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MULTI-ITEM PRODUCTION PLANNING AND MANAGEMENT SYSTEM BASED ON UNFULFILLED ORDER RATE IN SUPPLY CHAIN

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Abstract In the automobile industry, a usual business model has a problem to realize mass customization, because it is difficult to satisfy the diversified customer needs. This paper proposes a multi-item production and inventory planning method of the mass customization with the consideration of the restriction of daily manufacturing capacity and so on. This model is formulated as a stochastic programming problem, and then the sub problem as a linear programming problem. An efficient and practical algorithm for the multi-item model is developed.

Keywords: Inventory, production planning, mass-customization, mathematical programming, stochastic model

1. Introduction

In recent years, the automobile industry has pushed forward with reduction in cost, induction of foreign capital, competition between suppliers, and so on. It becomes very important to remove waste of the production activities and to meet the demands of an individual customer and a changing market. In particular, we must satisfy the variety of customer specification in product and service, without dropping the productive efficiency in mass customization [1,5,10].

For example, in case of a certain car manufacturer, it is said that there is possibility of 300,000,000 ways of specification for a "customized product" in one model of a car. On manufacturing of this "customized product", leveling of production load becomes very difficult because manufacturers have to meet orders of customers based on each specification. Then the productivity decreases, finished goods in stock increase, and it becomes difficult to deal with customer specification.

There have already been some manufacturing strategies for mass customization. However, the design method of production planning and management system for it has not yet been established. Under the precondition that delivery lead time which is expected by customers is longer than production lead time which is necessary for manufacturers, Make-to-Order management system in which manufacturing starts after receiving an order has been applied for a variety of customer specifications as production planning and management system for mass customization. However, it is often the case that delivery lead time becomes shorter than production lead time. Therefore manufacturers have to start manufacturing before they receive an order from customers.

For a variety of customer specifications, Make-to-Order management system and Parts Oriented Production System (POPS) type module manufacturing system are proposed [9].

The Material Requirement Planning (MRP) [11] and Advanced Planning & Scheduling (APS) [7] are presented to plan and manage such systems. This MRP satisfies various demands from customers by promoting modularization in production. By making use of modularization, the MRP makes production lead time shorter, and it manages Make-to-Order management system on precondition that delivery lead time is longer than production lead time. When delivery lead time is shorter than production lead time, we consider the minimum stock as necessary beforehand. But in these management systems, the mechanism between substitutability of unfulfilled order and stocks on mass customization has not been discussed quantitatively.

On precondition that delivery lead time is shorter than production lead time, manufacture seat system for both prospective stocks and order stocks has been proposed [13]. A study on the analysis of manufacture seat system coverage has also been done [6]. However, it has been pointed out that further theoretical work about the decision method of proper manufacture seat should be done [13].

In this paper, we take up production planning on the supplier side [4,14] among manufacturer and assembly suppliers in automobile industry. On precondition that delivery lead time is shorter than production lead time, our targets are both prospective stocks and order stocks. Our production planning and management system does neither discriminate between them nor depend on production seat. We regard mass customization as fluctuation of order quantities from manufacturer to supplier, and we deal with order quantities from manufacturer, that is, the demand for supplier, stochastically.

At first, we define the production planning and management system as the stochastic programming problem [2] to find production plan minimizing manufacture and inventory cost under both unfulfilled order constraint and production constraint. Next we show how to get practical and effective algorithm to the problem. Finally, we describe a design procedure of the whole production planning and management system for implementing mass customization.

2. Mass Customization Environment

The conceptual figure of collaboration model between manufacturer and suppliers for implementing mass customization for each item (part) is shown in Figure 1. The example in Figure 1 shows that there are 3 types of order information (forecast order) from manufacturer to supplier. The monthly forecast order gives the prospected order 3 months before, the weekly forecast order gives the prediction value 2 weeks before, and the delivery instruction to supplier is given as the firm order 1-3 days before delivery due date. In the mass customization environment, the increase of customer specifications means the increase of production specifications in manufacturer. This implies that the quantities of firm order reflecting customer needs have large fluctuation like the bottom image of Figure 1.

For supplier, the standard production lead time for 100-200 kinds of products (or parts) usually takes about 1 week. Therefore production lead time becomes longer than delivery lead time. Supplier must start production in advance based on production plan by using MRP, for example, according to weekly forecast order. Furthermore the supplier must perform the production planning while at the same time avoiding unfulfilled order to the firm order which is given 1-3 days before delivery due date.

We regard mass customization as fluctuation of order from customer to manufacturer and from manufacturer to supplier. Then it can be grasped from the supplier's point of view as the fluctuation of order quantities of target part from manufacturer to supplier. This

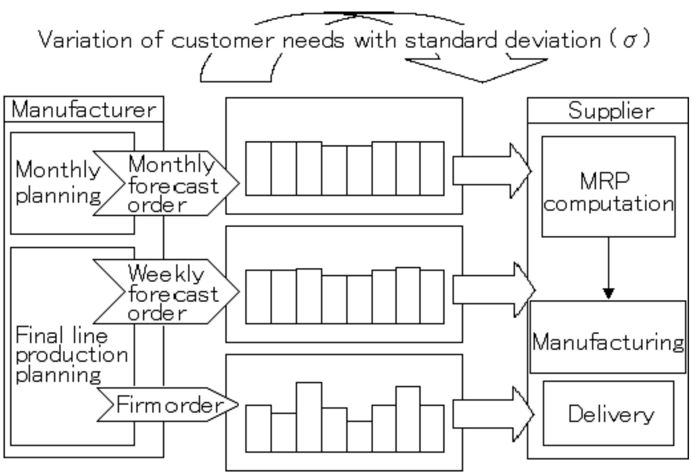


Figure 1: Future collaboration model between manufacturer and supplier for implementing mass customization

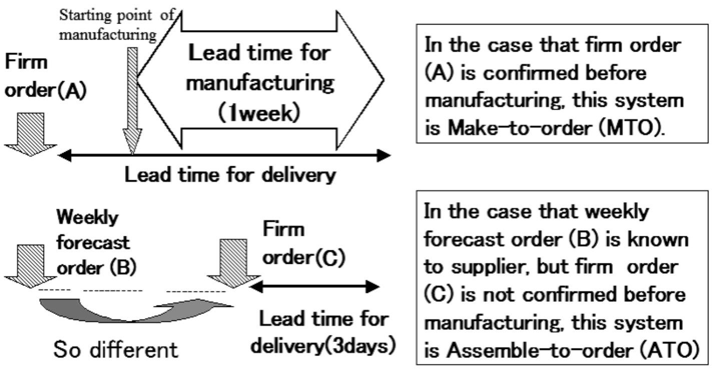


Figure 2: Difference between MTO and ATO

means that the fluctuation of order quantities can be represented by standard deviation around average that forecast order gives. Figure 2 shows the difference between make-to-order (MTO) and assemble-to-order (ATO) in environment under mass customization of production management system. We focus on this case, and supplier should manufacture according to weekly forecast order.

As is shown in Figure 3, assuming that σ_0 denotes the order fluctuation of concerned model of car from customer to manufacturer and σ denotes the order fluctuation about concerned production (or parts) from manufacturer to suppliers. Then the production planning and management system implementing mass customization in suppliers side can be considered by the problem such as how to product and to stock individual parts in advance in order not to run short of supplies for σ which gives fluctuation of firm order from manufacturer. In this paper, we assume that σ is given and demand is defined as a normal distribution with time variant average and time variant deviation of order. We here focus on the supplier's production planning system, so the effect of σ only are discussed in this model. However it is important to investigate the impact of σ_0 on the model, which should be included in the future research.

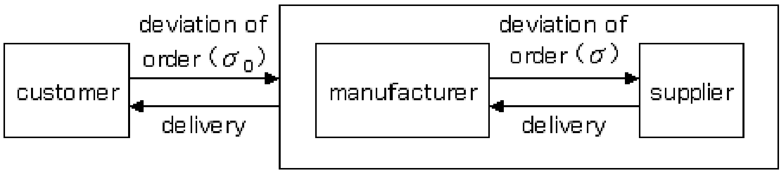


Figure 3: Flow of order among customer, manufacturer and supplier

3. Unfulfilled Order Rate for Supplier's Production Planning and Management System

3.1. Derivation of unfulfilled order rate

In mass customization environment, we formulate problem which determines proper production quantities among n periods to optimize inventory change at final production stage of supplier, corresponding to fluctuation of demand from manufacturer. In this section, we discuss about one certain item.
[Notation]

i **:** Period ($i \leq n$).
d_i **:** Firm order of manufacturer at period i.
x_i **:** Production quantity at period i of supplier.
S_i **:** Inventory quantity at period i of supplier.
p_i **:** Manufacturing cost per module at period i of supplier.
h_i **:** Inventory holding cost per module at period i of supplier.
r **:** Total product quantity until period n of supplier.
R **:** Set of linear production constraints of supplier.
SO_i **:** Unfulfilled order rate until period i of supplier.
β **:** Unfulfilled order rate of planned target of supplier.

It is assumed that d_i obeys an average $\bar{d}_i$ and standard deviation ω_i where d_i, d_j ($i \neq j$) are independent each other, and $\omega_i = \sigma \cdot \bar{d}_i$, where σ is the deviation of order. Let initial inventory be S_0. R is denoted by $(x_1, x_2, \cdots, x_n) \in R$ where R denotes linear production constraint, and is convex. SO_i is probability function which is in short of delivery to the firm order at least until period i.

The inventory quantity S_i at period i is given by

$$S_i = S_0 + \sum_{t=1}^{i} x_t - \sum_{t=1}^{i} d_t \tag{3.1}$$

where d_i is random variable so that S_i becomes random variable. It obeys normal distribution which has the following time variant average and variance.

$$Average \qquad m_i = S_0 + \sum_{t=1}^{i} x_t - \sum_{t=1}^{i} \bar{d}_t, \tag{3.2}$$

$$Variance \qquad \sigma_i^2 = \sum_{t=1}^{i} \omega_t^2. \tag{3.3}$$

S_i can be replaced by

$$y_i = \frac{S_i - m_i}{\sigma_i}. \tag{3.4}$$

The probability function M_i which satisfies inventory quantity $S_i \geq 0$ at period i is derived by

$$M_i = \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(S_i-m_i)^2}{2\sigma_i^2}} dS_i = \int_{-\frac{m_i}{\sigma_i}}^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{y_i^2}{2}} dy_i. \quad (3.5)$$

Thus

$$(i) \qquad M_i = 0.5 + \int_0^{\frac{m_i}{\sigma_i}} \frac{1}{\sqrt{2\pi}} e^{-\frac{y_i^2}{2}} dy_i \qquad (m_i \geq 0), \quad (3.6)$$

$$(ii) \qquad M_i = 0.5 - \int_0^{-\frac{m_i}{\sigma_i}} \frac{1}{\sqrt{2\pi}} e^{-\frac{y_i^2}{2}} dy_i \qquad (m_i < 0). \quad (3.7)$$

These equations can be represented by

$$M_i = 0.5 + sgn(m_i) \int_0^{|-\frac{m_i}{\sigma_i}|} \frac{1}{\sqrt{2\pi}} e^{-\frac{y_i^2}{2}} dy_i. \quad (3.8)$$

We consider an unfulfilled order rate SO_n which represents probability that $S_1 \geq 0, S_2 \geq 0, S_3 \geq 0, \cdots, S_n \geq 0$ is not satisfied. Then, the upper bound of the unfulfilled order rate SO_n is given by $1 - \prod_{t=1}^{n} M_t$. Since it is difficult to obtain the true value of the unfulfilled order rate, we define the unfulfilled order rate SO_n by

$$SO_n = 1 - \prod_{t=1}^{n} M_t. \quad (3.9)$$

By considering such unfulfilled order rate, we can construct the production planning on safety side. In addition, SO_n can be rewritten by applying the integration by parts to M_t as follows;

$$SO_n = 1 - A \quad (3.10)$$

where

$$A = \prod_{t=1}^{n} \left\{ 0.5 + sgn(m_t) \frac{e^{-\frac{1}{2}\left(\frac{m_t}{\sigma_t}\right)^2}}{\sqrt{2\pi}} \sum_{k=1}^{\infty} \frac{|-\frac{m_t}{\sigma_t}|^{2k+1}}{1 \cdot 3 \cdot 5 \cdots (2k+1)} \right\}. \quad (3.11)$$

3.2. Property of unfulfilled order rate

We clarify the property of unfulfilled order rate to understand effect of some variables.

Lemma 1 SO_n is monotonous decrease function of x_i.
Lemma 2 SO_n is monotonous decrease function of S_0.
Lemma 3 SO_n is monotonous increase function of $\bar{d}_i$.
Lemma 4 SO_n is monotonous increase function of ω_i in $m_i > 0$ $(i \leq n)$.

Proof

We apply partial derivative SO_n with respect to x_i, S_0, $\bar{d}_i$ and ω_i. Let α be arbitrary variable.

$$\frac{\partial SO_n}{\partial \alpha} = - \frac{\partial M_1}{\partial \alpha} M_2 \cdots M_n - M_1 \frac{\partial M_2}{\partial \alpha} M_3 \cdots M_n - M_1 M_2 \cdots M_{n-1} \frac{\partial M_n}{\partial \alpha}. \quad (3.12)$$

Since $M_i > 0$ and

$$\frac{\partial}{\partial \alpha}\int_{g(\alpha)}^{\infty} f(y_i)dy_i = -f(g(\alpha))\frac{\partial g(\alpha)}{\partial \alpha}, \tag{3.13}$$

by substituting α into x_i, S_0, $\bar{d}_i$ and ω_i, we can derive the following properties.

$$\frac{\partial M_i}{\partial \alpha} = \frac{1}{\sqrt{2\pi}\sigma_i}e^{-\frac{1}{2}\left(-\frac{m_i}{\sigma_i}\right)^2} > 0 \qquad (\alpha \leftarrow x_i, S_0), \tag{3.14}$$

$$\frac{\partial M_i}{\partial \alpha} = -\frac{1}{\sqrt{2\pi}\sigma_i}e^{-\frac{1}{2}\left(-\frac{m_i}{\sigma_i}\right)^2} < 0 \qquad (\alpha \leftarrow \bar{d}_i), \tag{3.15}$$

$$\frac{\partial M_i}{\partial \alpha} = -\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(-\frac{m_i}{\sigma_i}\right)^2}\frac{m_i}{\sigma_i^2}\cdot\frac{\omega_i}{\sigma_i} < 0 \quad (\alpha \leftarrow \omega_i,\ m_i > 0). \tag{3.16}$$

Equations (3.14)∼(3.16) and $M_i > 0$ $(i \leq n)$ lead lemmas 1 ∼ 4.

4. Production Planning for Implementing Mass Customization with Multi-item

4.1. Problem formulation of mass customization with multi-item

In this section, we define supplier's production planning and management system as the stochastic programming problem to find production plan minimizing manufacturing and inventory holding cost under both unfulfilled order rate constraint and production constraint. In order to consider the multi-item case, previous notations are extended by adding information about jth part of product $(j \leq m)$. For example, d_i^j, x_i^j and S_i^j denote the firm order about jth part of manufacturer at period i with time-variant average $\bar{d}_i^j$, the production quantity of jth part at period i and inventory quantity of jth part at period i respectively. Also p_i^j and h_i^j denote manufacturing cost per module about jth part of supplier at period i and inventory holding cost per module about jth part of supplier at period i, respectively. Those parameters, p_i^j and h_i^j, will be regarded as the same value about each part without losing generality and practicality in order to simplify a transformation of objective function. And S_0^j denotes initial inventory quantity of jth part, β^j is targeted unfulfilled order rate of jth part, and Q_i denotes upper bound of total product quantity at period i.

[Formula 1]

$$\text{minimize} \quad \mathrm{E}[\sum_{j=1}^{m}\{\sum_{i=1}^{n} p_i^j x_i^j + \sum_{i=1}^{n} h_i^j S_i^j\}] \tag{4.1}$$

$$\text{s.t.} \quad S_0^j + \sum_{t=1}^{i} x_t^j - \sum_{t=1}^{i} \bar{d}_t^j \geq 0 \quad (\forall i, j) \tag{4.2}$$

$$\sum_{i=1}^{n} x_i^j = r^j \quad (\forall j) \tag{4.3}$$

$$SO_n^j \leq \beta^j \quad (\forall j) \tag{4.4}$$

$$\sum_{j=1}^{m} x_i^j \leq Q_i \quad (\forall i) \tag{4.5}$$

$$(x_1^j, x_2^j, \cdots, x_n^j) \in R^j \quad (\forall j) \tag{4.6}$$

$$x_i^j \geq 0 \quad (\forall i, j) \tag{4.7}$$

The evaluation function (Equation (4.1)) expresses that we must find optimal solutions $(x_1^j, x_2^j, \cdots, x_n^j)$, $(j = 1, 2, \cdots, m)$ that minimize the expected value (expectation) of the sum of production cost and inventory cost. Equation (4.2) is non negative condition about inventory quantity that the demand is not over the forecast order. Equation (4.3) is constraint about target production quantity. Equation (4.4) gives constraint about unfulfilled order rate, Equation (4.5) denotes limit of production capacity at each period and Equation (4.6) represents linear constraint about production.

The above problem is a manufacture/inventory problem where demand changes stochastically [2]. Although the (s, S) strategy [3,12] is known as an effective tool for such a problem, it is not applicable when the problem has many conditions about production. Thus we propose practical and effective algorithm by solving linear problem as partial problems repeatedly.

4.2. Procedure of algorithm

We first substitute $\mathrm{E}[S_i^j] = S_0^j + \sum_{t=1}^{i} x_t^j - \sum_{t=1}^{i} \bar{d}_t^j$ into total cost MC of Equation (4.1),

$$\begin{aligned} MC &= \sum_{j=1}^{m}\left(\sum_{i=1}^{n} p_i^j \cdot x_i^j\right) + \sum_{j=1}^{m}\left(\sum_{i=1}^{n} h_i^j \cdot S_0^j\right) \\ &\quad + \sum_{j=1}^{m}\sum_{i=1}^{n}\left\{h_i^j\left(\sum_{t=1}^{i} x_t^j\right)\right\} - \sum_{j=1}^{m}\sum_{i=1}^{n}\left\{h_i^j\left(\sum_{t=1}^{i} \bar{d}_t^j\right)\right\} \\ &= \sum_{j=1}^{m}\left(\sum_{i=1}^{n} p_i^j \cdot x_i^j\right) + \sum_{j=1}^{m}\sum_{i=1}^{n}\left\{h_i^j\left(\sum_{t=1}^{i} x_t^j\right)\right\} + MC_1 \end{aligned} \tag{4.8}$$

where

$$MC_1 = \sum_{j=1}^{m}\sum_{i=1}^{n} h_i^j \cdot S_0^j - \sum_{j=1}^{m}\sum_{i=1}^{n}\left\{h_i^j\left(\sum_{t=1}^{i} \bar{d}_t^j\right)\right\} \tag{4.9}$$

Assuming that $h_i^j = h^j$, $p_i^j = p^j$, $i = 1, \cdots, n$ without losing generality and practicality on simplifying a transformation, first term of Equation (4.8) can be rewritten by $\sum_{j=1}^{m} p^j \cdot r^j$ because of $\sum_{j=1}^{m} p^j \sum_{i=1}^{n} x_i^j$. Then by using

$$MC_2 = \sum_{j=1}^{m} h^j \sum_{t=1}^{n} (n - t + 1) \cdot x_t^j, \tag{4.10}$$

MC is calculated by

$$MC = MC_1 + MC_2 + \sum_{j=1}^{m} p^j \cdot r^j \tag{4.11}$$

where MC_1 and $\sum_{j=1}^{m} p^j \cdot r^j$ are constants.

Here we formulate **Multi-item P problem** applying relaxation strategy by excepting Equation (4.4) in Formula 1.

[**Multi-item P problem**]

$$\text{minimize} \quad \sum_{j=1}^{m} h^j \sum_{t=1}^{n} (n - t + 1) x_t^j \tag{4.12}$$

$$\text{s.t.} \quad S_0^j + \sum_{t=1}^{i} x_t^j - \sum_{t=1}^{i} \bar{d}_t^j \geq 0 \qquad (\forall i, j) \tag{4.13}$$

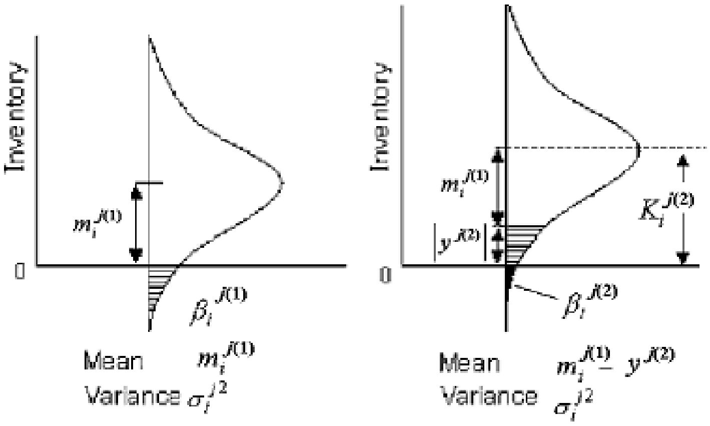


Figure 4: Basic idea of proposed algorithm for a multi-item production planning model

$$\sum_{i=1}^{n} x_i^j = r^j \quad (\forall j) \tag{4.14}$$

$$\sum_{j=1}^{m} x_i^j \leq Q_i \quad (\forall i) \tag{4.15}$$

$$(x_1^j, x_2^j, \cdots, x_n^j) \in R^j \quad (\forall j) \tag{4.16}$$

$$x_i^j \geq 0 \quad (\forall i, j) \tag{4.17}$$

When we obtain $(x_1^{j*}, x_2^{j*}, \cdots, x_n^{j*}), (j = 1, 2, \cdots, m)$ as optimal solution of multi-item P problem, it corresponds to global optimal solution if it satisfies Equation (4.4). If it does not satisfy Equation (4.4) then we consider the following **multi-item MP problem** where the unfulfilled order rate holds smaller value than certain target β_i^j about jth part at period i.

[Multi-item MP problem]

$$\text{minimize} \quad \sum_{j=1}^{m} h^j \sum_{t=1}^{n} (n - t + 1) x_t^j \tag{4.18}$$

$$\text{s.t.} \quad S_0^j + \sum_{t=1}^{i} x_t^j - \sum_{t=1}^{i} \bar{d}_t^j \geq 0 \quad (\forall i, j) \tag{4.19}$$

$$S_0^j + \sum_{t=1}^{i_j} x_t^j - \sum_{t=1}^{i_j} \bar{d}_t^j \geq K_{i_j}^j \quad (\forall i_j \in B^j, \ \forall j) \tag{4.20}$$

$$\sum_{i=1}^{n} x_i^j = r^j \quad (\forall j) \tag{4.21}$$

$$\sum_{j=1}^{m} x_i^j \leq Q_i \quad (\forall i) \tag{4.22}$$

$$(x_1^j, x_2^j, \cdots, x_n^j) \in R^j \quad (\forall j) \tag{4.23}$$

$$x_i^j \geq 0 \quad (\forall i, j) \tag{4.24}$$

Compared with the Multi-item P problem, the Multi-item MP problem involves Equation (4.20) newly. From Figure 4, $K_{i_j}^j$ and B^j in Equation (4.20) can be understood as follows.

We define the target unfulfilled order rate β_i^j for each period in plan from unfulfilled order rate of jth part β^j as follows;

$$1-\beta_i^j = \sqrt[n]{1-\beta^j} \tag{4.25}$$

Let unfulfilled order rate which is calculated at the first iteration on the multi-item MP problem be $\beta_i^{j(1)}$. By using $\beta_i^{j(1)}$, we derive

$$\beta_i^{j(2)} = \beta_i^{j(1)} - sgn(\beta_i^{j(1)} - \beta_i^j) \times \frac{|\beta_i^{j(1)} - \beta_i^j|}{INC}, \tag{4.26}$$

$$i_j^{(2)} = \arg\max_{i \le n}(\beta_i^{j(2)}) \tag{4.27}$$

and $B^j = \{i_j^{(2)}\}$.

In order to hold generally that the probability, inventory quantity S_i^j is less than y_i^j at period i, is smaller than the unfulfilled order rate β_i^j in Figure 4, we obtain

$$\beta_i^j = \int_{-\infty}^{y_i^j} \frac{1}{\sqrt{2\pi}\sigma_i^j} e^{-\frac{(S_i^j - m_i^j)^2}{2{\sigma_i^j}^2}} dS_i^j, \tag{4.28}$$

$$K_i^j = m_i^j - y_i^j, \tag{4.29}$$

B^j denote the set of period indices, which the unfulfilled order rate of certain period becomes smaller value than β_i^j.

Form Equation (4.26), (4.27), (4.28) and (4.29), we find $K_{i_j}^{j(2)}$ satisfying

$$\beta_{i_j}^{j(2)} = \int_{-\infty}^{y_i^{j(2)}} \frac{1}{\sqrt{2\pi}\sigma_{i_j}^j} e^{-\frac{(S_i^j - m_{i_j}^{j(1)})^2}{2{\sigma_{i_j}^j}^2}} dS_i^j, \tag{4.30}$$

$$K_{i_j}^{j(2)} = m_{i_j}^{j(1)} - y_i^{j(2)}. \tag{4.31}$$

Thus we can reconstruct new multi-item MP problem that the probability, inventory quantity S_i^j which is less than 0 of jth part at period $i_j^{(2)}$, takes smaller value than $\beta_{i_j}^{j(2)}$ by applying $K_{i_j}^{j(2)}$ and $i_j^{(2)} \in B^j$ to Equation (4.20).

We will show the reason why by solving multi-item MP problem repeatedly, we can find the approximate solution. Whenever Equation (4.20) is added sequentially, we can get the improved production quantity $x_i^{j(2)}$ about jth part at period i that the probability, which inventory quantity $S_i^{j(2)}$ of the jth part at period $i_j^{(2)}$ is less than 0, takes smaller value than $\beta_{i_j}^{j(2)}$ by $K_{i_j}^{j(2)}$ and $i_j^{(2)} \in \beta^j$ to Equation (4.20). Then improved production quantity $x_i^{j(2)}$ at one iteration makes both unfulfilled order rate $\beta_{i_j}^{j(2)}$ of jth part at period i_j and the unfulfilled order rate of jth part $\beta^{j(2)}$ monotonously decrease by Lemma 1 at section 3.2. These mechanism is repeatedly by changing next step targeted unfulfilled order rate $\beta_i^{j(l)}$ $(l > 2)$ of jth part at period i decreasing, which is defined like Equation (4.26) when $l = 2$.

Now that, we introduce update parameter INC to next step unfulfilled order rate in calculation (Equation (4.26)) of the target unfulfilled order rate β_i^j at each period in plan.

Figure 5 shows input factors between manufacturer and supplier. Purpose of research is to develop a supplier's production planning system which is minimizing total cost subject to unfulfilled order rate and production constraints when weekly forecast order and deviation of order σ are given.

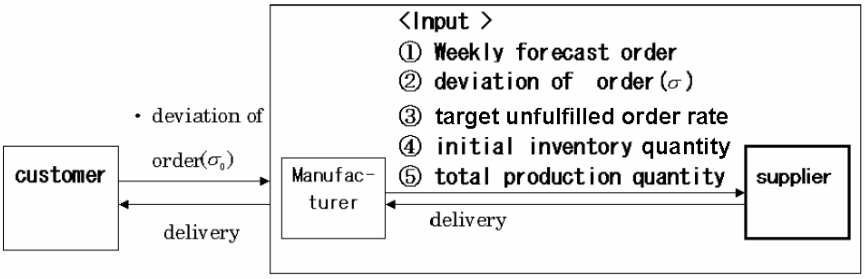


Figure 5: Input factors between manufacturer and supplier

4.3. Algorithm

In this section, we describe algorithm for finding the approximate solution of Formula 1.

Step 1

Let $l = 1$, $T^j = \{1, 2, \cdots, n\}$ $(\forall j)$, $B^j = \{\}$ $(\forall j)$ and $\epsilon = 0.0001$. T^j denotes set of period index.

$$\beta_i^j = 1 - \sqrt[n]{1 - \beta^j} \qquad (\forall i \leq n, \forall j \leq m) \tag{4.32}$$

Step 2

Finding a solution of the multi-item P problem, then we obtain the solution $(x_1^{j*}, x_2^{j*}, \cdots, x_n^{j*})$, cost $C^{(l)}$ as the objective value of Equation (4.18), unfulfilled order rate $\beta^{j(l)}$ and unfulfilled order rate at each period $\beta_i^{j(l)}$.

Step 3

(i) If $\beta^{j(l)} \leq \beta^j$, $(\forall j)$ holds, then $(x_1^{j*}, x_2^{j*}, \cdots, x_n^{j*})$ is global approximate solution (END).
(ii) Otherwise, for certain j holding $\beta^{j(l)} > \beta^j$, it is updated by

$$\beta_i^{j(l+1)} = \beta_i^{j(l)} - sgn(\beta_i^{j(l)} - \beta_i^j) \times \frac{|\beta_i^{j(l)} - \beta_i^j|}{INC}, \tag{4.33}$$

$$i_j^{(l+1)} = \arg\max_{i \leq n}(\beta_i^{j(l+1)}) \tag{4.34}$$

then

$$\{B^j\} = \{B^j\} \cup \{i_j^{(l+1)}\}, \ \{T^j\} = \{T^j\} - \{i_j^{(l+1)}\} \tag{4.35}$$

and reconstruct the multi-item MP problem by using

$$\beta_{i_j}^{j(l+1)} = \int_{-\infty}^{y_i^{j(l+1)}} \frac{1}{\sqrt{2\pi}\sigma_{i_j}^j} e^{-\frac{(S_i^j - m_{i_j}^{j(l)})^2}{2\sigma_{i_j}^{j\,2}}} dS_i^j, \tag{4.36}$$

$$K_{i_j}^{j(l+1)} = m_{i_j}^{j(l)} - y_i^{j(l+1)}. \tag{4.37}$$

Step 4

If we get a solution $(x_1^{j*}, x_2^{j*}, \cdots, x_n^{j*})$, cost $C^{(l+1)}$, and the unfulfilled order rate $\beta^{j(l+1)}$ for the multi-item MP problem, and the following relation

$$|\beta^{j(l+1)} - \beta^{j(l)}| < \varepsilon^j \qquad (\forall j) \tag{4.38}$$

holds then procedure is end.

Table 1 $\bar{d}_i, \omega_i, \sigma_i$ used in numerical example

i	1	2	3	4	5	6	7	8
$\bar{d}_i$	4	8	0	20	16	12	12	12
ω_i	1.4	2.8	0	7	5.6	4.2	4.2	4.2
σ_i	1.4	3.13	3.13	7.67	9.5	10.38	11.2	11.96

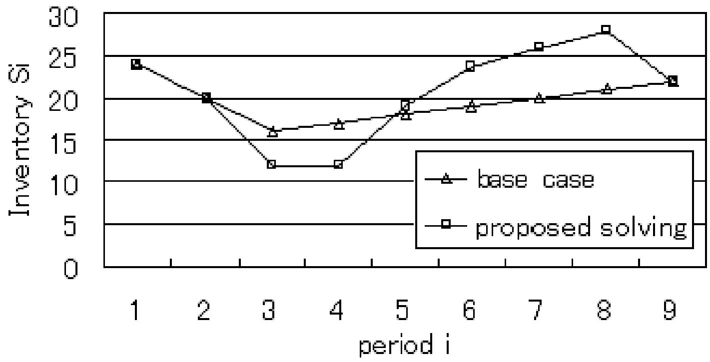


Figure 6: Result of proposed algorithm

If we can not get a solution $(x_1^{j*}, x_2^{j*}, \cdots, x_n^{j*})$, cost $C^{(l+1)}$, and the unfulfilled order rate $\beta^{j(l+1)}$ for the multi-item MP problem, then procedure is end.

Step 5

(i) If $\beta^{j(l+1)} \leq \beta^j$, $(\forall j)$ holds, then $(x_1^{j*}, x_2^{j*}, \cdots, x_n^{j*})$ is global approximate solution (END).
(ii) Otherwise, let $l = l + 1$, then go to **Step 3** (ii).

5. Numerical Experiments

5.1. Effect of proposed unfulfilled order rate as criteria

In this section, in order to confirm the effectiveness of criteria of unfulfilled order rate we propose in this paper, we apply our algorithm to actual operation data in auto parts supplier. Table 1 shows $\bar{d}_i$, ω_i and σ_i about forecast order. It is assumed that $S_0 = 24$, $p_i = 1$, $h_i = 1$, $r = 82$, $\omega_i = \bar{d}_i \times 0.35$ for each i and $\beta = 0.05$.

We set some parameters based on the actual data such as manufacturing and inventory holding costs to show the efficiency of this approach. Figure 6 shows comparison of inventory change between the result of proposal method in case of $m = 1$, $INC = 2$ and the actual production plan. We call the result of real operation planned by production manager in the supplier "base case". Here base case is consequence of actual operation data in supplier. From the results of Figure 6, we can find that our method provide lower inventory change because of considering inventory cost in first half, and provide higher inventory change for responding to fluctuation of demand in last half.

Figure 7 shows the transition of both unfulfilled order rate and the cost at each iteration by using proposed algorithm. The unfulfilled order rate SO_n decreases monotonously and solution is obtained after 21 iterations. The cost is plotted by relative comparison for setting the cost of initial state to 100%. We can find that the unfulfilled order rate is improved at each iteration. We can also find that the existence of trade-off between the unfulfilled order rate and the cost is shown in Figure 7.

Figure 8 shows improvement of unfulfilled order rate by proposed technique when initial inventory is changed. When initial inventory is given by 24, then unfulfilled order rate takes

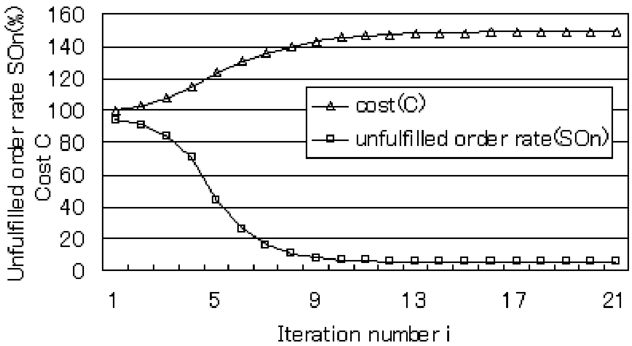
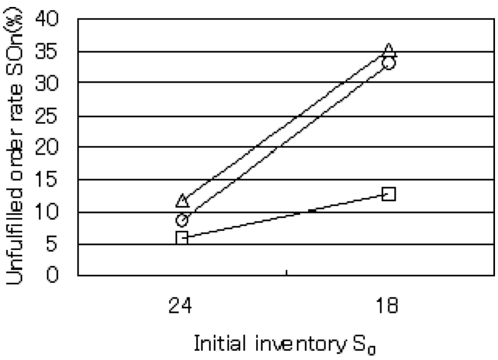


Figure 7: Result of unfulfilled order rate and cost for each iteration



(△ : base case, □ : proposed algorithm, ○ : proposed algorithm with same cost of base case)

Figure 8: Performance of proposal algorithm

11.7% by base case, but it takes 5.8% by proposal algorithm. Particularly, from the result about proposed method with same cost of base case, it takes 8.5% then we can find our method improving both unfulfilled order rate and cost.

5.2. Behavior of unfulfilled order rate in two-item case

Here we consider the case that supplier's product consists of two items. Table 2 shows $\bar{d}_i^j$ used in an example system of 2 items, where $S_0^j = 24$, $p_i^j = 1$, $h_i^j = 1$, $r_1 = 82$, $r_2 = 135$, $Q_i = 50$, $\omega_i^j = \bar{d}_i^j \times 0.35$ and $\beta^j = 0.05$.

Figure 9 shows results of inventory by proposed algorithm at 1st and 7th iterations, and final solution. Compared with obtained result at 1st iteration, we can find the tendency of becoming higher about inventory level when iteration increases. Figure 9 shows typical result.

Figure 10 shows result of production quantity by proposed algorithm. Compared with base case, we can find that proposed algorithm plans necessarily the precedence production at 4th, 5th, 6th and 7th for item number 1, also it plans necessarily the precedence production at 2nd for item number 2.

Table 2 $\bar{d}_i^j$ used in an example system of 2 items

i	1	2	3	4	5	6	7	8
$\bar{d}_i^1$	4	8	0	20	16	12	12	12
$\bar{d}_i^2$	20	20	20	15	15	15	15	15

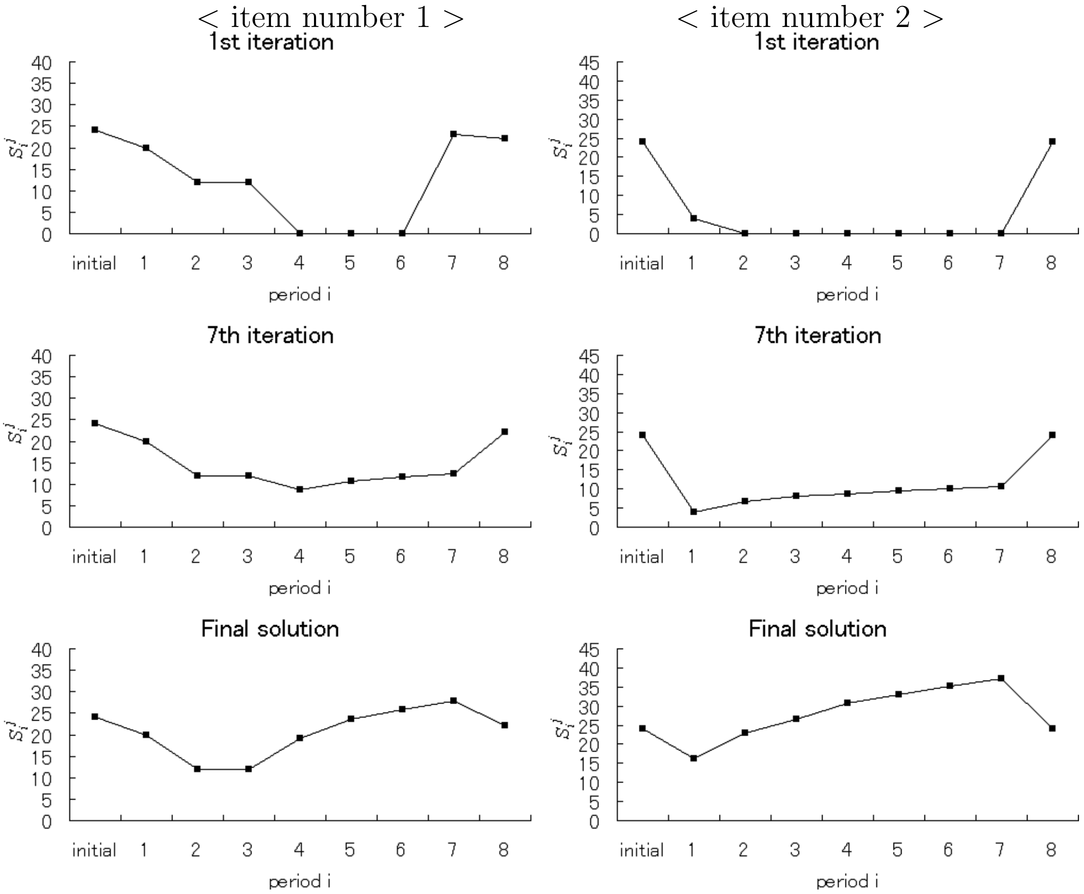


Figure 9: Results of inventory by proposed algorithm for each item

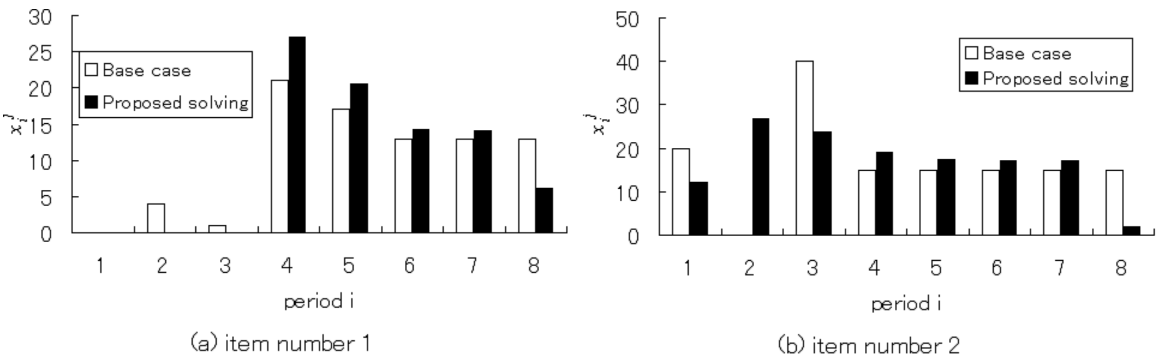


Figure 10: Result of production quantity by proposed algorithm

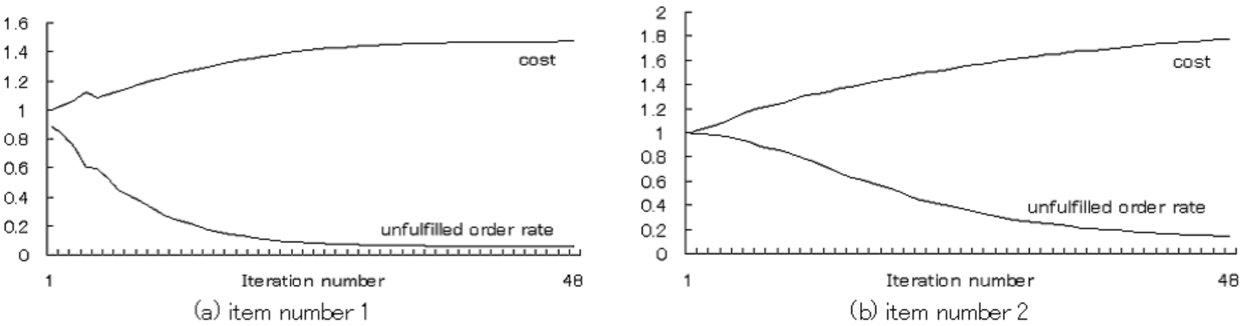


Figure 11: Result of cost and unfulfilled order rate for each iteration

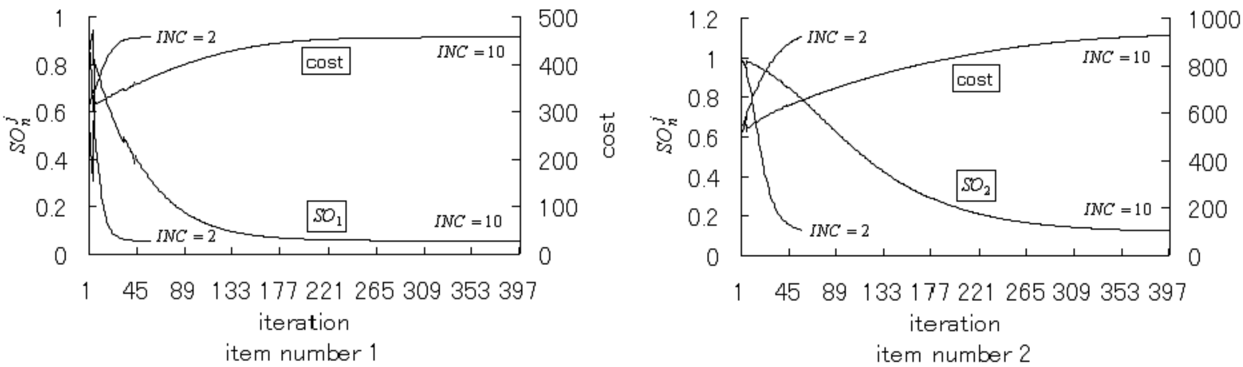


Figure 12: Results for each iteration by changing a step size for update rule

Figure 11 shows result of cost and unfulfilled order rate for each iteration in the case that INC =2. The unfulfilled order rate decreases gradually, and solution is obtained after 48th iteration. Cost is represented by rate compared with the cost obtained at 1st iteration. From the result, we can find that proposed algorithm can improve the unfulfilled order rate, and we can understand existence of alternative between cost and the unfulfilled order rate.

Figure 12 shows results for each iteration by changing a step size for update rule INC where $Q_i = 50$. It can be found that algorithm, where INC=2, is converged faster than another.

Table 3 shows results by changing Q_i and INC. Result when $INC = 2$ is quite nearly converged same as one when $INC = 10$ for each Q_i.

Figure 13 shows performance of proposed algorithm for each item. For the item number 1, the unfulfilled order rate of base case takes 11.7%, but the unfulfilled order rate obtained by proposed method becomes 5.8%. Especially, when we compare it under the same cost of base case, proposed method realizes 8.3% about the unfulfilled order rate. Similarly, for item number 2, the unfulfilled order rate is improved from 52.1% in base case to 14.5% in

Table 3 Results by changing Q_i and INC

	SO_n^1		SO_n^2		cost	
Q_i	$INC = 2$	$INC = 10$	$INC = 2$	$INC = 10$	$INC = 2$	$INC = 10$
60	0.0576	0.0576	0.1276	0.1276	1380.5	1380.5
50	0.0576	0.0576	0.1283	0.1283	1379.3	1379.6
40	0.0576	0.0576	0.1285	0.1286	1387.2	1385.2

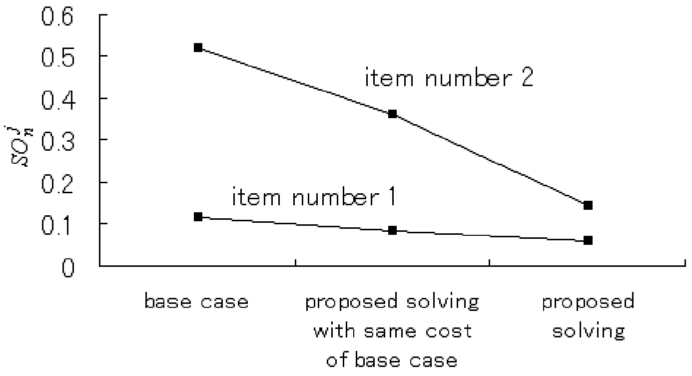


Figure 13: Performance of proposed algorithm for each item

Table 4 $\bar{d}_i^j$ used in an example system of 5 items

i	1	2	3	4	5	6	7	8
$\bar{d}_i^1$	4	8	0	20	16	12	12	12
$\bar{d}_i^2$	20	20	20	15	15	15	15	15
$\bar{d}_i^3$	16	16	16	20	20	8	8	8
$\bar{d}_i^4$	24	24	24	12	12	12	12	12
$\bar{d}_i^5$	20	20	20	16	16	16	16	16

proposed method. Furthermore, even if we compare it under the same cost of base case, proposed method realizes 36.1% about the unfulfilled order rate. Proposed algorithm is better than base case from the point of both unfulfilled order rate and cost.

From these results, compared with base case, we can find that the proposed method can improve both cost and unfulfilled order rate under keeping manufacturing capacity restrictions and so on.

5.3. Behavior of unfulfilled order rate in multi-item case

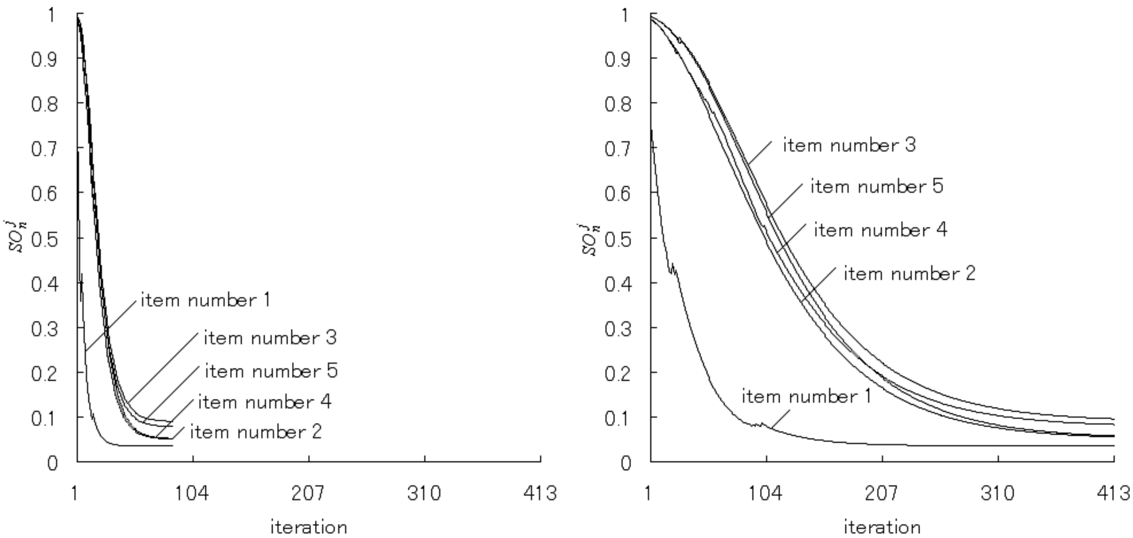
In this section, the behavior of unfulfilled order rate in multi-item case is investigated. Table 4 shows $\bar{d}_i^j$ used in this numerical example with 5 items. The other parameters are assumed to be $S_0^1 = 18$, $S_0^2 = 24$, $S_0^3 = 14$, $S_0^4 = 24$, $S_0^5 = 18$, $p_i^j = 1$, $h_i^j = 1$, $r_1 = 82$, $r_2 = 135$, $r_3 = 112$, $r_4 = 132$, $r_5 = 140$, $Q_i = 100$, $\omega_i^j = \bar{d}_i^j \times 0.20$ and $\beta^j = 0.05$.

Figure 14 shows results when system applies $INC = 2$ and $INC = 10$ as the step size for update rule. From its results, we can understand that the unfulfilled order rate changes decrease monotonously and converges also in case of 5 items.

5.4. Proposal design method in actual case

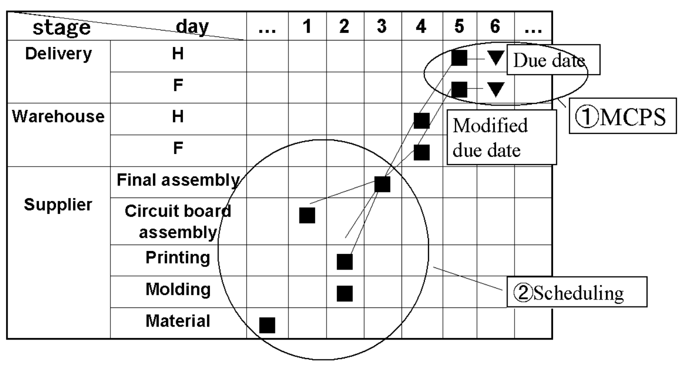
In this section, we show proposal design method with 2 steps in actual case. As the design method is illustrated in Figure 15, we first solve the production planning problem by the proposed algorithm in the previous section, and we call this algorithm MCPS (Masscustomization Production Planning and Management System), and by this result we modify original due date for delivery. Almost all the case it means "precedence production". Next we execute scheduling based on modified due date for delivery.

So, we first determine the production planning of final stage for each item in order to obtain minimum inventory level while keeping target unfulfilled order rate. For the due date of delivery that is given by each order, it can be realized by improving a schedule that finishes the final stage. For example, final stage is set by the final process about supplier's assembly, or the delivery to manufacturer's warehouse from supplier. That is, such schedule



(a) INC=2 (b) INC=10

Figure 14: Result of unfulfilled order rate and cost for each iteration



(■ : date that order is assigned and to be manufactured)

Figure 15: Proposal design method in actual case

aims both the proper necessary precedence production for each item and the holding of minimum inventory.

Next we schedule upstream process from final state by using commercial software that supports system based on Make-to-Order production system. We will select the best software that can take into account about the condition of supplier's production restriction in detail.

In order to realize mass customization, in the case that we apply the existing technique, for example, MRP II [11], APS [7] directly, it is necessary to set up safety stock in advance. However, safety stock method is not realistic from the fact that forecast order changes in the planned period usually happen in the automobile industry.

By applying the proposed algorithm, it is not necessary to give the safety stock in advance. When the planned unfulfilled order rate is given in advance, then the nearly optimal production quantity and simultaneously the minimum inventory for planned term, instead of safety stock, are decided.

In sum, both MCPS and scheduling software solve sequentially, and realize mass customization.

6. Concluding Remarks

In this paper, we took up production planning of parts assembly supplier in auto industry, and defined "supplier's multi-item production planning and management system for implementing mass customization ". The problem about proper inventory quantity of multi-item production was formulated by stochastic programming problem. We proposed the effective algorithm to obtain nearly optimal inventory quantity for the production planning and management system implementing mass customization.

Further work is an application of the decomposition principle [8] to find the solution of the multi-item MP problem which was derived, because if the number of items becomes large then the size of sub problem for iteration become huge.

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