The basic objective of the problem is to determine an investment of new technology and to operate power plants to ensure an economic and reliable supply to electricity demand. The load patterns are modeled by load duration curves. For long-range planning, block approximations of load duration curves are used. A load duration curve represents the number of hours in which the load equals or exceeds the given load value. A typical load duration curve is illustrated in Figure 6. Since the long-term planning problems involve uncertain data, mathematical programming models which can deal with stochastic factors have been developed. In Murphy, Sen, and Soyster [16], the uncertainty in demand is incorporated into a mathematical programming model. In this paper, we consider a stochastic programming model in which the demand in each load level is uncertain.

We assume that there are n_1 generators and the demand is given by the load duration curve with n_2 load levels. Suppose the demand of load level j is defined as a random variable $\tilde{\xi}_j$, and t_j, the duration of load level j, is fixed. Let $\xi_1, \ldots, \xi_{n_2}$ be the realizations of random variables $\tilde{\xi}_1, \ldots, \tilde{\xi}_{n_2}$, and $\Xi_1, \ldots, \Xi_{n_2}$ be their supports. These random variables are integrated as a random vector $\tilde{\xi} = (\tilde{\xi}_1, \ldots, \tilde{\xi}_{n_2})^\top$, and the support Ξ of $\tilde{\xi}$ is described as $\Xi = \Xi_1 \times \cdots \times \Xi_{n_2}$.

The first stage decision variable is the available capacity of an existent generator i for the load level j denoted by x_{ij}, $i = 1, \ldots, n_1, j = 1, \ldots, n_2$. Let a_{ij} and r_i be the fuel

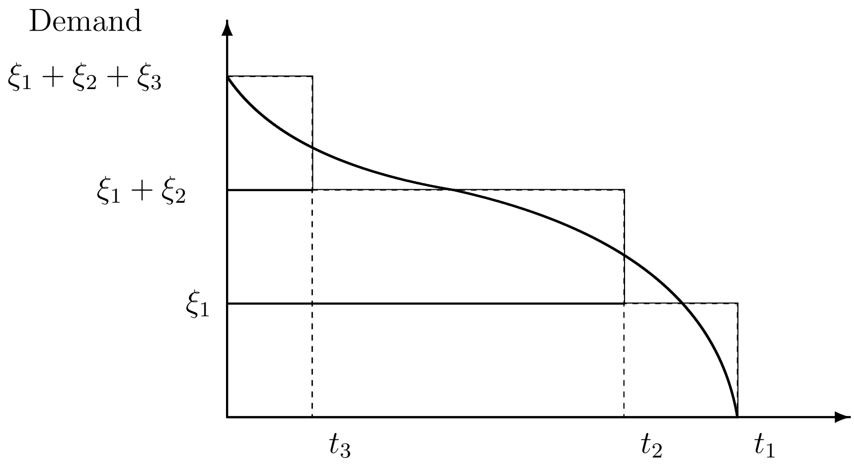

Figure 6: Load duration curve

consumption rate of generator i at load level j and the fuel price for generator i. Here, the first stage cost of power generation is described as $c_{ij} = a_{ij}t_j r_i$. For the first stage constraints, let b_i be the upper bound for the amount of fuel consumption of generator i.

Given a first stage decision x, the realization of random demand ξ of $\tilde{\xi}$ becomes known. After observing the realization ξ, the second stage decisions $y_j(\xi_j)$ and $z_j(\xi_j)$ are taken to meet the electricity demand. The amount of unserved electricity demand has to be supplied by a new plant constructed in the second stage. The recourse variables $y_j(\xi_j)$ and $z_j(\xi_j)$ denote the power supplied for load level j and the binary decision which represents whether a new generator is constructed or not for load level j. The recourse costs q_j and f_j are the operating cost and the construction cost. The formulation of the problem is described as (PGP). The first constraint of the second stage problem to define $\psi_j(\chi_j, \xi_j)$ says the demand must be satisfied, whereas the second constraint for the recourse problem expresses that power is not supplied from the new plant if it is not constructed.

(PGP):

$$
\begin{array}{rl}
\min & \displaystyle\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} c_{ij}x_{ij} + \Psi(\chi) \\
\text{subject to} & \displaystyle\sum_{j=1}^{n_2} a_{ij}x_{ij} \le b_i, i = 1,\dots,n_1 \\
& x_{ij} \ge 0, i = 1,\dots,n_1, j = 1,\dots,n_2 \\
& \chi_j = \displaystyle\sum_{i=1}^{n_1} x_{ij}, j = 1,\dots,n_2 \\
\text{where} & \Psi(\chi) = \displaystyle\sum_{s=1}^{S} p^s \psi(\chi, \xi^s) \\
& \psi(\chi, \xi^s) = \displaystyle\sum_{j=1}^{n_2} \psi_j(\chi_i, \xi_j^s), s = 1,\dots,S \\
& \psi_j(\chi_j, \xi_j^s) = \min\left\{ q_j y_j(\xi_j^s) + f_j z_j(\xi_j^s) \;\middle|\; \begin{array}{l} y_j(\xi_j^s) + \chi_j \ge \xi_j^s \\ 0 \le y_j(\xi_j^s) \le M z_j(\xi_j^s) \\ z_j(\xi_j^s) \in \{0,1\}^{n_2} \end{array} \right\}, \\
& \qquad s = 1,\dots,|\Xi_j|, j = 1,\dots,n_2
\end{array}
$$

Table 1 explains how to set up problem data.

Table 1: Problem data

First stage cost	$c_{ij} = U[500, 1000] \times 0.1(j/n_2), i = 1, \ldots, n_1, j = 1, \ldots, n_2$, where $U[500, 1000]$ is a number drawn from a uniform distribution between 500 and 1000.
Second stage cost	$q_j = U[500, 1000] \times 0.2, j = 1, \ldots, n_2$.
Fixed cost rate	$f_j/q_j = 500, 1000, 1500, j = 1, \ldots, n_2$.
Fuel consumption rate	$a_{ij} = U[500, 1000], i = 1, \ldots, n_1, j = 1, \ldots, n_2$.
Fuel limit	$b_i = U[0.5, 1.0] \times \sum_{j=1}^{n_2} a_{ij} x^*_{ij}$, where x^* is the optimal solution of the linear relaxation of (PGP).
Power demand	$\xi_j^s = \xi_j^1 - s \times (\xi_j^1 - \xi_j^{\lvert\Xi_j\rvert})/\lvert\Xi_j\rvert, s = 1, \ldots, \lvert\Xi_j\rvert, j = 1, \ldots, n_2$, where $\xi_j^1 = U[500, 1000]$ and $\xi_j^{\lvert\Xi_j\rvert} = 0.5\xi_j^1$.
Probability	$p_j^s = 1/\lvert\Xi_j\rvert, s = 1, \ldots, \lvert\Xi_j\rvert, j = 1, \ldots, n_2$.

The branch-and-cut method for the electric power generation problem was implemented using ILOG OPL Development Studio on DELL DIMENSION 8300 (CPU: Intel Pentium(R)4, 3.20GHz). The simplex optimizer of CPLEX 9.0 was used to solve the subproblem. We compare the branch-and-cut method with the traditional branch-and-bound method. For the branch-and-bound method, the mixed integer optimizer of CPLEX 9.0 was used to solve the deterministic equivalent problem of (PGP). In both the branch-and-bound and branch-and-cut algorithms, depth-first search plus backtracking was exploited for node selection.

The problems considered in this section consist of 10, 15 and 20 load levels and 10 generators. Each load has 10, 15 and 20 scenarios. The results of the numerical experiments appear in Table 2, where LB, UB and OPT denote the optimal objective value of the linear programming problem M_0, the best objective obtained by the dynamic slope scaling procedure(DSSP) and the optimal objective value obtained by the branch-and-cut(BC) method or the branch-and-bound(BB) method, respectively. The gap described as $(UB - LB)/LB$ seems to be relatively large, while the relative errors of DSSP described as $(UB - OPT)/OPT$ are all within 2 %. It is observed that in all cases the CPU time of DSSP is less than that of BC and BB. The heuristic approach DSSP is efficient in solution time.

In order to see the efficiency of the exact algorithm, we compare BC with BB. It is noticed that BC requires less branchings than BB, and the same objective value is obtained. The results show that the branch-and-cut method performs reasonably well on relatively large problems. The computing time of BB tends to rise as the size of the problem increases. Especially in the cases with 20 load levels and 20 scenarios, the traditional branch-and-bound method did not terminate within 10,000 seconds. The results indicate that problems become more difficult when the number of scenarios in each load level becomes larger. This can be explained as follows. The upper bound for the number of subproblems generated in BB is $O(2^{\sum_{j=1}^{n_2} \lvert\Xi_j\rvert})$ since the number of binary variables involved in (PGP) is $\sum_{j=1}^{n_2} \lvert\Xi_j\rvert$. Similarly, the upper bound for the number of subproblems generated in BC is $O(\Pi_{j=1}^{n_2} \lvert\Xi_j\rvert)$ since the domain of tender variable χ_j is divided into at most $\lvert\Xi_j\rvert$ intervals in the branch-and-cut procedure. Thus, there are at the maximum $2^{10\times 10} \approx 1000^{10}$ subproblems to compare for traditional BB, whereas there are 10^{10} subproblems for BC with $n_j = 10$ and $\lvert\Xi_j\rvert = 10$.

Table 2: Computational results

Number of load levels n_2	Number of scenarios $\lvert\Xi_j\rvert$	Fixed cost rate f_j/q_j	Gap (%) $\frac{UB-LB}{LB}\times 100$	Relative Error(%) $\frac{UB-OPT}{OPT}\times 100$	CPU time (sec) DSSP	CPU time (sec) BC	CPU time (sec) BB	Number of subproblems BC	Number of subproblems BB
10	10	500	5.42	1.11	0.32	2.43	1.56	338	1454
10	10	1000	7.65	1.46	0.32	1.59	1.81	219	1442
10	10	1500	9.59	1.73	0.32	2.31	3.28	328	3348
10	15	500	3.95	1.76	0.50	4.36	15.10	449	12003
10	15	1000	6.60	1.77	0.50	3.36	24.01	395	17054
10	15	1500	8.05	1.68	0.52	4.72	16.22	560	11402
10	20	500	3.78	1.46	0.20	3.63	75.45	474	26927
10	20	1000	6.39	1.13	0.73	6.99	82.42	705	36550
10	20	1500	7.83	1.44	0.75	8.52	205.82	870	98307
15	10	500	3.48	0.58	0.45	8.12	31.62	811	26687
15	10	1000	4.38	0.72	0.46	5.67	73.07	578	44444
15	10	1500	5.01	1.25	0.48	7.86	52.53	793	35499
15	15	500	3.14	0.30	0.28	21.74	1182.05	789	492685
15	15	1000	3.11	0.55	0.73	14.60	718.30	1245	281900
15	15	1500	5.60	0.84	0.25	10.95	1084.13	1009	377704
15	20	500	1.67	0.45	0.31	13.39	3878.72	1117	1019114
15	20	1000	3.18	0.63	0.32	9.16	2901.50	756	657882
15	20	1500	5.37	0.76	0.28	8.90	3191.11	742	709039
20	10	500	2.71	0.37	0.35	10.80	223.78	836	158993
20	10	1000	4.21	0.95	0.32	20.36	36.22	1583	21119
20	10	1500	5.63	0.87	0.31	9.77	56.40	745	32604
20	15	500	2.09	0.34	0.37	27.04	4279.36	1952	1313029
20	15	1000	3.74	0.69	0.39	17.61	1328.03	1256	423012
20	15	1500	5.07	1.00	0.37	40.70	1427.94	2784	345240
20	20	500	1.67	0.29	0.38	204.46	–	13005	–
20	20	1000	3.80	0.68	0.37	135.27	–	8771	–
20	20	1500	4.73	0.80	0.38	49.03	–	3232	–

The symbol (−) means that branch-and-bound did not terminate within 10,000 seconds.

7. Concluding Remarks

In this paper, we have introduced a class of stochastic programming problem with fixed charge recourse in which a fixed cost is imposed if the value of the continuous recourse variable is strictly positive. The algorithm of a branch-and-cut method to solve the problem is developed and numerical results for a power generating system are presented. This mathematical programming model is quite useful for a variety of design and operational problems.

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