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AN EOQ MODEL WITH NONINSTANTANEOUS RECEIPT UNDER SUPPLIER CREDITS

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Abstract This paper tries to incorporate all Huang and Chung [4], Chung and Huang [2] and Teng [7] to develop the retailer's inventory model. That is, we want to investigate the retailer's optimal replenishment policy with noninstantaneous receipt under trade credit, cash discount and the retailer's unit selling price is not lower than the unit purchasing price. Mathematical models have been derived for obtaining the optimal cycle time for item so that the annual total relevant cost is minimized. One easy-to-use theorem is developed to efficiently determine the optimal cycle time for the retailer. Some previously published results of other researchers are deduced as special cases. Furthermore, numerical examples are given to illustrate the results and managerial insights are drawn.

Keywords: Inventory, EOQ, trade credit, permissible delay in payments, cash discount

1. Introduction

In the real world, the supplier often makes use of the trade credit policy to promote his/her commodities. Before the end of trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of trade credit period. Therefore, it makes economic sense for the retailer to delay the settlement of the replenishment account up to the last moment of the permissible period allowed by the supplier. In the credit card market, we can easily find the above situation. We can buy any items and do not pay the payment immediately. However, we must pay higher interest if the payment is not settled by the end of the payment time assigned by credit card issuers (or banks).

From the viewpoint of the supplier, the supplier hopes that the payment is paid from retailer as soon as possible. It can avoid the possibility of resulting in bad debt. So, in most business transactions, the supplier will offer the credit terms mixing cash discount and trade credit to the retailer. The retailer can obtain the cash discount when the payment is paid before cash discount period offered by the supplier. Otherwise, the retailer will pay full payment within the trade credit period. For example, the supplier provides r discount off the price if the payment is made within M_1 period, otherwise the full payment is due within M_2 period, this usually denoted as "$r/\ M_1,\ M_2$". Many articles related to the inventory policy under trade credit and cash discount can be found in Chang [1], Ouyang et al. [5, 6] and Huang and Chung [4].

Recently, Teng [7] assumed that the selling price not equal to the purchasing price to modify the Goyal's [3] model. Chung and Huang [2] investigated the topic of permissible delay in payments with noninstantaneous receipt. Therefore, this paper tries to incorporate all Huang and Chung [4], Chung and Huang [2] and Teng [7] to extend and develop the retailer's inventory model. That is, we want to investigate the retailer's optimal replenishment policy with noninstantaneous receipt under trade credit, cash discount and the retailer's unit selling price is not lower than the unit purchasing price within the EPQ framework. Mathematical models have been derived for obtaining the optimal cycle time for item so that the annual total relevant cost is minimized. One easy-to-use theorem is developed to efficiently determine the optimal cycle time for the retailer. Finally, numerical examples are given to illustrate the results and managerial insights are drawn.

2. Model Formulation and Convexity

2.1. Notation

A = cost of placing one order
c = unit purchasing price per item
D = demand rate per year
h = unit stock holding cost per item per year excluding interest charges
I_e = interest which can be earned per \$ per year
I_k = interest charges per \$ investment in inventory per year
M_1 = the period of cash discount in years
M_2 = the period of trade credit in years, $M_1 < M_2$
P = replenishment rate per year, $P > D$
$\rho = 1 - \dfrac{D}{P} > 0$
r = cash discount rate, $0 \le r < 1$
s = unit selling price per item
T = the cycle time in years (decision variable)

$$TVC_1(T) = \text{the annual total relevant cost when payment is paid at time } M_1 \text{ and } T > 0$$
$$= \begin{cases} TVC_{11}(T) & \text{if } M_1 \le PM_1/D \le T \\ TVC_{12}(T) & \text{if } M_1 \le T \le PM_1/D \\ TVC_{13}(T) & \text{if } 0 < T \le M_1 \end{cases}$$

$$TVC_2(T) = \text{the annual total relevant cost when payment is paid at time } M_2 \text{ and } T > 0$$
$$= \begin{cases} TVC_{21}(T) & \text{if } M_2 \le PM_2/D \le T \\ TVC_{22}(T) & \text{if } M_2 \le T \le PM_2/D \\ TVC_{23}(T) & \text{if } 0 < T \le M_2 \end{cases}$$

$$TVC(T) = \text{the annual total relevant cost when } T > 0$$
$$= \begin{cases} TVC_1(T) & \text{if the payment is paid at time } M_1 \\ TVC_2(T) & \text{if the payment is paid at time } M_2 \end{cases}$$

T^* = the optimal cycle time of $TVC(T)$.

2.2. Assumptions

(1) Demand rate, D, is known and constant.
(2) Replenishment rate, P, is known and constant.
(3) Shortages are not allowed.
(4) Time horizon is infinite.
(5) $s \ge c$; $I_k \ge I_e$.
(6) Supplier offers a cash discount after settlement of an order if payment is paid within M_1, otherwise the full payment is paid within M_2. The account is settled when the payment

is paid.

(7) During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. At the end of the period, the retailer pays off all units sold and keeps his/her profits, and starts paying for the higher interest charges on the items in stock.

2.3. Mathematical model

The annual total relevant cost consists of the following elements: (1) annual ordering cost, (2) annual stock holding cost (excluding interest charges), (3) annual purchasing cost (cash discount earned if the payment is made at M_1), (4) annual cost of interest charges for unsold items when the account is settled, and (5) annual interest earned from sales revenue during the permissible period.

(1) Annual ordering cost $= \dfrac{A}{T}$.

(2) Annual stock holding cost (excluding interest charges) =

$$\frac{hT(P-D)\dfrac{DT}{P}}{2T} = \frac{DTh}{2}(1-\frac{D}{P}) = \frac{DTh\rho}{2}.$$

Since the supplier offers a cash discount if payment is paid within M_1, there are two payment policies for the retailer. First, the payment is paid at time M_1 to get the cash discount, **Case 1**. Second, the payment is paid at time M_2 not to get the cash discount, **Case 2**. So purchasing cost, interest payable and interest earned, we shall discuss these two cases as follows.

(3) Annual purchasing cost:

Case 1 : Payment is paid at time M_1, the annual purchasing cost $= c(1-r)D$.

Case 2 : Payment is paid at time M_2, the annual purchasing cost $= cD$.

(4) Annual cost of interest charges for the items kept in stock:

Case 1 : Payment is paid at time M_1

Case 1.1: $\underline{M_1 \le \dfrac{PM_1}{D} \le T}$, as shown in Figure 1.

In this case, the retailer pays the payment at M_1 to get cash discount and the account is settled. Hence, the retailer must pay the cost of interest charges for unsold items behind M_1. Therefore, the annual interest payable

$$= cI_k(1-r)[\frac{DT^2\rho}{2} - \frac{(P-D)M_1^2}{2}]/T = cI_k(1-r)\rho(\frac{DT^2}{2} - \frac{PM_1^2}{2})/T.$$

Case 1.2: $\underline{M_1 \le T \le \dfrac{PM_1}{D}}$, as shown in Figure 2.

Same discussion as above case 1.1, the annual interest payable

$$= cI_k(1-r)[\frac{D(T-M_1)^2}{2}]/T.$$

Case 1.3: $\underline{T \le M_1.}$

In this case, all items have sold when the payment is paid at time M_1. Therefore, there is no interest charges are paid for the items.

Case 2 : Payment is paid at time M_2

Case 2.1: $\underline{M_2 \le \dfrac{PM_2}{D} \le T}$, as shown in Figure 1.

In this case, the retailer cannot get the cash discount since the retailer pays the payment at M_2, then the account is settled. Hence, the retailer must pay the cost of interest charges for unsold items behind M_2. Therefore, the annual interest payable
$= cI_k[\frac{DT^2\rho}{2} - \frac{(P-D)M_2^2}{2}]/T = cI_k\rho(\frac{DT^2}{2} - \frac{PM_2^2}{2})/T.$

Case 2.2: $\underline{M_2 \le T \le \frac{PM_2}{D}}$, as shown in Figure 2.

Same discussion as above case 2.1, the annual interest payable
$= cI_k[\frac{D(T-M_2)^2}{2}]/T.$

Case 2.3: $\underline{T \le M_2}$.

In this case, all items have sold when the payment is paid at time M_2. Therefore, there is no interest charges are paid for the items.

(5) Annual interest earned:

Case 1 : Payment is paid at time M_1

During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. Hence, the retailer can earn the interest from sales revenue during (0, M_1].

Case 1.1: $\underline{M_1 \le \frac{PM_1}{D} \le T}$.

Annual interest earned $= sI_e(\frac{DM_1^2}{2})/T.$

Case 1.2: $\underline{M_1 \le T \le \frac{PM_1}{D}}$.

Annual interest earned $= sI_e(\frac{DM_1^2}{2})/T.$

Case 1.3: $\underline{T \le M_1}$, as shown in Figure 3.

Annual interest earned $= sI_e[\frac{DT^2}{2} + DT(M_1 - T)]/T.$

Case 2 : Payment is paid at time M_2

During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. Hence, the retailer can earn the interest from sales revenue during (0, M_2].

Case 2.1: $\underline{M_2 \le \frac{PM_2}{D} \le T}$.

Annual interest earned $= sI_e(\frac{DM_2^2}{2})/T.$

Case 2.2: $\underline{M_2 \le T \le \frac{PM_2}{D}}$.

Annual interest earned $= sI_e(\frac{DM_2^2}{2})/T.$

Case 2.3: $\underline{T \le M_2}$, as shown in Figure 3.

Annual interest earned $= sI_e[\frac{DT^2}{2} + DT(M_2 - T)]/T.$

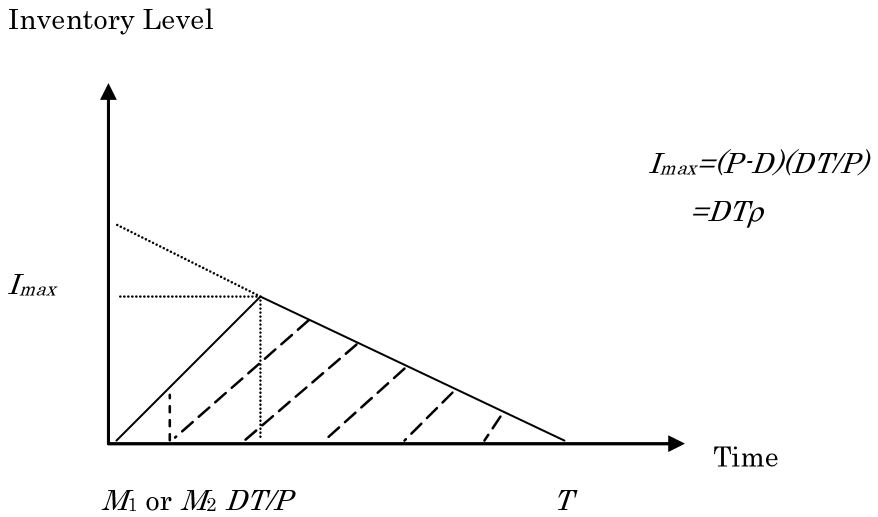

Figure 1: The total accumulation of interest payable when $PM_1/D \le T$ or $PM_2/D \le T$

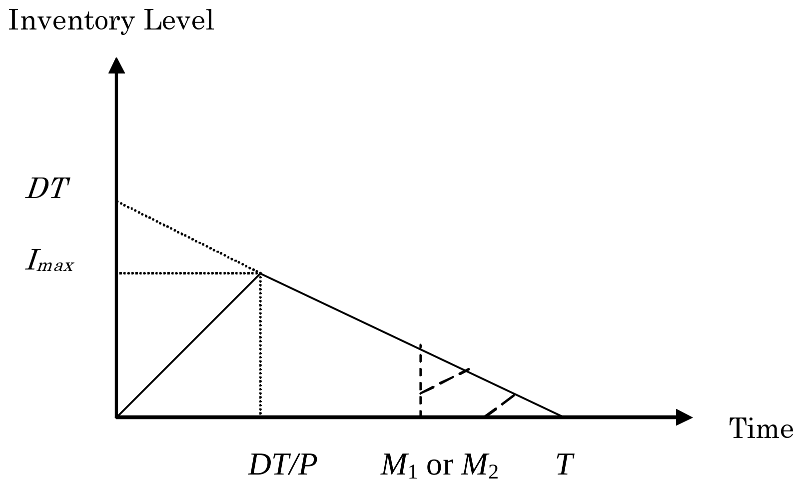

Figure 2: The total accumulation of interest payable when $M_1 \le T \le PM_1/D$ or $M_2 \le T \le PM_2/D$

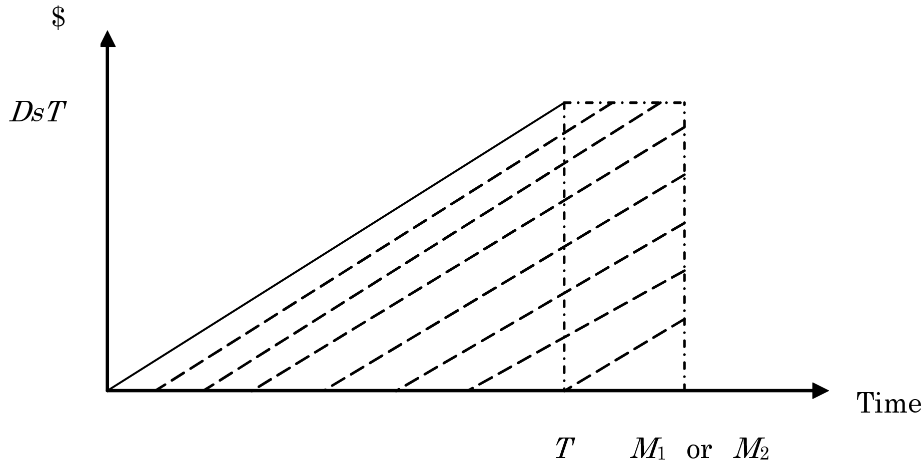

Figure 3: The total accumulation of interest earned when $T \le M_1$ or $T \le M_2$

The annual total relevant cost for the retailer can be expressed as:

$$TVC(T) = \text{ordering cost} + \text{stock-holding cost} + \text{purchasing cost} + \text{interest payable} - \text{interest earned.}$$

We show that the annual total relevant cost is given by
Case 1 : Payment is paid at time M_1

$$TVC_1(T) = \begin{cases} TVC_{11}(T) & \text{if } M_1 \le PM_1/D \le T & (1.1)\\ TVC_{12}(T) & \text{if } M_1 \le T \le PM_1/D & (1.2)\\ TVC_{13}(T) & \text{if } 0 < T \le M_1 & (1.3)\end{cases}$$

where :

$$TVC_{11}(T) = \frac{A}{T} + \frac{DTh\rho}{2} + c(1-r)D + cI_k(1-r)\rho\left[\frac{DT^2}{2} - \frac{PM_1^2}{2}\right]/T - sI_e\left(\frac{DM_1^2}{2}\right)/T, \quad (2)$$

$$TVC_{12}(T) = \frac{A}{T} + \frac{DTh\rho}{2} + c(1-r)D + cI_k(1-r)\left[\frac{D(T-M_1)^2}{2}\right]/T - sI_e\left(\frac{DM_1^2}{2}\right)/T \quad (3)$$

and

$$TVC_{13}(T) = \frac{A}{T} + \frac{DTh\rho}{2} + c(1-r)D - sI_e\left[\frac{DT^2}{2} + DT(M_1-T)\right]/T. \quad (4)$$

Then, we find $TVC_{11}(PM_1/D) = TVC_{12}(PM_1/D)$ and $TVC_{12}(M_1) = TVC_{13}(M_1)$. Hence $TVC_1(T)$ is continuous and well-defined. All $TVC_{11}(T)$, $TVC_{12}(T)$, $TVC_{13}(T)$ and $TVC_1(T)$ are defined on $T > 0$.

Case 2 : Payment is paid at time M_2

$$TVC_2(T) = \begin{cases} TVC_{21}(T) & \text{if} \quad M_2 \le PM_2/D \le T & (5.1) \\ TVC_{22}(T) & \text{if} \quad M_2 \le T \le PM_2/D & (5.2) \\ TVC_{23}(T) & \text{if} \quad 0 < T \le M_2 & (5.3) \end{cases}$$

where :

$$TVC_{21}(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cD + cI_k\rho\left(\frac{DT^2}{2} - \frac{PM_2^2}{2}\right)/T - sI_e\left(\frac{DM_2^2}{2}\right)/T, \quad (6)$$

$$TVC_{22}(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cD + cI_k\left[\frac{D\left(T - M_2\right)^2}{2}\right]/T - sI_e\left(\frac{DM_2^2}{2}\right)/T \quad (7)$$

and

$$TVC_{23}(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cD - sI_e\left[\frac{DT^2}{2} + DT\left(M_2 - T\right)\right]/T. \quad (8)$$

Then, we find $TVC_{21}(PM_2/D) = TVC_{22}(PM_2/D)$ and $TVC_{22}(M_2) = TVC_{23}(M_2)$. Hence $TVC_2(T)$ is continuous and well-defined. All $TVC_{21}(T)$, $TVC_{22}(T)$, $TVC_{23}(T)$ and $TVC_2(T)$ are defined on $T > 0$.

2.4. Optimality conditions:

From equations (2)–(4) and (6)–(8) yield :

$$TVC'_{11}(T) = -\left\{\frac{2A - M_1^2\left[cI_k\left(1-r\right)P\rho + sI_eD\right]}{2T^2}\right\} + \frac{D\rho\left[h + cI_k\left(1-r\right)\right]}{2}, \quad (9)$$

$$\begin{aligned} TVC''_{11}(T) &= \frac{2A - M_1^2\left[cI_k\left(1-r\right)P\rho + sI_eD\right]}{T^3} \\ &= \frac{2A - cM_1^2PI_k\left(1-r\right) + DM_1^2\left[cI_k\left(1-r\right) - sI_e\right]}{T^3}, \end{aligned} \quad (10)$$

$$TVC'_{12}(T) = -\left\{\frac{2A + DM_1^2\left[cI_k\left(1-r\right) - sI_e\right]}{2T^2}\right\} + \frac{D\left[h\rho + cI_k\left(1-r\right)\right]}{2}, \quad (11)$$

$$TVC''_{12}(T) = \frac{2A + DM_1^2\left[cI_k\left(1-r\right) - sI_e\right]}{T^3}, \quad (12)$$

$$TVC'_{13}(T) = -\frac{A}{T^2} + \frac{D\left(h\rho + sI_e\right)}{2}, \quad (13)$$

$$TVC''_{13}(T) = \frac{2A}{T^3} \quad > \quad 0, \quad (14)$$

$$TVC'_{21}(T) = -\left[\frac{2A - M_2^2\left(cI_kP\rho + sI_eD\right)}{2T^2}\right] + \frac{D\rho\left(h + cI_k\right)}{2}, \quad (15)$$

$$TVC''_{21}(T) = \frac{2A - M_2^2\left(cI_k\rho P + sI_eD\right)}{T^3} = \frac{2A - cM_2^2PI_k + DM_2^2\left(cI_k - sI_e\right)}{T^3}, \quad (16)$$

$$TVC'_{22}(T) = -\left[\frac{2A + DM_2^2\left(cI_k - sI_e\right)}{2T^2}\right] + \frac{D\left(h\rho + cI_k\right)}{2}, \quad (17)$$

$$TVC''_{22}(T) = \frac{2A + DM_2^2\left(cI_k - sI_e\right)}{T^3}, \quad (18)$$

$$TVC'_{23}(T) = -\frac{A}{T^2} + \frac{D\left(h\rho + sI_e\right)}{2} \quad (19)$$

and

$$TVC''_{23}(T) = \frac{2A}{T^3} > 0. \tag{20}$$

Equations (14) and (20) imply that both $TVC_{13}(T)$ and $TVC_{23}(T)$ are convex on $T > 0$. However, equation (10) implies that $TVC_{11}(T)$ is convex on $T > 0$ if $2A - cM_1^2 PI_k (1-r) + DM_1^2 [cI_k (1-r) - sI_e] > 0$; equation (12) implies that $TVC_{12}(T)$ is convex on $T > 0$ if $2A + DM_1^2 [cI_k (1-r) - sI_e] > 0$; equation (16) implies that $TVC_{21}(T)$ is convex on $T > 0$ if $2A + DM_2^2 (cI_k - sI_e) - cM_2^2 PI_k > 0$ and equation (18) implies that $TVC_{22}(T)$ is convex on $T > 0$ if $2A + DM_2^2 (cI_k - sI_e) > 0$. Furthermore, we have

$$TVC'_{11}(\frac{PM_1}{D}) = TVC'_{12}(\frac{PM_1}{D}), \quad TVC'_{12}(M_1) = TVC'_{13}(M_1),$$
$$TVC'_{21}(\frac{PM_2}{D}) = TVC'_{22}(\frac{PM_2}{D})$$

and $TVC'_{22}(M_2) = TVC'_{23}(M_2)$.

3. Decision Rule of the Optimal Cycle Time T^*

The main purpose of this section is to develop a solution procedure to determine the optimal cycle time T^*.

Let $TVC'_{ij}(T) = 0$, for all $i = 1 \sim 2$ and $j = 1 \sim 3$. Then we can obtain

$$T^*_{11} = \sqrt{\frac{2A - cM_1^2 PI_k (1-r) + DM_1^2 [cI_k (1-r) - sI_e]}{D\rho [h + cI_k (1-r)]}}$$
$$\text{if } 2A - cM_1^2 PI_k (1-r) + DM_1^2 [cI_k (1-r) - sI_e] > 0, \tag{21}$$

$$T^*_{12} = \sqrt{\frac{2A + DM_1^2 [cI_k (1-r) - sI_e]}{D [h\rho + cI_k (1-r)]}} \quad \text{if } 2A + DM_1^2 [cI_k (1-r) - sI_e] > 0, \tag{22}$$

$$T^*_{13} = \sqrt{\frac{2A}{D (h\rho + sI_e)}}, \tag{23}$$

$$T^*_{21} = \sqrt{\frac{2A + DM_2^2 (cI_k - sI_e) - cM_2^2 PI_k}{D\rho (h + cI_k)}} \quad \text{if } 2A + DM_2^2 (cI_k - sI_e) - cM_2^2 PI_k > 0, \tag{24}$$

$$T^*_{22} = \sqrt{\frac{2A + DM_2^2 (cI_k - sI_e)}{D (h\rho + cI_k)}} \quad \text{if } 2A + DM_2^2 (cI_k - sI_e) > 0 \tag{25}$$

and

$$T^*_{23} = \sqrt{\frac{2A}{D (h\rho + sI_e)}}. \tag{26}$$

From equation (21) the optimal value of T for the case of $T \geq PM_1/D$ is $T^*_{11} \geq PM_1/D$. We can substitute equation (21) into $T^*_{11} \geq PM_1/D$ to obtain the optimal value of T

$$\text{if and only if } -2A + \frac{M_1^2}{D} \left[cI_k (1-r) \left(P^2 - D^2\right) + sI_e D^2 + hP (P-D)\right] \leq 0.$$

Similar disscussion, we can obain following results:

$M_1 \leq T^*_{12} \leq PM_1/D$
if and only if $-2A + \dfrac{M_1^2}{D}\left[cI_k(1-r)\left(P^2 - D^2\right) + sI_eD^2 + hP(P-D)\right] \ \geq \ 0$ and
if and only if $-2A + DM_1^2(h\rho + sI_e) \ \leq \ 0$.

$T^*_{13} \leq M_1$ if and only if $-2A + DM_1^2(h\rho + sI_e) \ \geq \ 0$.

$T^*_{21} \geq PM_2/D$ if and only if $-2A + \dfrac{M_2^2}{D}\left[cI_k\left(P^2 - D^2\right) + sI_eD^2 + hP(P-D)\right] \ \leq \ 0$.

$M_2 \leq T^*_{22} \leq PM_2/D$
if and only if $-2A + \dfrac{M_2^2}{D}\left[cI_k\left(P^2 - D^2\right) + sI_eD^2 + hP(P-D)\right] \ \geq \ 0$ and
if and only if $-2A + DM_2^2(h\rho + sI_e) \ \leq \ 0$.

$T^*_{23} \leq M_2$ if and only if $-2A + DM_2^2(h\rho + sI_e) \ \geq \ 0$.

Let

$$\Delta_1 = -2A + \frac{M_1^2}{D}\left[cI_k(1-r)\left(P^2 - D^2\right) + sI_eD^2 + hP(P-D)\right], \tag{27}$$

$$\Delta_2 = -2A + DM_1^2(h\rho + sI_e), \tag{28}$$

$$\Delta_3 = -2A + \frac{M_2^2}{D}\left[cI_k\left(P^2 - D^2\right) + sI_eD^2 + hP(P-D)\right] \tag{29}$$

and

$$\Delta_4 = -2A + DM_2^2(h\rho + sI_e). \tag{30}$$

From equations (27)–(30), we can obtain $\Delta_3 > \Delta_1 > \Delta_2$ and $\Delta_3 > \Delta_4 > \Delta_2$ since $M_2 > M_1$. Summarized above arguments, we can obtain following results.

Theorem 1 :

(A) If $\Delta_2 \ \geq \ 0$, then $TVC(T^*) = \min\{TVC_1(T^*_{13}),\ TVC_2(T^*_{23})\}$. Hence T^* is T^*_{13} or T^*_{23} associated with the least cost.
(B) If $\Delta_1 \ \geq \ 0$, $\Delta_2 \ < \ 0$ and $\Delta_4 \ \geq \ 0$, then $TVC(T^*) = \min\{TVC_1(T^*_{12}),\ TVC_2(T^*_{23})\}$. Hence T^* is T^*_{12} or T^*_{23} associated with the least cost.
(C) If $\Delta_1 \ \geq \ 0$, $\Delta_2 \ < \ 0$ and $\Delta_4 \ < \ 0$, then $TVC(T^*) = \min\{TVC_1(T^*_{12}),\ TVC_2(T^*_{22})\}$. Hence T^* is T^*_{12} or T^*_{22} associated with the least cost.
(D) If $\Delta_1 \ < \ 0$ and $\Delta_4 \ \geq \ 0$, then $TVC(T^*) = \min\{TVC_1(T^*_{11}),\ TVC_2(T^*_{23})\}$. Hence T^* is T^*_{11} or T^*_{23} associated with the least cost.
(E) If $\Delta_1 \ < \ 0$, $\Delta_3 \ > \ 0$ and $\Delta_4 \ < \ 0$, then $TVC(T^*) = \min\{TVC_1(T^*_{11}),\ TVC_2(T^*_{22})\}$. Hence $T^*$is T^*_{11} or T^*_{22} associated with the least cost.
(F) If $\Delta_3 \ \leq \ 0$, then $TVC(T^*) = \min\{TVC_1(T^*_{11}),\ TVC_2(T^*_{21})\}$. Hence T^* is T^*_{11} or T^*_{21} associated with the least cost.

Theorem 1 immediately determines the optimal cycle time T^* after computing the numbers Δ_1, Δ_2, Δ_3 and Δ_4. Theorem 1 is really very simple.

4. Special Cases

In this section, we want to deduce some previously published models as special cases.

(I) Huang and Chung's [4] model

When $P \to \infty$ and $s = c$, it means that the items instantaneously receive and the retailer's unit selling price and the unit purchasing price are equal. Let

$$\overline{TVC}_{11}(T) = \frac{A}{T} + \frac{DTh}{2} + c(1-r)D + \frac{c(1-r)I_kD(T-M_1)^2}{2T} - \frac{cI_eDM_1^2}{2T},$$

$$\overline{TVC}_{12}(T) = \frac{A}{T} + \frac{DTh}{2} + c(1-r)D - DcI_e(M_1 - \frac{T}{2}),$$

$$\overline{TVC}_{21}(T) = \frac{A}{T} + \frac{DTh}{2} + cD + \frac{cI_kD(T-M_2)^2}{2T} - \frac{cI_eDM_2^2}{2T}$$

and

$$\overline{TVC}_{22}(T) = \frac{A}{T} + \frac{DTh}{2} + cD - DcI_e(M_2 - \frac{T}{2}).$$

Equations (1.1–1.3) and (5.1–5.3) will be reduced as follows:

$$TVC_1(T) = \begin{cases} \overline{TVC}_{11}(T) & \text{if } M_1 \le T & (31.1) \\ \overline{TVC}_{12}(T) & \text{if } 0 < T \le M_1 & (31.2) \end{cases}$$

and

$$TVC_2(T) = \begin{cases} \overline{TVC}_{21}(T) & \text{if } M_2 \le T & (32.1) \\ \overline{TVC}_{22}(T) & \text{if } 0 < T \le M_2. & (32.2) \end{cases}$$

Equations (31.1–31.2) and (32.1–32.2) will be consistent with equations 1(a, b) and 4(a, b) in Huang and Chung [4], respectively. Hence, Huang and Chung [4] will be a special case of this paper.

(II) Chung and Huang's [2] model

When $r = M_1 = 0$, $M_2 = M$ and $s = c$, it means that the cash discount policy does not offered and the retailer's unit selling price and the unit purchasing price are equal. Let

$$TVC_4(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cI_k\rho(\frac{DT^2}{2} - \frac{PM^2}{2})/T - cI_e(\frac{DM^2}{2})/T,$$

$$TVC_5(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cI_k[\frac{D(T-M)^2}{2}]/T - cI_e(\frac{DM^2}{2})/T$$

and

$$TVC_6(T) = \frac{A}{T} + \frac{DTh\rho}{2} - cI_e[\frac{DT^2}{2} + DT(M-T)]/T.$$

Equations (1.1–1.3) and (5.1–5.3) will be reduced as follows:

$$TVC(T) = \begin{cases} TVC_4(T) & \text{if } T \ge \frac{PM}{D} & (33.1) \\ TVC_5(T) & \text{if } M \le T \le \frac{PM}{D} & (33.2) \\ TVC_6(T) & \text{if } 0 < T \le M. & (33.3) \end{cases}$$

Equations (33.1–33.3) will be consistent with equations 6(a, b, c) in Chung and Huang [2], respectively. Hence, Chung and Huang [2] will be a special case of this paper.

(III) Teng's [7] model

When $P \to \infty$, $r = M_1$=0 and$M_2 = M$, it means that the items instantaneously receive and the cash discount policy does not offered. Let

$$TVC_7(T) = \frac{A}{T} + \frac{DTh}{2} + cI_k[\frac{D(T-M)^2}{2}]/T - sI_e(\frac{DM^2}{2})/T$$

and

$$TVC_8(T) = \frac{A}{T} + \frac{DTh}{2} - sI_e[\frac{DT^2}{2} + DT(M-T)]/T.$$

Equations (1.1–1.3) and (5.1–5.3) will be reduced as follows:

$$TVC(T) \quad = \quad \begin{cases} TVC_7(T) & \text{if} \quad M \le T & (34.1) \\ TVC_8(T) & \text{if} \quad 0 < T \le M & (34.2) \end{cases}$$

Equations (34.1–34.2) will be consistent with equations (1) and (2) in Teng [7], respectively. Hence, Teng [7] will be a special case of this paper.

5. Numerical Examples

To illustrate the results, let us apply the proposed method to solve the following numerical examples. From Table 1 and Table 2, we can observe the optimal cycle time with various parameters of s and M_1, respectively. The following inferences can be made based on Table 1 and Table 2.

(1) The larger the value of s is, the smaller value of the optimal cycle time and the lower value of the annual total relevant cost will be. Table 1 shows the computed results.

(2) The larger the value of M_1 is, the smaller value of the optimal cycle time and the lower value of the annual total relevant cost will be. Table 2 shows the computed results.

Table 1: Optimal cycle time with various value of s

Let A = \$150/order, D = 500units/year, c=\$50/unit, P=800units/year, r=0.1, h=\$20/unit/year, I_e = \$0.15/\$/year, I_k=\$0.2/\$/yaer, M_1 = 0.1 year, M_2 = 0.15 year.

s(\$/unit)	Δ_1	Δ_2	Δ_3	Δ_4	Theorem 1	T^*	$TVC\,(T^*)$
100	< 0	< 0	> 0	< 0	E	$T^*_{11} = 0.191$	23538
150	< 0	< 0	> 0	> 0	D	$T^*_{11} = 0.172$	23496
200	> 0	< 0	> 0	> 0	B	$T^*_{12} = 0.154$	23318
250	> 0	< 0	> 0	> 0	B	$T^*_{12} = 0.138$	23190

Table 2: Optimal cycle time with various value of M_1

Let A=\$150/order, D=800units/year, c=\$80/unit, P=900units/year, r=0.1, s=\$100/unit, h=\$10/unit/year, I_e=\$0.1/\$/year, I_k=\$0.15/\$/yaer, M_2=0.4 year.							
M_1	Δ_1	Δ_2	Δ_3	Δ_4	Theorem 1	T^*	$TVC\,(T^*)$
0.05	< 0	< 0	> 0	> 0	D	$T^*_{11} = 0.387$	58316
0.1	< 0	< 0	> 0	> 0	D	$T^*_{11} = 0.336$	58222
0.15	< 0	< 0	> 0	> 0	D	$T^*_{11} = 0.228$	58020
0.2	> 0	> 0	> 0	> 0	A	$T^*_{13} = 0.184$	57633

6. Conclusions

The supplier offers the trade credit policy to stimulate the demand of the retailer. However, the supplier can also use the cash discount policy to attract retailer to pay the full payment of the amount of purchasing cost to shorten the collection period. This paper investigates the retailer's replenishment policy with noninstantaneous receipt under trade credit and cash discount and assumes that the retailer's unit selling price and the purchasing price per unit are not necessarily equal. The major contribution of this paper is provided a very efficient solution procedure to help the decision-maker to quickly determine the optimal replenishment policy. In addition, some previously published results of other researchers are deduced as special cases. Finally, numerical examples are given to illustrate the results. There are some managerial phenomena as follows:

(1) The retailer will order less quantity to take the benefits of the delay payments more frequently when the larger the differences between the unit selling price per item and the unit purchasing price per item.

(2) The retailer will order less quantity to take the benefits of the cash discount more frequently when the smaller the differences between the period of cash discount and the period of trade credit.

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