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LOCATING MULTIPLE FACILITIES IN A PLANAR COMPETITIVE ENVIRONMENT

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Abstract We investigate the location of one or more facilities anywhere in an area in which several competing facilities already exist. The attractiveness of each facility is modeled by a utility function. Each customer selects the facility with the greatest utility function value. The objective is to find the locations for one or more facilities which attract the maximum buying power. We generate a set of candidate locations and solve the single facility problem by evaluating the buying power attracted to the new facility at each candidate location. We then solve the location of multiple facilities by converting the problem to a maximum covering problem. The solution procedure is illustrated on an example problem with 100 demand points and seven existing facilities. As a case study we find the best locations of new convenience stores in the city of Seto, Japan.

Keywords: Facility planning, maximum covering problem, convenience stores

1. Introduction

Consider demand originating at a given number of demand points in the plane. Each demand point generates demand in proportion to the cumulative buying power of all residents represented by this point. A number of competing facilities already exist in the area and the objective is to locate a chain of one or more new facilities so as to capture the maximum buying power. The appeal of a facility (whether existing or new) is determined by a utility function defined as the difference between the attractiveness of the facility, which may be different for different facilities, and the distance between the demand point and the facility [4]. Each customer patronizes the facility with the highest utility value whether existing or new. The location objective for the chain is to attract the maximum buying power.

1.1. Literature review

Hotelling introduced the competitive facility location problem [12]. His assumption was that each demand point is attracted to and patronizes the closest facility. A large body of literature is based on Hotelling's proximity assumption. Hakimi solved the competitive location problem on a network [9–11] . He introduced both the location of one or more facilities in the presence of competing facilities and the Stackelberg equilibrium or the leader-follower problem which seeks the best location (or locations) for the leader considering that a future competitor (the follower) will locate in the area one or more competing facilities. ReVelle defined the competitive facility location problem as a maximum covering problem [17]. Ghosh and Rushton provide a review of location-allocation problems that are also based on the premise that each demand point is attracted to the closest facility [8]. Okabe and Suzuki solved the competitive facility location in the plane assuming continuous demand [15]. Their solution procedure is based on iteratively heuristically solving Voronoi diagrams of the locations of the facilities. Serra and ReVelle provide a review of competitive location problems in discrete space [18]. This proximity assumption is predicated on the premise that the facilities are equally attractive. In reality customers prefer some facilities over others and are willing to travel an extra distance to a more attractive facility. Drezner suggested to incorporate attractiveness of competing facilities into the model by defining a utility function for each competing facility [4]. Thus, a customer selects the facility with the highest utility value rather than the closest one.

Another approach to modeling competing facilities was proposed by Huff [13, 14]. Huff followed the gravity model in [16] and proposed that the probability that a customer patronizes a facility is proportional to its attractiveness and inversely proportional to an increasing function of the distance, such as a power of the distance.

The paper is organized as follows. In Section 2 the problem is formulated and solution properties are proven. An algorithm for finding the set of candidate solution locations is described in Section 3. In Section 4 we suggest solution algorithms and in Section 5 we report the results of computational experiments. We conclude the paper in Section 6.

2. Formulation

2.1. Notation and definitions

n	Number of demand points
k	Number of existing competing facilities
p	Number of new facilities to be located
$L_i = (a_i, b_i)$	Location of demand point i for $i = 1, \ldots, n$
$E_j = (u_j, v_j)$	Location of existing facility j for $j = 1, \ldots, k$
$X_m = (x_m, y_m)$	Unknown location of new facility m for $m = 1, \ldots, p$
B_i	Buying power at demand point i for $i = 1, \ldots, n$
A_j	The attractiveness of existing facility j for $j = 1, \ldots, k$
A	The attractiveness of the new facilities (we assume that all new facilities are equally attractive)
$d(G, H)$	Euclidean distance between points G and H (two points in the plane)

The utility function of facility j by demand point i is defined as $A_j - d(L_i, E_j)$. The utility function for a new facility m by demand point i is $A - d(L_i, X_m)$. Customers located at demand point i patronize the facility with the maximum utility among all facilities. Note that the model does not change if a constant is added to all A_j and A.

The attractiveness values A and A_j can be determined as follows: A facility k is arbitrarily selected (or the least attractive facility is selected) as the benchmark facility and is assigned $A_k = 0$. The attractiveness A_j (or A) is the *extra distance customers are willing to travel* to facility j rather than facility k. Some of these attractiveness values may be negative. If one does not like negative attractiveness values, a constant may be added to all attractiveness values at the end of the process so that the attractiveness of all facilities is positive.

2.2. Analysis

Demand point i prefers new facility m over existing facility j if

$$A - d(L_i, X_m) \ > \ A_j - d(L_i, E_j)$$

or (and defining a radius R_{ij}):

$$d(L_i, X_m) \ < \ d(L_i, E_j) + A - A_j \ = \ R_{ij} \tag{1}$$

For demand point i we calculate a distance (radius) R_i:

$$R_i \;=\; \min_{j=1,\ldots,k}\{R_{ij}\} \tag{2}$$

Demand point i will patronize the new facility m (or possibly another new facility) if and only if $d(L_i, X_m) < R_i$. To simplify the analysis we assume that in case of a tie, the customer patronizes the existing facility. Note that otherwise, if a new facility is located at the same location of an existing facility, it will attract all the demand points attracted to the existing facility. By subtracting a small ϵ from the covering radius R_i we can apply the rule defined by $\leq$ rather than $<$.

A diagram of n circles is constructed. Demand point i generates a circle centered at L_i with a radius $R_i - \epsilon$ [4]. If a new facility is located at a point at which the discs defined by the circles intersect, the new facility attracts all the demand points defining these discs. For locating one new facility we wish to find a location in the plane which maximizes the total buying power captured from all discs covering it. That means, finding a point in the plane that is in the intersection of circles capturing the maximum possible total buying power.

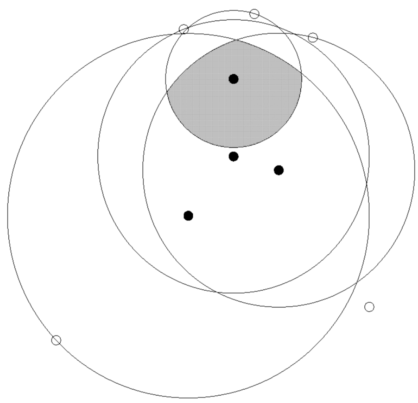


Figure 1: An intersection diagram of circles
• Demand point; ◦ Existing facility

Figure 1 depicts a diagram of four demand points and five existing facilities. For this diagram we assume the same utility function (attractiveness) for the existing facilities and the new one. Four circles are centered at demand points and their circumferences cross the existing facility closest to them. The shaded area (surrounding the top demand point) represents the intersection of all four circles thus if the new facility is located anywhere in that area, it will attract all the buying power.

Geometrically, two circles in the plane can be either incontiguous (i.e. their interiors do not intersect), or intersect at one or two points, or one circle can be inside the other without touching it. In the following theorem we show that in our intersection diagram of circles it is impossible for one circle to be completely inside another circle unless it is tangent to it from the inside.

Theorem 1: *Two circles generated by two demand points are either incontiguous or have one or two intersection points.*

Proof: Suppose that circle r is inside circle s and we reach a contradiction. Since circle r is inside circle s, $R_s > R_r + d(L_r, L_s)$. For circle r there is a facility j such that $R_r = R_{rj}$.

By (2) $R_s = \min_{l=1,\dots,k} \{R_{sl}\} \le R_{sj}$. Therefore, $R_{sj} > R_{rj} + d(L_r, L_s)$. Substituting (1) we get

$$d(L_s, E_j) + A - A_j > d(L_r, E_j) + A - A_j + d(L_r, L_s)$$

or

$$d(L_s, E_j) > d(L_r, E_j) + d(L_r, L_s)$$

which is a contradiction by the triangle inequality. □

Consider a set of n circles in the plane, each with an associated weight. If a facility is located inside a circle, it attracts the buying power associated with the demand point defining that circle. These circles partition the plane into disjoint areas. The union of these areas is the whole plane and the intersection of the interiors of two areas is empty. Each area is defined by a set of arcs (similar to sides defining a polygon). An arc is "convex" (with respect to the given area) if it belongs to a circle which covers the area. An arc is "concave" if it belongs to a circle that does not cover the area. For each area there is a set of circles covering it and a set of circles that do not. The total buying power attracted by a facility located at any interior point in the area is the total buying power associated with the demand points centered at the circles that cover that area. If a facility is located at any point inside an area, it is covered by exactly the same set of circles. Therefore, the whole plane is divided to areas, each representing equivalent coverage.

There is no reason to place two facilities in the same area because they will capture exactly the same buying power. Therefore, for each area we define a "candidate location", which is a point in the interior of the area, representing that area. Any configuration of locations for new facilities is equivalent to a selection from the set of candidate locations. Each facility is in an area and can be replaced by the candidate location in that area.

We prove that a candidate location in a non-convex area is inferior to at least one other candidate location. This means, that there is another candidate location that captures the same buying power and additional buying power from at least one extra demand point.

Theorem 2: *A candidate location in a non-convex area cannot be an optimal location because there exists another candidate location which attracts more buying power.*

Proof: Consider a candidate location X in a non-convex area. Since the area is not convex, one of the arcs defining it is part of a circle which is not covered (the area is outside that disc). This circle covers the adjacent area. The candidate location in that adjacent area is covered by all the circles covered by X but it also must include this additional circle defining the non-convex arc. Therefore, the candidate location in the adjacent area attracts additional buying power and must be superior. □

Theorem 2 suggests that the set of candidate locations should include only locations in convex areas.

Drezner, Mehrez, and Wesolowsky proved that there are at most $2n(n-1)$ distinct areas (either convex or non-convex) [7]. Since the candidate locations are determined only by convex areas, this bound can be improved by a proof similar to the one in [7].

Theorem 3: *The number of candidate locations is bounded by* $\frac{n(n-1)}{2}$.

Proof: There are $\frac{n(n-1)}{2}$ pairs of circles. Each pair of circles intersects at two or fewer points for a total number of intersections not exceeding $n(n-1)$ intersection points. If more than two circles intersect at the same point we treat this point as several intersection points. Each intersection point is a vertex of four areas. Three of the four areas are not convex because they contain the exterior of at least one circle. Therefore, the number of vertices of convex areas is bounded by $n(n-1)$. Since each convex area has at least two vertices, the number of convex areas is bounded by $\frac{n(n-1)}{2}$. $\square$

Consider the diagram in Figure 1. There are 13 distinct areas but only one of them (the shaded one) is convex. Since $n = 4$, the bounds are 24 for the total number of areas, and 6 for the number of convex areas.

3. Finding the Candidate Locations

In this section we propose an algorithm which finds a list of candidate locations in $O(n^2 \log n)$ time. Some of the areas may be represented more than once. Therefore, a second phase in which the candidate locations are compared is required in order to remove duplications.

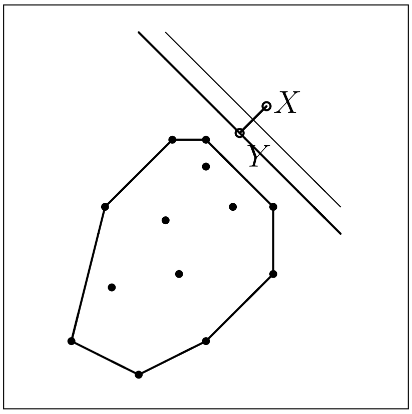
Consider one circle of the diagram. There are at most $2(n-1)$ intersection points between the selected circle and all other circles. Each intersection point defines an angle between the line connecting the intersection point with the center of the circle and the line parallel to the x-axis through the center of the selected circle. (In other words, the angle defined by the polar coordinates with its origin at the center of the selected circle.) The list of intersection points is sorted by these angles in $O(n \log n)$ time. The first sorted point is duplicated at the end of the list. The arcs of the selected circle which are part of the diagram are all the arcs between consecutive intersection points. The center of the segment connecting two consecutive intersection points is inside the circle. The "other" side of the arc bounds a non-convex area and thus can be ignored. Some of the centers of the segments can be eliminated from consideration as well. The center of the segment must be *inside* the two circles intersecting at the end points of the arc. Otherwise, the area bounded by the arc is outside at least one of the circles and must be non-convex.

This process is repeated for each circle, generating a list of candidate locations in $O(n^2 \log n)$ time. A second phase comparing pairs of candidate locations and removing duplicates or inferior locations will result in the shortest possible list of candidate locations. This phase may well require more than $O(n^2 \log n)$ time. One can skip this phase if the extended list which includes duplications is usable.

One should note, however, that the extended list of candidate locations is at least twice as long as as the shortest possible list. If an area is defined by a polygon with $K > 2$ sides, the K centers of the sides of the polygon are all in the extended list. If an area is an intersection of two circles, the candidate location is at the center of the segment connecting the two intersection points, and will be represented twice in the extended list (once for each intersection circle).

Theorem 4: *For each convex area, there is a candidate location in the area which is in the convex hull of the demand points.*

Proof: Let X be a point outside a convex hull (see Figure 4). There exist a line separating X from the convex hull of the demand points and there is a point Y on the intersection between that line and the perpendular line through X. The perpendicular bisector of the segment XY divides the plane into two half planes. One half plane contains all the points closer to X and the other half plane contains all the points closer to Y. All the points in the convex hull are closer to Y. Therefore, if X is in an area, Y cannot be outside a circle covered by X because it is closer to all demand points (which are the centers of all circles). Therefore, Y can only be a better candidate location than X. The process continues constructing a series of points converging to the boundary of the convex hull. The limit of this sequence is on the boundary of the convex hull and can only be a better candidate location than point X. $\square$



• Demand point

Figure 2: Proof of Theorem 4

Table 1: Number of candidate locations

Existing	$n = 100$			$n = 200$			$n = 300$			$n = 400$			$n = 500$		
Facilities	Min.	Max	Ave.	Min.	Max	Ave.	Min.	Max	Ave.	Min.	Max	Ave.	Min.	Max	Ave.
1	1	55	18.6	1	100	35.8	1	149	55.2	1	197	65.5	1	253	81.9
2	9	244	125.0	24	830	436.6	8	1703	872.7	22	3210	1610.1	146	4454	2438.0
3	46	248	162.0	89	840	546.3	204	1794	1077.9	657	3076	2016.9	420	4494	3022.8
4	80	255	166.7	218	766	551.6	453	1695	1148.2	535	2987	1988.1	715	4153	2954.5
5	58	212	155.5	186	791	516.9	509	1586	1117.4	938	2721	1960.9	1325	4032	2873.4
6	82	211	153.7	226	687	500.8	429	1468	1024.0	565	2400	1780.2	1564	3746	2800.8
7	96	200	151.8	302	640	482.8	654	1426	1012.9	1181	2308	1717.4	1757	3619	2622.7
8	97	205	145.3	342	606	476.3	474	1262	939.3	1098	2175	1598.6	1677	3150	2415.7
9	89	201	139.3	306	597	452.7	674	1163	947.9	1028	2331	1563.5	1408	3250	2266.3
10	89	185	133.5	301	586	428.4	598	1118	874.9	827	1957	1424.4	1537	3256	2217.6
20	87	141	108.9	265	422	325.1	503	873	632.3	802	1265	1039.4	1171	2106	1515.9
30	60	135	96.8	211	342	271.0	413	642	522.1	702	1117	837.8	1034	1506	1234.6
40	66	113	89.3	188	287	240.9	387	545	456.6	587	884	724.9	886	1224	1059.7
50	67	100	81.7	173	270	220.2	346	469	410.0	560	807	645.4	815	1101	929.8
100	49	80	63.8	144	206	169.3	268	347	301.3	415	522	463.8	593	737	650.4

It follows from Theorem 4 that the search for optimal locations for locating p new facilities can be restricted to the convex hull of the demand points.

In order to investigate the relationship between the number of demand points, the number of facilities, and the number of candidate locations we generated problems with $n = 100, \ldots, 500$ demand points and $1, \ldots, 10, 20, 30, 40, 50, 100$ existing facilities, all randomly generated in a unit square. Each combination was generated 100 times and the minimum, maximum, and average number of candidate locations is reported in Table 1. Note that when there is only one existing facility, the minimum number of candidate locations was always 1. If the existing facility is generated in a certain region (for example, outside the convex hull of the demand points) then there is a location for the new facility at which it attracts all the demand. Chances are that in 100 trials the existing facility is located in that region at least once. The number of candidate locations depends on the number of existing facilities. It is smallest for one existing facility, reaches a maximum at about 3-4 existing facilities and declines after that value. The average number of candidate locations is plotted as a function of the number of existing facilities for each value of n in Figure 3. Figure 3 confirms this behavior. The number of candidate locations seems to increase at a rate greater than linear but less than quadratic in n. The bound of Theorem 3 is quite generous. For example, for $n = 500$ the bound is 124,750 while the maximum observed value is only 4,494.

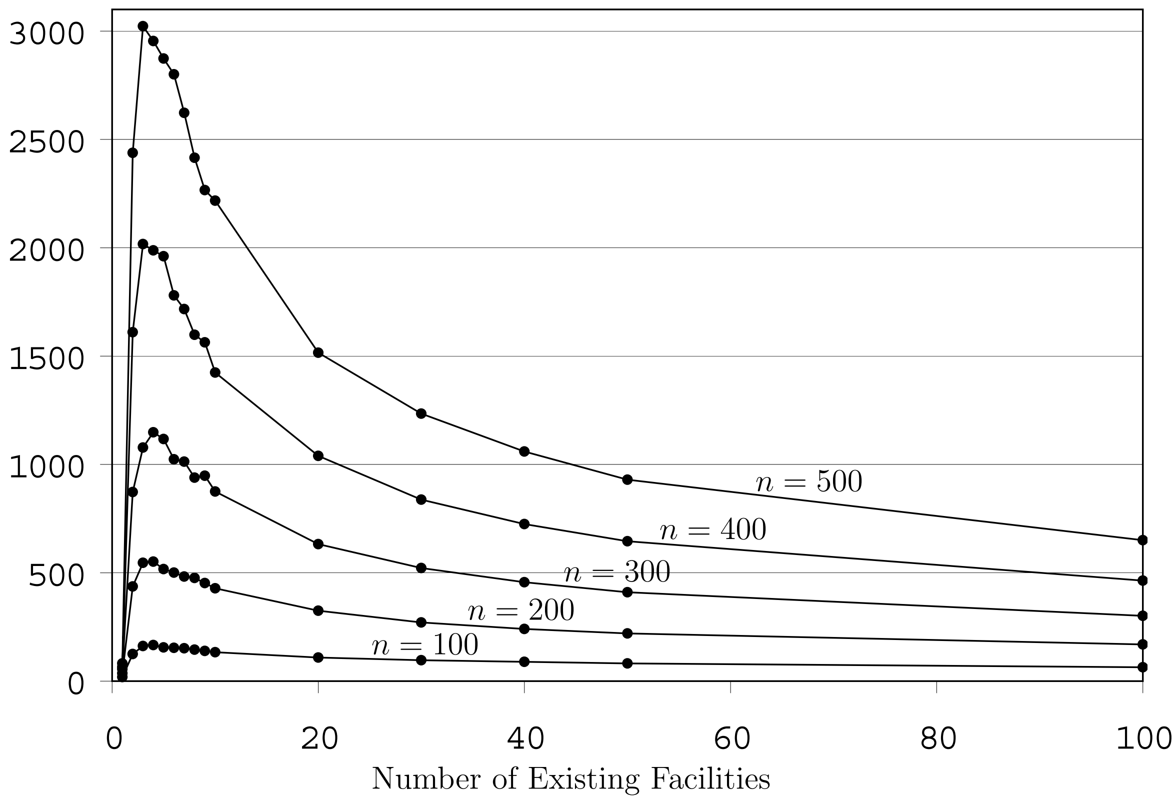


Figure 3: Average number of candidate locations (horizontal axis represents the number of existing facilities and vertical axis represents the number of candidate points)

Note that there exist algorithms which can be applied to find the list of candidate locations more efficiently (for example, see [1]). However, the comlexity of the solution approach for the location of multiple facilities is dominated by solving the max-covering problem which is NP-hard. In all our experimentations it took a fraction of a second to calculate all candidate points. Much larger problems (when the potential reduction in run time may be meaningful) are intractable because the resulting max-covering problem is too large.

4. Solution Approaches

4.1. Solving the single facility problem

In order to solve the single facility location problem we can scan all distinct convex areas (area bounded by circumferences of circles) in the circles' intersection diagram described above (see, for example, Figure 1). This can be achieved by evaluating the buying power attracted by the new facility at each candidate location and selecting the best one. In [6] an algorithm of complexity $O(n^2 \log n)$ was proposed for scanning all such distinct areas and evaluating the cumulative buying power attracted to each, thus finding the optimal area for locating the new facility. Since generating all candidate locations as described above requires $O(n^2 \log n)$ time, we cannot improve this approach for the single facility location problem.

4.2. Solving the multiple facilities problem

To locate p facilties we need to select a subset of p of the candidate locations and evaluate the total buying power attracted by all of them. Obviously, if a demand point is captured by more than one new facility, it should be counted only once. Therefore, the solution is not necessarily to select the p candidate locations which attract the most buying power. Each candidate location attracts a known set of demand points. Therefore, the problem is converted to a maximum cover problem with p facilities [2, 3]. We propose to solve the multiple facilities problem as follows:

1. Calculate the candidate locations.
2. Construct a list of captured demand points for each candidate location.
3. Solve the resulting p-Maxcover problem.

The p-Maxcover problem can be efficiently solved using Sitation ([19]). However, the p-Maxcover problem is NP-hard, thus the complexity of the algorithm cannot be established. A total enumeration approach bounds the complexity of the algorithm by $O(n^{p+1})$ for $p > 1$ which is polynomial in n for a fixed p.

Alternatively, the problem can be formulated and solved as an integer program. In order to find the locations for p facilities we generate a matrix $A = \{a_{ij}\}$ of n columns (a column for each demand point) and c rows (a row for each candidate location). In row j and column i of the matrix we enter $a_{ij} = 0$ if demand point i is not captured by the candidate location j, and enter a "1" if it is captured. We define n 0-1 variables x_i for $i = 1, \ldots, n$ which obtain the value of "1" if demand point i is captured by at least one new facility and the value of "0" otherwise, and c 0-1 variables y_i, for $j = 1, \ldots, c$ indicating whether candidate location j is selected or not. The objective function coefficient for column i is B_i. The formulation is:

$$\max\left\{\sum_{i=1}^{n} B_i x_i\right\} \tag{3}$$

Subject to:

$$x_i \le \sum_{j=1}^{c} a_{ij} y_j \text{ for } i = 1, \ldots, n \tag{4}$$

$$\sum_{j=1}^{c} y_j = p \tag{5}$$

$$x_i, y_j \in \{0, 1\} \tag{6}$$

4.3. Extensions

Varying Attractiveness Values:

Suppose that there is a list of attractiveness values that the new facilities can assume (we require that all of the new facilities assume the same value). For each attractiveness value there is a cost of building and maintaining the facility. We need to select the attractiveness value for all new facilities and the locations for p new facilities. This variant can be solved by finding the best solution for each attractiveness value and selecting the solution with the highest overall profit.

Varying Attractiveness Values of New Facilities:

Suppose, for example, that we wish to locate five facilities. One facility with one attractiveness value and four facilities with a different value. In general, we wish to locate p_1 facilities with attractiveness A^1, p_2 facilities with attractiveness A^2, and so on for a total of g groups. The number of new facilities is $p = p_1 + \ldots + p_g$. In order to solve such a problem we need to generate candidate locations for each group. The number of y variables is the

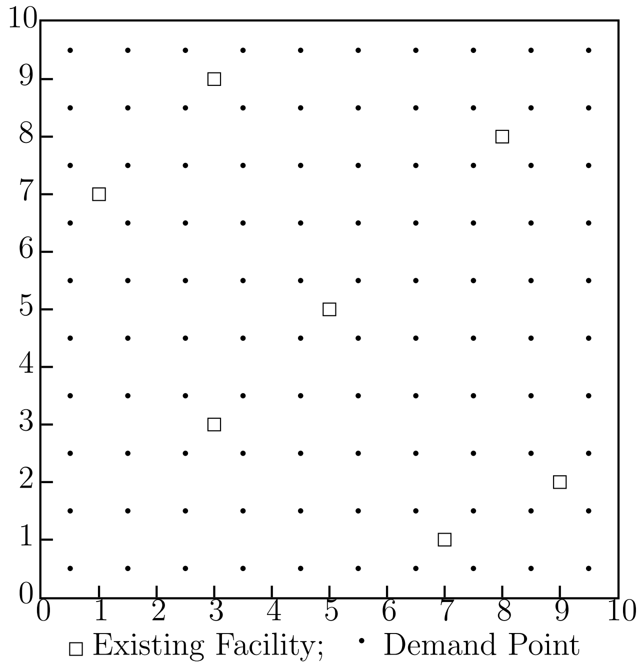


Figure 4: Locations of demand points and existing facilities for the example problem

total number of candidate locations for all groups. We solve it as an integer programming problem by replacing the constraint (5) $\sum_{i=1}^{c} y_j = p$ with g constraints, one for each group.

Suppose we have $h = 1, \ldots, g$ different attractiveness levels. Let $a_{ij}^h = 1$ if demand point i is covered by candidate location j using attractiveness h and is equal to 0, otherwise. The variable $y_j^h = 1$ if a facility of attractivenes A^h is located at candidate location j, and is equal to 0, otherwise. The integer programming formulation is:

$$\max \left\{ \sum_{i=1}^{n} B_i x_i \right\} \tag{7}$$

Subject to:

$$x_i \leq \sum_{h=1}^{g} \sum_{j=1}^{c} a_{ij}^h y_j^h \text{ for } i = 1, \ldots, n \tag{8}$$

$$\sum_{j=1}^{c} y_j^h \ = \ p_h \text{ for } h = 1, \ldots, g \tag{9}$$

$$x_i, y_j^h \in \{0, 1\} \tag{10}$$

5. Computational Experiments

We experimented with an example problem and a real application of locating new convenience stores in the city of Seto, Japan. A program that finds the list of candidate locations was coded in Fortran and ran on a 2.8 GHz Pentium IV PC. The resulting list of candidate locations and the demand points each of them captures was prepared in a file for the What's best! 7.0 software which was run on an identical computer. What's best solved the p-Maxcover problem as an integer program.

Table 2: Demand points attracted to candidate locations for the example problem

Demand	Freq.	Demand	Freq.
3	1	10	50
4	1	11	29
5	10	12	14
6	11	13	11
7	23	14	4
8	38	15	6
9	30		

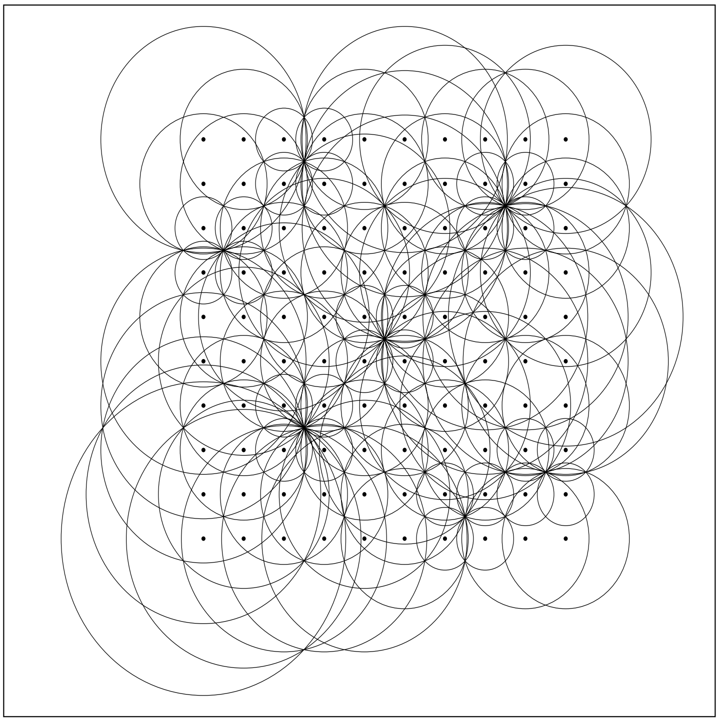


Figure 5: The diagram of intersecting circles for the example problem

5.1. An example problem

As an illustrative example we solved the problem in [4, 5]. The example problem consists of 100 demand points arranged in a grid of 10 by 10 and seven existing facilities. See Figure 4. The buying power at each demand point is equal to 1, and all facilities (whether existing and new ones) are equally attractive. Every demand point defines a circle whose center is at the demand point and its circumference passes through the closest existing facility. The diagram of intersecting circles is depicted in Figure 5.

There are 228 candidate locations depicted in Figure 6. It took a fraction of a second of computer time to calculate these candidate locations. In Table 2 we depict the frequency distribution of the number of demand points captured by each candidate location. The best candidate locations (there are 6 tying candidate locations) capture 15 demand points as reported in [5]. The results for the p-Maxcover problem for $p = 1, \ldots, 14$ are given in Table 3. The problems were solved in about 10 seconds each. Note that fourteen facilities attract all the demand points because two facilities are located on both sides of each existing facility attracting all the demand from that facility. In figure 7 the solution for $p = 5$ facilities that

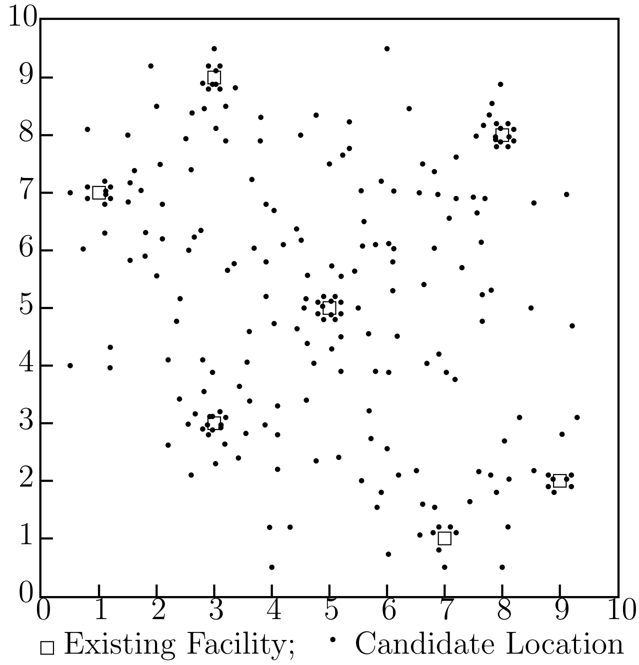


Figure 6: Candidate locations for the example problem

Table 3: Solutions to the p-maxcover example problem

p	Demand	p	Demand
1	15	8	78
2	28	9	83
3	39	10	87
4	49	11	91
5	58	12	95
6	66	13	98
7	73	14	100

capture 58 demand points is depicted.

5.2. An application: the city of Seto, Japan

The city of Seto, Japan has 131,177 residents partitioned into 348 demand areas. There are 32 convenience stores in the city, and the problem is where to locate one or more new convenience stores. Figure 8 depicts the 348 demand points, the 32 locations of the convenience stores, and the best candidate location which attracts 10,122 residents. There are 661 candidate locations. Calculating these candidate locations took a few seconds of computer time. The top 20 candidate locations are depicted in Table 4. This list can be used to select the best location for one convenience store. In Table 5 we list the solutions for the p-Maxcover problems for $p = 1, \ldots, 14$. Each problem was solved in about 14 seconds of computer time.

6. Conclusions

In this paper we proposed algorithms for the location of one or more competing facilities in a competitive environment. Each facility has a utility function which represents its attractiveness. Rather than assuming that customers patronize the closest facility, we assume that customers select the facility with the highest value of the utility function.

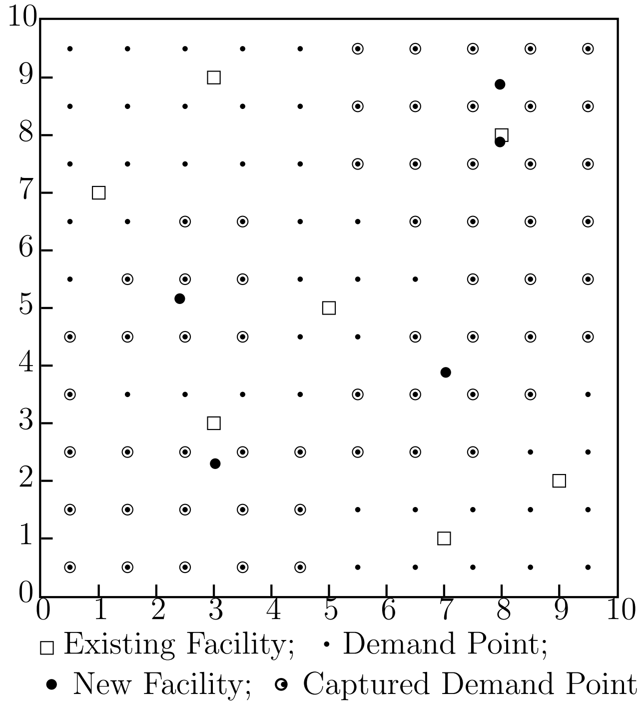


Figure 7: Optimal location of five facilities for the example problem

Table 4: Top 20 candidate locations in Seto

x	y	Pop.	x	y	Pop.
56.347	33.217	10,122	56.833	32.460	7,451
57.149	37.313	10,029	60.839	37.509	7,370
56.183	33.748	8,402	64.046	34.350	7,353
60.635	37.389	7,811	56.133	33.375	7,287
56.813	32.723	7,778	49.144	48.971	7,267
57.803	31.372	7,645	50.112	48.974	7,259
57.767	31.846	7,584	49.503	48.785	7,225
49.174	49.195	7,497	60.193	40.492	7,095
49.413	49.117	7,489	60.999	32.731	7,066
58.067	30.056	7,477	60.084	32.310	7,015

The problem is converted to a p-Maxcover problem by creating a list of candidate locations and maximizing the buying power captured by locating p facilities in the area. The proposed solution approach was tested successfully on an example problem and on real data for convenience stores located in the city of Seto, Japan.

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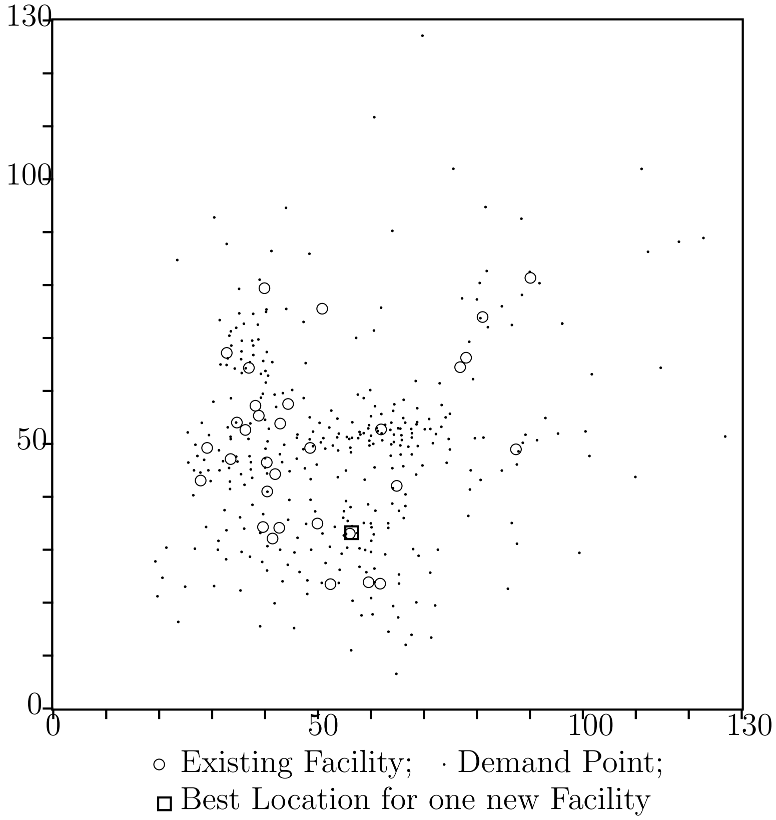

Figure 8: Locations of demand points and convenience stores in Seto, Japan

Table 5: Solutions to the p-maxcover Seto problem

p	Demand	p	Demand
1	10,122	8	52,122
2	17,726	9	56,936
3	24,610	10	61,564
4	30,622	11	66,111
5	36,359	12	70,391
6	41,731	13	74,655
7	46,974	14	78,701

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