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ANALYSIS AND OPTIMIZATION FOR AUTOMATED VEHICLE ROUTING ON A SINGLE LOOP

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Abstract This paper treats with an automated material handling system called a permutation circulation-type vehicle routing system (denoted PCVRS). In the PCVRS a fleet of vehicles unidirectionally and repeatedly circulate on a single loop to carry items to stations located along by the loop where items are served. No passing is allowed between vehicles on the loop. This may induce interferences or blocking between vehicles which may deteriorate the performance of the PCVRS. One of the most serious interferences is the one lap behind (denoted OLB) interference which occurs when the first vehicle is interfered by the last vehicle in a fleet of vehicles. Once the OLB interference occurs, the system can not reach the steady state in which no interference occur. This paper theoretically analyzes the steady state with no interference and the transient state with interferences including the OLB one. This paper considers both the infinite acceleration and deceleration and the finite ones on the vehicles, and four vehicle routing rules by which each job (and each vehicle) is allocated to a processing station for service. Two of them are existing ones and the other two are newly introduced to improve the existing ones. This paper adopts the throughput and the mean interference time for evaluating the vehicle routing rules. This paper confirms the theoretically obtained results by means of numerical simulation.

Keywords: Transportation, automated material handling system, interference, vehicle routing, throughput

1. Introduction

This paper treats with an automated material handling system (MHS) referred to as a permutation circulation-type vehicle routing system (denoted PCVRS). It consists of a fleet of (robotic) vehicles, multiple stations including an input/output one and a guide path of a single loop (see Figure 1). Each vehicle unidirectionally and repeatedly circulates on the loop to serve a job on stations located along the loop. In the PCVRS no passing is allowed between vehicles on the loop (from which the system is named permutation circulation-type). We can find such PCVRSs in many real MHSs in automated storage and retrieval systems (AS/RSs) and flexible manufacturing systems (FMSs) among others. The main reason for practical use of the PCVRS is the simplicity in design and control [8]. An important issue of the PCVRS is the interference (or blocking) between vehicles which occurs when a vehicle is obliged to stop by the preceding one for collision-avoidance. Such interference induces wasteful waiting time and wasteful energy consumptions by deceleration and acceleration, and may deteriorate system performances. Thus, many previous papers on the PCVRS avoid the interference by taking sufficient distance between vehicles. However, optimal vehicle routing we consider does not always mean the same distance between vehicles as assumed in many previous papers and a PCVRS which guarantees interference-free may not be realistic when the number of vehicles is increased under a fixed loop length in order to increase the throughput. For example, in big automated warehouses (by which our study was motivated) a lot of items simultaneously supplied by big tracks from the outside have to be stored in short time (e.g. a couple of hours) and a lot of items stored have to be shipped out in very short time. In these situations high throughput is desired even if interferences are unavoidable. The objective of this paper is to discuss vehicle routing rules for minimizing the interference and maximizing the throughput. A vehicle routing rule decides a station on which each vehicle in each lap serves.

This paper is organized as follows. Chapter 2 briefly reviews the previous research related to the PCVRS. Chapter 3 describes basic assumptions for " parallel and bottleneck-free " PCVRS. It discusses the dynamics of the PCVRS with infinite acceleration and deceleration of vehicles and introduces the " steady state " and the " one lap behind " (denoted OLB) interference, and introduces the " throughput " and the " mean interference time " as performance measures. Chapter 4 explains two basic vehicle routing rules (Random rule and Order rule) introduced in literature and applies the results in Section 3 to them. Chapter 5 introduces two vehicle routing rules (" E-order " rule and " D-order " rule) and shows that the E-order rule is optimal in the steady state, but the D-order rule is better than the E-order rule under the OLB interference. Chapter 6 discusses the PCVRS with finite acceleration and deceleration of vehicles, and introduces the " virtual stop " for the acceleration/deceleration by which results in the infinite case can approximately be applied. Chapter 7 reports results obtained by simulation and shows that they confirm the theoretical results. Chapter 8 is a concluding remark.

2. Literature Review

There are many design, planning and control problems to be solved for guided vehicle routing systems (denoted GVRS), that is, (1) guide path layout and location of pickup/delivery points, (2) scheduling and dispatching of vehicles, (3) traffic control and vehicle routing, and (4) determination of the fleet size (the number of vehicles). The better these problems are resolved, the more efficient system operates, and this leads to a closer realization of the system operational goals (Egebelu (1993) [5]). Thus, a number of researches on GVRSs have been published, especially in manufacturing systems such as FMSs and automated distribution centers such as AS/RSs (see excellent reviews of Ganesharajab, et al, 1998 [7] on FMS and Rowenhorst, et al, 2000 [14] and Van de Berg(1999) [17] on AS/RS). Nevertheless, there have been not many papers on the PCVRS. As far as we know, only the following papers are closely related to the PCVRS.

Bartholdi and Platzman (1988) [1] analyzed First-Encounted-First-Served (FEFS) dispatching rule for a PCVRS with a single vehicle and extended to the one with multiple vehicles, in which a vehicle does not stop on the first encounted request if the stop induces an interference. Bozer and Srinivasan (1991)[3] proposed to divide a guide path of network type into multiple disjoint single loops, on each of which only a single vehicle can travel. Any item can be moved between arbitrary stations on different loop by changing vehicles and following the FEFS rule. Needless to say, no interference occurs in this GVRS. Blazewicz, et al (1991) [2] treated with a real FMS which consists of m parallel machines and $k(\leq m)$ AGVs traveling on a single loop without passing. This is a special case of our PCVRS. They consider a scheduling problem which asks to schedule $n(n > m)$ jobs and k vehicles simultaneously such that the whole jobs are processed in the minimum time (or the maximum throughput). The assumption $n > m > k$ can make vehicles congestion free by taking sufficient start interval between vehicles. Our PCVRS can afford to have more vehicles than stations (i.e.,$k > m$) for increasing the throughput, but is exposed to danger of interference. Sinrich and Tanchoco (1993) [16] addressed the problem of designing an optimal single loop in a PCVRS with a single vehicle, i.e., the one of finding a loop that passes through all the stations and that minimizes the total time the vehicle has to travel to complete its assignments. They argued that empty vehicle traffic has a small impact on the single guided path than on more general network. Egbelu (1993) [5] first addressed the problem of determining the home position of m vehicles in a PCVRS. The home position is a location where a vehicle stays while being idle. He showed that the problem of minimizing the maximum response time (i.e., the maximum travel time to the request station from its nearest home position) is solvable in the case of the single vehicle. Gademann and van de Velde (2000) [6] extended it into a multi-vehicle case and strengthened the Egbelu's result by using a dynamic programming algorithm. These problems avoid interference by setting a side path near each station or by setting a consecutive and disjoint territory on the loop for each vehicle to response to a request. Furthermore, these problems are static, i.e., every vehicle is idle when a request occurs. Miyamoto, et al(1995) [13] discussed a physical distribution problem for a PCVRS. The objective is to assign each request for traveling and to determine its route in real time so as to avoid interference between vehicles. They formulated the problem as a constraint satisfaction problem, and developed a knowledge processing system based on PROLOG for solving the problem. They adopted a strategy which decreases the number of vehicles by one, every time an interference occurs. Subuncouglu, et al (1998) [15] treated with an FMS in which the transportation system is a PCVRS, and studied, through simulation, sensitivity analysis of vehicle priority schemes as well as scheduling rules. Kim et al (2000) [9] also treated with an FMS similar to the Sabuncouglu, et al's one and evaluated several machine dispatching rules and vehicle allocation rules by means of simulation. In both FMSs a part is sequentially processed on more than one machines, thus interferences may occur, but no explicit analysis on the interference is given. We realized after reviewing previous researches that very little basic research has been devoted to the interference in PCVRS we consider.

Lu et al (2001) [10] first discussed the PCVRS we consider. They developed a new simulation system for the PCVRS and theoretically and empirically analyzed two basic vehicle routing rules (respectively referred to as Random and Order in this paper). Lu et al (2002) [11] discussed the PCVRS with different job processing times. They discussed two types of interferences (respectively referred to as Serial and Parallel in this paper) and empirically analyzed some scheduling rules for releasing jobs into the PCVRS. Lu, Hu and Kise (2003) [12] developed a new vehicle routing rules (referred to as Exchange Order in this paper) for the PCVRS which much improves the Order rule to increase the throughput of the PCVRS.

3. Basic Assumptions and Basic Analysis

This section provides some basic assumptions on the PCVRS and basic analysis on the dynamics of the PCVRS.

3.1. Parallel and bottleneck-free PCVRS

As shown in Figure 1, the PCVRS consists of a loop of length L, a set of n_V identical vehicles $V = \{V_1, V_2, \ldots, V_{n_V}\}$ (the vehicles are indexed in the order of their circulations), a single input/output station (denoted I/O or S_0) where items are loaded or unloaded by vehicles, a set of n_S processing stations $SP = \{S_1, S_2, \ldots, S_{n_S}\}$ (the stations are indexed in the clockwise direction). It is implied by a saving job that a vehicle receives an (unit) item at station I/O, sends it to a certain processing station, unloads it there and then returns

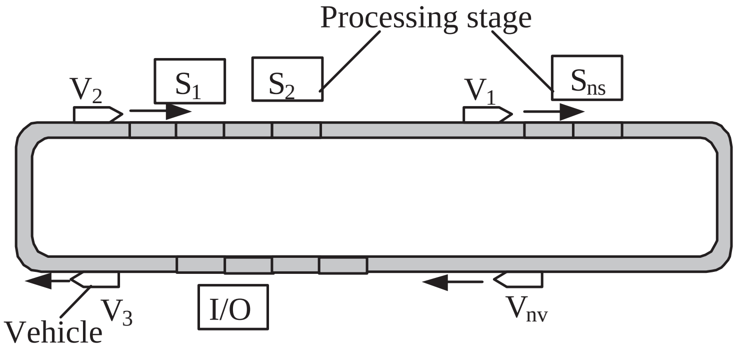

Figure 1: Permutation circulation-type vehicle routing system, PCVRS

to station I/O emptily. It is implied by a retrieval job that an empty vehicle goes to a certain processing station, loads an item there, gets back to station I/O and output it there. This exclusiveness of these two jobs is more effective than a job which includes both saving and retrieving operations for avoiding interferences with a big fleet of vehicles, and hence is found in many real systems such as AS/RS. In the following we assume only the saving jobs, but the results obtained hold in the retrieving jobs. There is a set of n_J (saving) jobs, $J = \{J_1, J_2, \ldots, J_{n_J}\}$ to be served in a given planning horizon, where the jobs are indexed according to the input order at station I/O. In the following we assume that every vehicle serves a job in each lap, i.e., no empty circulation is allowed. Let $V_{k(i)}$ be the vehicle serving job J_i, then

$$k(i) = i - \lfloor (i-1)/n_V \rfloor n_V, i = 1, 2, \ldots, n_J \tag{3.1}$$

where $\lfloor x \rfloor$ stands for the largest integer not greater than x [10]. We assume that all processing stations (excluding I/O) can equally serve any job to keep the load balance. This means a parallel system which could, in general, give the highest system efficiency and the highest system reliability, and means that processing time p_P on any station of SP and p_0 on loading or unloading station are, repectively, constant, i.e., let $p_m(i)$ be the processing time of job J_i on station S_m, then

$$\begin{aligned} p_m(i) = p_P > 0 \quad & m = 1, 2, \ldots, n_S,\ i = 1, 2, \ldots, n_J, \text{ if } S_m \text{ really serves } J_i \\ p_m(i) = p_0 > 0 \quad & m = 0,\ i = 1, 2, \ldots, n_J \\ p_m(i) = 0 \quad & else \end{aligned} \tag{3.2}$$

We assume that station I/O is never bottleneck, meaning that

$$p_0 \leq p_P / n_S \tag{3.3}$$

The constant processing times are realistic in many AS/RSs, when each vehicle carries a unit load. We also assume in the following that the number of jobs is a multiple of the one of vehicles, unless saying otherwise.

3.2. Interferences and steady state

This section assumes that the acceleration and the deceleration of the vehicle are infinite (the finite case is considered in Section 6), and every distance between any two points is measured by time units a vehicle takes to run without stop. It also assumes that every

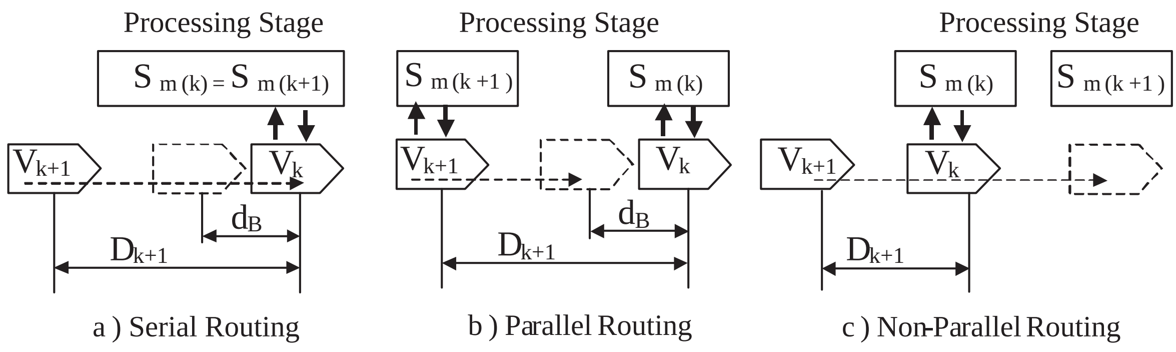

Figure 2: Types of routings

vehicle runs with same and constant speed and hence the corresponding metric distance is obtained by multiplying the speed of vehicle. We employ the so called zone control policy which keeps the distance between two adjacent vehicles at least d_B for collision avoidance and assume that the distance between every two adjacent vehicles is d_B when they leave at the I/O station in the first lap (referred to as the minimum start interval).

Let D_{k+1} be a distance from vehicle V_{k+1} to its preceding vehicle $V_k (k = 1, 2, \ldots, n_V)$, where D_{n_V+1} stands for one from V_1 to V_{n_V} (in the clockwise direction). Obviously,

$$L = \sum_{k=1}^{n_V} D_{k+1} \tag{3.4}$$

and

$$\Delta_{k+1} \equiv D_{k+1} - d_B \geq 0 \tag{3.5}$$

Let $S_{m(k)}$ be a station where V_k serves a job in a certain lap, then routings of two vehicles V_k and V_{k+1} are called serial if $m(k) = m(k+1)$, parallel if $m(k) > m(k+1)$ and non-parallel if $m(k) < m(k+1)$ (see Figure 1 and Figure 2). Note that the routing on station I/O for every vehicle is serial. V_{k+1} is interfered (or blocked) by V_k in non-parallel or serial routing, if V_k stays p time units on a station (e. g., $p = p_P$ on a processing station and $p = p_0$ on the I/O one) and

$$\Delta_{k+1} < p \tag{3.6}$$

then V_{k+1} has to wait behind V_k for time given by

$$W_{k+1} \equiv p - \Delta_{k+1} = p + d_B - D_{k+1} \tag{3.7}$$

In other words V_{k+1} is not blocked if

$$D_{k+1} \geq D^*_{k+1} \equiv p_P + d_B \quad \text{on a processing station} \tag{3.8}$$

$$D_{k+1} \geq D^*_{k+1} \equiv p_0 + d_B \quad \text{on station I/O} \tag{3.9}$$

The interference in the serial routing is referred to as serial and the one in the non-parallel routing to as non-parallel. Note that under the assumption of constant processing times

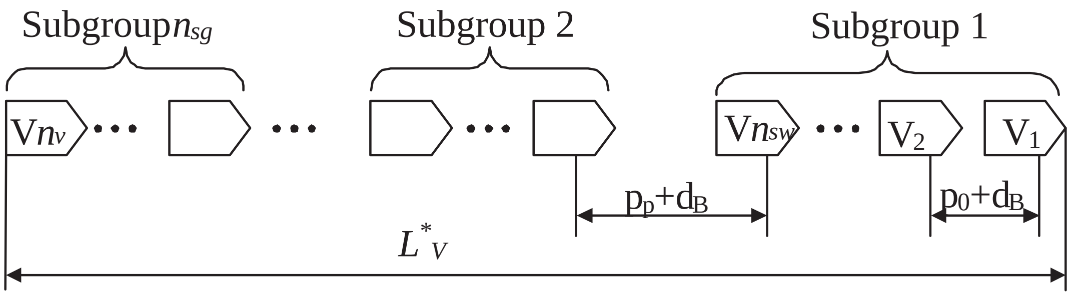

Figure 3: Minimum fleet length MFL in steady state

(3.2) no interference in parallel routing occurs. An interference with a vehicle may be infectious to the successors. Vehicle $V_{k+l}(l = 2, 3, \ldots, n_V - k)$ waits for

$$W_{k+l} = max\{0, W_{k+l-1} - \Delta_{k+l}\} = max\{0, p - \sum_{h=1}^{l} \Delta_{k+h}\} \tag{3.10}$$

where $\Delta_{k+l} = D_{k+l} - d_B$ [10].

LetD'_{k+1} be the distance from V_{k+1} to V_k after their serial or non-parallel services in a certain lap, then

$$D'_{k+1} = max\{p + d_B, D_{k+1}\} \geq D_{k+1} \tag{3.11}$$

Thus V_{k+1} is not blocked on the same station unless V_k is interfered afterward. Let $D_{k+1} = D^*_{k+1}$ is the minimum interference-free (denoted MIF) distance with which V_{k+1} is not blocked by V_k (by (3.8) and (3.9)), and

$$L^*_V \equiv D^*_1 + D^*_2 + \ldots + D^*_{n_V} \tag{3.12}$$

be the minimum fleet length (denoted MFL) of the n_V vehicles which guarantees the interference-free (see (3.4)). The fleet of n_V vehicles with the MFL is divided into n_{sg} subgroups, each having n_{sw} vehicles except the last subgroup as shown in Figure 3 [10]. The distance between two adjacent vehicles within a subgroups is $p_0 + d_B$, and the one between two adjacent subgroups is $p_P + d_B$. n_{sg} and n_{sw} depend on vehicle routing rule used as discussed in the next sections. This fact may eventually lead to a state referred to as (deterministic) steady state in which no interference at any station occurs in the remaining laps (a state before steady state is referred to as transient). This means that the vehicles should circulate in a steady state from the first lap, if possible. However, any steady state may be impossible when the one lap behind interference occurs as discussed in the following.

3.3. One lap behind interference

We say that the one lap behind (denoted OLB)occurs, if V_1 is blocked by V_{n_V} (the last vehicle in the latest lap) with the MFL. That is, the OLB occurs on a processing station, if

$$L - (L^*_V - p) < D^*_{n_V+1} = p_P + d_B \tag{3.13}$$

or on the I/O station if

$$L - (L^*_V - p) < D^*_{n_V+1} = p_0 + d_B \tag{3.14}$$

where L satisfies (3.4) and p stands for processing time of V_1 on station $S_{m(1)}$ on which V_1 serves immediately before the OLB, (i.e.,$p = p_0$ for on station I/O, $p = p_P$ on every processing station). Note that when V_1 is blocked on a station, the MFL is reduced to $L_V^* - p$ by its service on the immediately preceding station. When the vehicles start with the minimum start interval d_B on station I/O , MIF distances (3.8) and (3.9) and the MFL (3.12) are eventually realized by (3.11) unless the OLB interference occurs, but, it does not mean that once a vehicle realizes the MIF distance, the vehicle keeps it in the remaining laps. For example, even if $D_{k+1} = D_{k+1}^*$ is realized in a certain lap, $D_{k+1} < D_{k+1}^*$ occurs when V_k is blocked by its preceding vehicles in following laps (e.g., by infection (3.10)). However, once V_{k+1} takes the MIF distance after every preceding vehicle takes its MIF distance, then V_{k+1} is never disturbed in the remaining laps unless the OLB interference occurs. This means that the vehicles reach a steady state, according to (3.12),(3.13) and (3.3), if and only if

$$L - L_V^* \geq p_P + d_B - p_0 \tag{3.15}$$

otherwise, they suffer from interferences including the OLB one in every lap [11].

3.4. Throughput and mean interference time

One of the most important performance measures for the PCVRS is average throughput rate (simply referred to as the throughput) which is defined by the average number of jobs output from station I/O per unit time. Let F_{max} be time period to process the n_J jobs (i.e., makespan), then the throughput is defined by

$$T_p \equiv n_J / F_{max} \tag{3.16}$$

Let the n_V vehicles take n_J/n_V laps to process n_J jobs, then,

$$F_{max} \geq (n_J/n_V)(L + p_P + p_0)$$

by neglecting the interference time to occur. Thus,

$$T_p \leq T_p^V \equiv \frac{n_V}{L + p_P + p_0} \tag{3.17}$$

T_p^V is a vehicle-based upper bound which linearly increases with n_V [11]. If no OLB interference occurs,

$$\lim_{n_J \to \infty} T_p = T_p^V \tag{3.18}$$

Let L_{n_S} be the time units for a vehicle to move from S_1 to S_{n_S} and consider a situation in which the n_S stations are always busy to serve n_S jobs except $(L_{n_S} + d_B)$ time units to simultaneously exchange the next n_S vehicles which wait behind S_1. This situation makes the throughput maximum, then

$$T_p^S \equiv n_S/(p_P + L_{n_S} + d_B) \tag{3.19}$$

is a station-based upper bound which increases with n_S. Thus

$$UB_{T_p} \equiv min\{T_p^V, T_p^S\} \tag{3.20}$$

is an upper bound of the throughput.

Another important measure is the total flow time (denoted TFT) that is the total sum of traveling time over n_J jobs or equivalently the mean flow time per job(denoted MFT). TFT and MFT are, respectively, given by

$$\begin{aligned} TFT &= (L + p_P + p_0)n_J + TIT \\ MFT &= L + p_P + p_0 + MIT \end{aligned} \tag{3.21}$$

where TIT and MIT, respectively, stand for the total interference time over n_J jobs and the mean interference time per job. Thus to minimize TIT (MIT) is equivalent to minimizing TFT (MFT). So the MIT is used for the evaluation in the following. Let the fleet of the vehicles take the minimum start interval in the first lap, then the distance from V_1 to V_{k+1} is expanded (by interferences) from $(k-1)d_B$ to $(D_2^* + \ldots + D_{k+1}^*)$ when it reaches a steady state (see Figure 3). Then,

$$MIT^* \equiv \frac{\sum_{k=2}^{n_V}(n_V - k + 1)(D_k^* - d_B)}{n_J^*} \tag{3.22}$$

is the MIT with the minimum start interval, where n_J^* stands for the number of jobs with which the steady state is realized. Let n_V^* be the maximum number of vehicles with no OLB interference and T_p^* be the maximum throughput which is realized by n_v^* in a steady state. The following two sections will show that these performance measures strongly depends on vehicle routing rules employed.

4. Two Basic Vehicle Routings

This section analyzes two basic vehicle routing rules which keep the load balance to all processing stations for the parallel and bottleneck-free system to be effective. (see 3.1)

4.1. Random rule

A vehicle routing rule is referred to as Random, if every vehicle serves a job on a processing station at random with the same probability [10]. Every two vehicles take serial and/or non-parallel routing under the Random rule. Then, when a steady state is reached, although it is stochastic, the vehicles are equally apart at interval according to(3.8),

$$D_{k+1}^R \equiv D_{k+1}^* = p_P + d_B, \quad k = 1, 2, \ldots, n_V - 1$$

and the MFL becomes, according to (3.12)

$$L_V^R \equiv L_V^* = (n_V - 1)(p_P + d_B) \tag{4.1}$$

thus, the steady state is realized by (3.14), if

$$L \geq n_V(p_P + d_B) - p_0 \tag{4.2}$$

The maximum number of vehicles satisfying (4.2), the vehicle-based upper bound of the throughput derived from (3.16) are given by

$$n_V^R \equiv n_V^* = \lfloor (L + p_0)/(p_P + d_B) \rfloor \tag{4.3}$$

$$T_p^R \equiv T_p^* = \frac{1}{(L + p_P + p_0)} \lfloor \frac{L + p_0}{p_P + d_B} \rfloor \tag{4.4}$$

and the MIT is given by, according to (3.22) and D_{k+1}^R,

$$MIT^R \equiv MIT^* = \frac{(n_V^R - 1)n_V^R(p_P + d_B)}{2n_J^*} \tag{4.5}$$

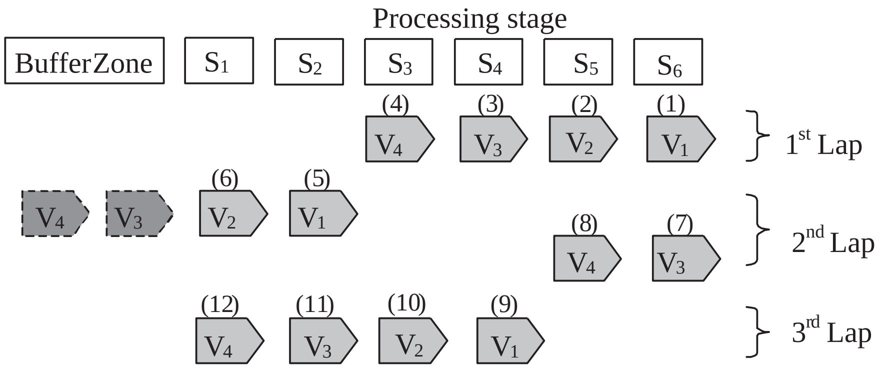

Figure 4: Order rule with $n_S = 6$, $n_V = 4$ and $n_J = 12$

4.2. Order rule

The vehicle routing rule is referred to as the Order rule, if it deterministically and repeatedly allocates vehicles (and jobs) to stations, $S_{n_S}, S_{n_S-1}, \dots, S_1$ in this order [10] (see Figure 1 and Figure 4). Let

$$m(i) = n_S + 1 - i + \lfloor (i-1)/n_S \rfloor n_S, i = 1, 2, \dots, n_J \tag{4.6}$$

then, job J_i is served on station $S_{m(i)}$ by $V_{k(i)}$ (see (3.1)) [10].

Figure 4 illustrates the Order rule with $n_S = 6$, $n_V = 4$, and $n_J = 12$ where the number in a parenthesis on each vehicle stands for the job served by that vehicle. In the 1^{st} lap the 4 vehicles, respectively, serve on stations S_6 to S_3 without interference. In the 2^{nd} lap V_1 and V_2, respectively, serve on stations S_2 and S_1 to keep the load balance on the processing stations. As a result, V_3 and V_4 are blocked behind S_1 and the 4 vehicles are divided into two subgroups;$\{V_1, V_2\}$ and $\{V_3, V_4\}$ (see Figure 3). In the 3^{rd} lap the vehicles serve on S_4 to S_1 to keep the load balance.

An outstanding feature of the Order rule is that no interference occurs in any station but S_1 and I/O under constant processing times (3.2). Let $g \equiv \gcd(n_S, n_V)$ be the greatest common divisor of n_S and n_V, then the Order rule divides the set of n_V vehicles into $a \equiv n_{sg} = n_V/g$ sub-groups $G_1, G_2, \dots, G_a$ under assumptions (3.2) and (3.3) so that each sub-group has $n_{sw} = g$ vehicles in Figure 3 (see [10] for the proof). Let $V_{p,q}$ be the q-th vehicle of the p-th sub-group ($p = 1, 2, \dots, a, q = 1, 2, \dots, g$), then, MIF distance (3.9) from $V_{p,q+1}$ to $V_{p,q}$ and the one (3.8) from $V_{p+1,1}$ to $V_{p,g}$ are, respectively, given by

$$D^O_{p,q+1} \equiv D^*_{p,q+1} = p_0 + d_B, \quad D^O_{p+1,1} \equiv D^*_{p+1,1} = p_P + d_B,$$

thus, MFL (3.12) becomes

$$\begin{aligned} L^O_V(n_V, g) \equiv L^*_V &= \sum_{p=1}^{a-1}\Big(\sum_{q=1}^{g-1} D^O_{p,q+1} + D^O_{p+1,1}\Big) + \sum_{q=1}^{g-1} D^O_{a,q+1} \\ &= \frac{n_V(g-1)p_0 + (n_V - g)p_P + (n_V - 1)gd_B}{g} \end{aligned} \tag{4.7}$$

and condition of the steady state (3.15) is satisfied, if

$$L \geq L^O_V(n_V, g) + p_P + d_B - p_0 = n_V\{(1 - \frac{1}{g})p_0 + \frac{1}{g}p_P + d_B\} - p_0 \tag{4.8}$$

The throughput (3.17) leads to

$$T_p^O(n_V, g) \equiv T_p^V = \frac{1}{(L + p_P + p_0)} \lfloor \frac{L + p_0}{(1 - 1/g)p_0 + p_P/g + d_B} \rfloor \tag{4.9}$$

if (4.8) is satisfied. The mean interference time (3.22) satisfying (4.8) is given by, according to $D^O_{p,q+1}$ and $D^O_{p+1,1}$,

$$MIT^O(n_V, g) \equiv MIT^* = \frac{(n_V/2)[(p_P + d_B)(n_V/g - 1) + p_0 n_V(1 - 1/g)]}{n_J^*} \tag{4.10}$$

These results mean that the Order rule depends on g=gcd(n_s, n_V) and hence the throughput does not always linearly increase with n_V. T_p^* and MIT^* of the Order rule is better than the ones of the Random rule except $g = 1$ where both have the same values. This dependence of the Order rule is overcome in new vehicle routing rules as introduced in the next section.

5. Optimal Vehicle Rules

This section shows two vehicle routing rules: one routing rule (referred to as Exchange-Order and denoted E-Order) is optimal as long as a steady state is realized, but not optimal when the OLB interference is inevitable. The E-Order rule refines the C-order rule introduced in [12]. The other is the Dynamic-Order rule (denoted D-Order) which is same as the E-order rule under the steady state and better under the OLB interference.

5.1. Exchange-order rule

Let the Order rule be applied to the first $(l-1)(l = 1, 2, \ldots)$ laps and the last vehicle, V_{n_V} serve job $J_{(l-1)n_V}$ on station S_{u+1}, then u is derived from (3.1) and (4.6).

$$u = \{\lfloor \frac{(l-1)n_V - 1}{n_S} \rfloor + 1\} n_S - (l-1)n_V \tag{5.1}$$

In the l-th lap V_1 serves on S_u (S_{n_V}, if $u = 0$) and V_{n_V} to S_w, where

$$w = \{\lfloor \frac{(ln_V - 1)}{n_S} \rfloor + 1\} n_S + 1 - ln_V \tag{5.2}$$

One of the following three exclusive conditions to keep the load balance holds.

1)$u < w - 1$: the number of vehicles allocated to stations $S_m(m = u+1, u+2, \ldots, w-1$ is less than the one to each of the remaining stations by one.

2)$u = w - 1$: each station has the same number of vehicles allocated (when n_V is a multiple of n_S)

3)$u > w - 1$: the number of vehicles allocated to stations $S_m(m = w, w+1, \ldots, u)$ is larger than the one to each of the remaining stations by one.

The E-Order rule uses the following two sub-rules in l-th lap and keeps the load balance as the Order rule does, although it may take different routing rule (the E-order rule is, of course, applied from scratch $(l = 1)$in the execution):

Changing rule: the Order rule is applied to a subset of n_V vehicles from station S_{n_S}. The Changing rule is optimal for the specified subset, because it gives no interference except unavoidable ones (e.g., if $n_V > n_S$, $(n_V - n_S)$ vehicles are blocked behind S_1 in any rule).

Unchanging rule: the Order rule is applied to a subset of n_V vehicles from S_u. The Unchanging rule is optimal, if u is not smaller than the number of the vehicles in the specified subset, because it gives no interference and is the Changing rule, if $u = n_S$.

The E-Order rule uses these two rules depending on the sign of $(u - v)$ as follows.

4) $u \leq w(u < n_V, n_S)$: The Changing rule is applied to the first $(n_V - u)$ vehicles, $V_1, V_2, \ldots, V_{n_V - u}$ and then the Unchanging rule is applied to the last u vehicles,V_{n_V-u+1}, $\ldots, V_{n_V}$. It is easily seen that V_{n_V-u} serves on S_w and V_{n_V-u+1} on S_u, thus the above conditions 1) and 2) are satisfied.

5) $u > w(u < n_V, n_S)$: The Changing rule is applied to the first $(n_V - u + w - 1)$ vehicles and then the Unchanging rule is applied to the last $(u - w + 1)$ vehicles, $V_{n_V-u+w}, \ldots, V_{n_V}$.

It holds by (5.1) and (5.2) that

$$n_v - u + w - 1 = \{\lfloor (l^{n_V} - 1)/n_S \rfloor - \lfloor ((l-1)n_V - 1)/n_S \rfloor\} n_S$$

is a multiple of n_S, i.e., each station serves the same number of jobs by the first$(n_V - u + w - 1)$ vehicles and the routing of the last$(u - w + 1)$ vehicles which starts from S_u satisfies the above condition 3), thus the E-order rule keeps the load balance as the Order rule does. The detail of the E-Order rule is as follows:

Algorithm E-Order

Step 1: $l \leftarrow 0, \ell_{max} \leftarrow \lceil n_J/n_V \rceil$

Step 2: $l \leftarrow l + 1$. If $l > l_{max}$, halt. If $l = l_{max}$ and $n_J < l_{max} n_V$, then $n_V \leftarrow n_J - (l_{max} - 1)n_V$, and go to Step 3.

Step 3: Compute u by (5.1).

Step 4: If $u \geq n_V$, or if $u = n_S$, apply the Unchanging rule to all n_V vehicles, and go to Step 2, otherwise, go to Step 5.

Step 5: Compute w by (5.2). If $u < w$, go to Step 6, otherwise, go to Step 7.

Step 6: Apply the Changing rule to the first $(n_V - u)$ vehicles, and apply the Unchanging rule to the last u vehicles,$V_{n_V-u+1}, \ldots, V_{n_V}$. Go to Step 2.

Step 7: Apply the Changing rule to the first $(n_V - u + w - 1)$ vehicles, V_1, …, $V_{n_V-u+w-1}$, and the Unchanging rule to the last $(u - w + 1)$ Vehicles, V_{n_V-u+w}, …, V_{n_V}. Go to Step 2.

Figure 5 illustrates the E-Order rule with the same condition as in Figure 4 to compare with the Order rule. In the 1^{st} lap the vehicles take the same routing as the Order rule by Step 4 ($u = 6 = n_S$). In the 2^{nd} lap V_1 and V_2 serve on S_6 and S_5, and V_3 and V_4 on S_2 and S_1, respectively by Step 6 ($u = 2 < n_S$ in Step 4 and $u < w = 5$ in Step 5). This routing is different from the one by the Order rule in Figure 4, but keep the same load balance as the Order rule does. As a result, no interference occurs. In the 3^{rd} lap the same routing as the one by the Order rule is taken by Step 4 ($u = 4 = n_V$ in Step 4).

It is easily seen that the E-order rule separates the n_V vehicles into $\lceil n_V/n_S \rceil$ sub-groups, each of the first $\lfloor n_V/n_S \rfloor$ groups consists of n_S vehicles and the last one has $(n_V - \lfloor n_V/n_S \rfloor n_S)$ vehicles in a steady state. Thus, MFL (3.12) is given by

$$\begin{aligned} L_V^E(n_V) \equiv L_V^* = (p_P + d_B)(\lceil n_V/n_S \rceil - 1) \\ +(p_0 + d_B)\{(n_S - 1)\lfloor n_V/n_S \rfloor + max(0, n_V - \lfloor n_V/n_S \rfloor ns - 1)\} \end{aligned} \tag{5.3}$$

Note that the max part of the right hand of equation takes 0, if n_V is a multiple of n_s, takes 1 otherwise. The steady state is realized by (3.15), if

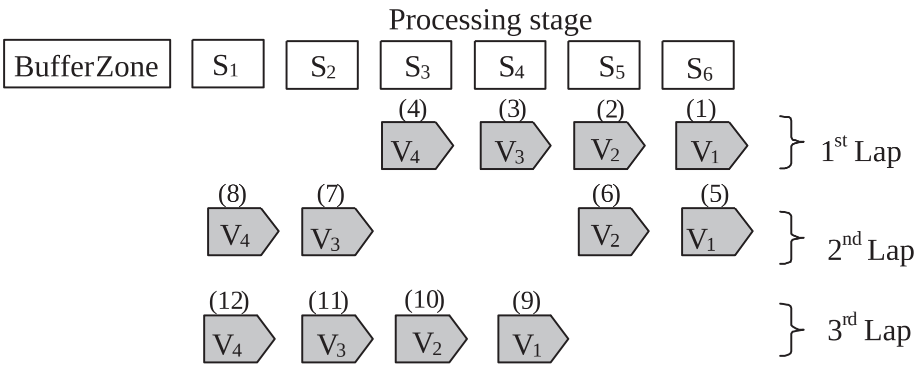

Figure 5: E-order rule with $n_S = 6$, $n_V = 4$ and $n_J = 12$

$$\begin{aligned} L &\geq L_V^E(n_V) + p_P + d_B - p_0 \\ &= qp_P + \{max(qn_S, n_V - 1) - q\}p_0 + max(qn_S, n_V - 1)d_B \end{aligned} \tag{5.4}$$

where $q = \lceil n_V/n_S \rceil - 1$. It is easily seen that$L_V^E$ does not depend on g=gcd(n_V, n_S) and by (4.7) and (5.3) that, $L_V^E \leq L_V^0$. This means by (3.17), (3.18) and (4.7) that

$$T_p^E(n_V) \geq T_p^0(n_V, g) \tag{5.5}$$

holds under the steady state. MIT (3.22) for the E-order rule is given by, according to (5.3),

$$MIT^E(n_V) \equiv MIT^* = \frac{q\{n_V - (q+1)n_S/2\}p_P + (n_V - q)p_0}{n_J} \tag{5.6}$$

It is also easily seen by (4.10) that

$$MIT^E(n_V) \leq MIT^0(n_V, g) \tag{5.7}$$

holds under the steady state.

5.2. Dynamic order rule

The E-order rule is not optimal when the OLB interference occurs. For example, consider when V_1 at the 2^{nd} lap is blocked by V_4 at the 1^{st} lap in Figure 5, then, the 4 vehicles are blocked, because V_1 and V_2 are scheduled to serve on S_6 and S_5, respectively and hence cannot serve on S_2 and S_1, though they stay there, while the Order rule can take more expedient routing as shown in Figure 4.

The Dynamic-order routing rule (denoted D-order routing) is obtained by modifying the E-order rule every time the OLB interference occurs. Let

$$D_{OLB} = L - L_V - p_0 \tag{5.8}$$

where L_V stands for the fleet length of the n_V vehicles. Then, an OLB interference occurs, if $D_{OLB} < D^*_{n_V+1} = p_P + d_B$ as shown in (3.15). The D-order rule is as follows:

Algorithm D-order
Step 1: the same as Step 1 of algorithm E-order.
Step 2: the same as Step 2 of the E-order.

Step 3:. Compute u by (5.1) and D_{OLB} by (5.8).
Step 4: If $u > n_V$,$u = n_S$, or $D_{OLB} < p_P + d_B$, apply the Unchanging rule to all n_V vehicles, and go to Step 2, otherwise, go to Step 5.
Step 5: the same as Step 5 of algorithm E-order.
Step 6: the same as Step 6 of algorithm E-order.
Step 7: the same as Step 7 of algorithm E-order.

It is obvious that algorithm D-order keeps the load balance as the Order and the E-order rules do and is same as algorithm E-order except Step 4 which allocates vehicles to idle stations as many as possible when the OLB interference occurs, resulting in not smaller throughput (and not larger interference time) than the E-order rule.

6. Finite Acceleration and Deceleration of Vehicle

We assumed so far that the acceleration and deceleration of the vehicle be infinite. Of course, they are finite in practice. However, it is very difficult to exactly grasp the dynamics of the PCVRS with finite acceleration/deceleration by mathematical means as long as interferences occur. For overcoming this difficulty this section approximates the PCVRS with finite acceleration and deceleration by the one with " virtual stop " for the acceleration/deceleration.

6.1. Virtual stop by finite acceleration/deceleration

Let v_{max} be the max. speed of the vehicles and $\alpha(< \infty)$ be the (same) value of the acceleration and the deceleration. The vehicle moves more slowly during acceleration /deceleration, resulting in moving distance shorter than the one by the max. speed. The difference between these moving distances is regarded as delay by the virtual stop. It is regarded that interference occurs when a vehicle is obliged to decelerate by the preceding one, even if it does not really stop. The virtual stop aims at applying the analysis of the PCVRS with infinite deceleration/acceleration in which a vehicle runs with a constant speed or really stops. In the following every distance is represented by not time but metric one (unlike the previous sections), aiming at easiness of understanding.

6.1.1. Virtual stop in acceleration process

When a vehicle accelerates for t time units from real stop, the moving distance during this time period is given by $d = \alpha t^2/2$. Then, the delay is defined and given by

$$d^A(t) \equiv v_{max}t - d = v_{max}t - \alpha t^2/2.$$

The virtual stop time is defined and given by

$$w^A(t) \equiv d^A(t)/v_{max} = t - \alpha t^2/2v_{max} \tag{6.1}$$

In particular, when $t = v_{max}/\alpha$, i.e., the vehicle reaches the max. speed,

$$w^A(v_{max}/\alpha) = v_{max}/2\alpha \tag{6.2}$$

6.1.2. Virtual stop in deceleration process

When a vehicle decelerates for t time units from the max. speed, the moving distance during this time period is given by $d = v_{max}t - \alpha t^2/2$, then the delay and the virtual stop time are, respectively, given by

$$\begin{aligned} d^D(t) &= \alpha t^2/2 \\ w^D(t) &= \alpha t^2/2v_{max} \end{aligned} \tag{6.3}$$

In particular, when $t = v_{max}/\alpha$, i.e., the vehicle reaches the real stop, then the virtual stop time reduces to (6.2)

6.1.3. Virtual stop in deceleration/acceleration process

When a vehicle decelerates for t_1 time units from the max. speed and then accelerates for $t_2(\leq t_1)$ time units with the switching time neglected, then, the delay and the virtual stop time are, respectively, given by

$$\begin{aligned} d^{DA}(t_1+t_2) &= \alpha(t_1^2/2 + t_1t_2 - t_2^2) \\ w^{DA}(t_1+t_2) &= \alpha(t_1^2/2 + t_1t_2 - t_2^2)/v_{max} \end{aligned} \tag{6.4}$$

In particular, when $t_1 = t_2 = v_{max}/\alpha$, i.e., the vehicle really stops and then reaches the max. speed,

$$w^{DA}(2v_{max}/\alpha) = v_{max}/\alpha \tag{6.5}$$

On the other hand, when the vehicle accelerates for t_1 time units from the real stop and decelerates for $t_2(\leq t_1)$ time units with the switching time neglected, the delay and the virtual stop times are, respectively, given by

$$\begin{aligned} d^{AD}(t_1+t_2) &= v_{max}(t_1+t_2) - \alpha(t_1^2/2 + t_1t_2 - t_2^2) \\ w^{AD}(t_1+t_2) &= t_1 + t_2 - \alpha(t_1^2/2 + t_1t_2 - t_2^2/2)/v_{max} \end{aligned} \tag{6.6}$$

In particular, when $t_1 = t_2 = v_{max}/\alpha$, the virtual stop time reduces to (6.5). Therefore, real processing time $p(> 0)$ on a station in steady state is replaced by virtual processing time $p + v_{max}/\alpha$. This means that throughputs (3.17) and (3.19) are simply replaced by

$$\begin{aligned} T_p^{V_\alpha} &= \frac{n_V}{L/v_{max} + p_P + p_0 + 2v_{max}/\alpha} \\ T_p^{S_\alpha} &= \frac{n_S}{(L_{n_S} + d_B)/v_{max} + p_P + v_{max}/\alpha} \end{aligned} \tag{6.7}$$

where L and L_{n_S} are the metric loop length and the metric distance between S_1 and S_{n_S} respectively. However, total flow time and mean flow time (3.21) are more complicated, because they include interferences as discussed below.

6.2. Interferences in finite acceleration/deceleration

Let D_{k+1} be the metric distance from V_{k+1} to V_k, then

$$\Delta_{k+1}^{\infty} \equiv (D_{k+1} - d_B)/V_{max} \tag{6.8}$$

is equivalent to Δ_{k+1} in (3.5). In the following the stop means sum of virtual and real stops unless saying otherwise. Let V_k stop for P_k time units, then V_{k+1} is interfered according to (3.6), if

$$\Delta_{k+1}^{\infty} < P_k \tag{6.9}$$

otherwise, V_{k+1} travels without deceleration unless the serial service is done there. When V_{k+1} is interfered, it really stops, if

$$\Delta_{k+1}^{\alpha} \equiv \frac{D_{k+1} - d_B}{V_{max}} + \frac{V_{max}}{2\alpha} = \Delta_{k+1}^{\infty} + \frac{V_{max}}{2\alpha} \leq P_k, \tag{6.10}$$

otherwise V_{k+1} only decelerates and accelerates. Note that (6.10) reduces to (3.6), since the virtual stop time in P_k is given by (6.2). The real stop time can be expressed by

$$w'_{k+1} = max(P_k - \Delta^{\alpha}_{k+1}, 0) = max(P_k - \Delta^{\infty}_{k+1} - v_{max}/(2\alpha), 0) \tag{6.11}$$

This is also equivalent to (3.7) and hence the real stop time is not affected by the acceleration/deceleration. Let t' be deceleration time of V_{k+1}, then time for which V_{k+1} runs with the max. speed is $(P_k - t' - w'_{k+1})$ and hence

$$D_{k+1} - d_B = v_{max}(P_k - t' - w'_{k+1}) + (v_{max} - \alpha t'/2)t'$$

thus,

$$t' = \sqrt{2\{v_{max}(P_k - w'_{k+1}) - D_{k+1} + d_B\}/\alpha}$$

Since the time for acceleration to the max. speed is also t' and the time for the deceleration and the acceleration is $t = 2t'$, the waiting (stop) time of V_{k+1} is given by, according to (6.5) and (6.11).

$$\begin{aligned} W^{\alpha}_{k+1} &= w^{DA}(t) + w'_{k+1} = \alpha(t/2)^2/v_{max} + w'_{k+1} \\ &= P_k - \Delta^{\alpha}_{k+1} + v_{max}/\alpha + min(p_k - \Delta^{\alpha}_{k+1}, 0) \end{aligned} \tag{6.12}$$

This equation gives W_{k+2}, the waiting time of the following vehicle V_{k+2} by setting $P_{k+1} = W_{k+1}$ in (6.12) as (3.10).

Now consider the MIF distance between V_k and V_{k+1} with the finite acceleration/deceleration. When V_k and V_{k+1} really stop, the distance between them is d_B and they start accelerating simultaneously. Assume that V_{k+1} next really stops for p time units at a station which is distance D off and V_k does not stop ($p = p_P$ on a processing station and $p = p_0$ on station I/O). Then, the distance between the vehicles is stretched from d_B to $D^{\alpha^*}_{k+1}$, where

$$D^{\alpha^*}_{k+1} = d_B + v_{max}p + v^2_{max}/\alpha, \quad \text{if } D \geq v^2_{max}/\alpha \tag{6.13}$$

$$D^{\alpha^*}_{k+1} = d_B + v_{max}p + 2v_{max}\sqrt{D/\alpha} - D, \quad \text{if } D < v^2_{max}/\alpha \tag{6.14}$$

Note that (6.13) and (6.14) are obtained by adding the virtual stop time to the real stop time in (3.8) or (3.9). The minimum fleet length $L^{\alpha*}_V$ and the mean interference time MIT^* are, respectively obtained by replacing $D^{\alpha^*}_{k+1}$ for D^*_{k+1} in (3.12) and (3.22). A steady state is realized according to (3.13) and (3.14), if

$$L - L^{\alpha^*}_V \geq (p_P - p_0)v_{max} + d_B \tag{6.15}$$

otherwise the OLB interference occurs as in the infinite case.

We can conclude from the above analysis that these performance measures for the vehicle routing rules except the D-order rule can be calculated by introducing virtual stops in Sections 4 and 5.

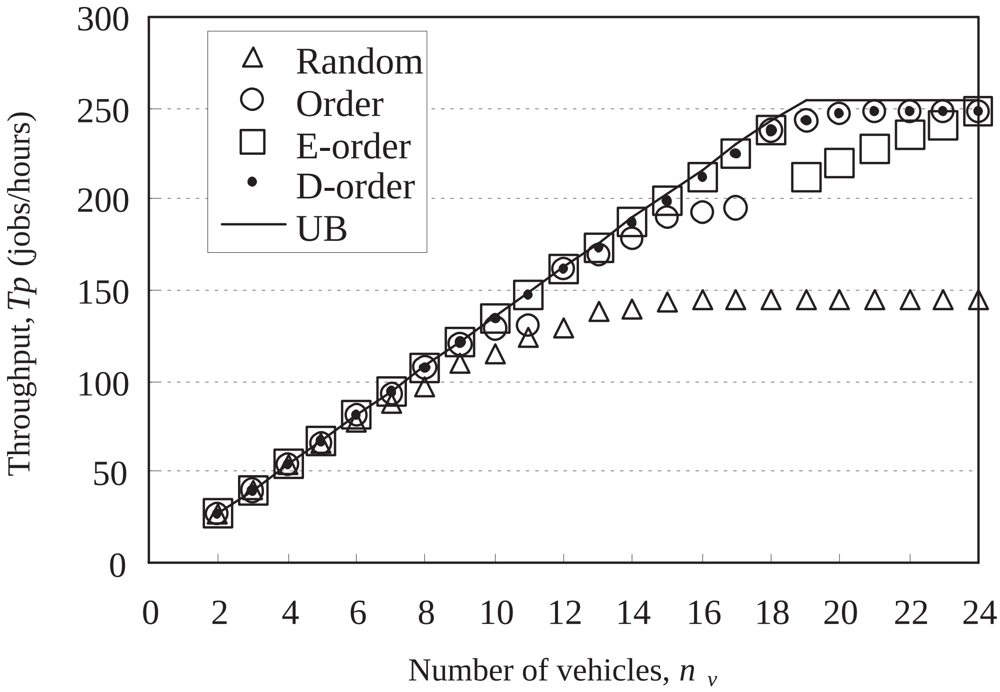

Figure 6: Effect of n_V on T_P
($n_S = 6$, $n_J = 600$, $p_P = 55(s)$, $p_0 = 1(s)$, $L = 210(m)$, $\alpha = \infty$)

7. Numerical Simulation

This section describes results obtained by numerical simulation which was executed to confirm theoretical results obtained so far.

A simulation system for the PCVRS with the infinite acceleration/deceleration was developed by Lu, et al [10]. In this system the schedule is calculated job by job (i.e., vehicle by vehicle in each lap) by a set of equations, taking advantage of the no passing between vehicles. The algorithm is of $O(ne)$ time where ne stands for the number of events (i.e., stops and starts) in the entire schedule and of $O(n_S n_V)$ memory which does not depend on n_J. It has been confirmed that it is faster than an existing event-driven simulation software which generates same results. We developed a simulation system for the finite acceleration/deceleration by introducing the formulation in 6.1 to the above one.

Figure 6 shows the effect of n_V (the number of vehicles) on T_P (the throughput) for each of the Random, the Order, the E-order and the D-order rules under conditions: $n_J = 600$ (the number of jobs), $n_S = 6$ (the number of processing stations), $p_P = 55(s)$(constant processing time of the processing stations), $p_0 = 1(s)$ (constant processing time of the I/O station), $d_B = 5(m)$(the minimum distance between vehicles), $v_{max} = 1(m/s)$(the max. speed of the vehicle), $\alpha = \infty$ (the value of acceleration/deceleration) and $L = 210(m)$ (the loop length). The real line stands for upper bound (3.20) (consisting of two lines T_P^V (3.17) and T_p^S (3.19), respectively). The Random rule saturates with lower T_P due to the early OLB interference. This fact is consistent with (4.2) and (4.3). The Order rule increases the throughput with n_V, but makes bumps with some n_Vs, depending on $\gcd(n_S, n_V)$ as discussed in 4.2. The E-order rule linearly increases T_P according to T_p^V till $n_V = 18$ and makes a big bump with $n_V \geq 19$, resulting in being behind the Order rule as suggested in

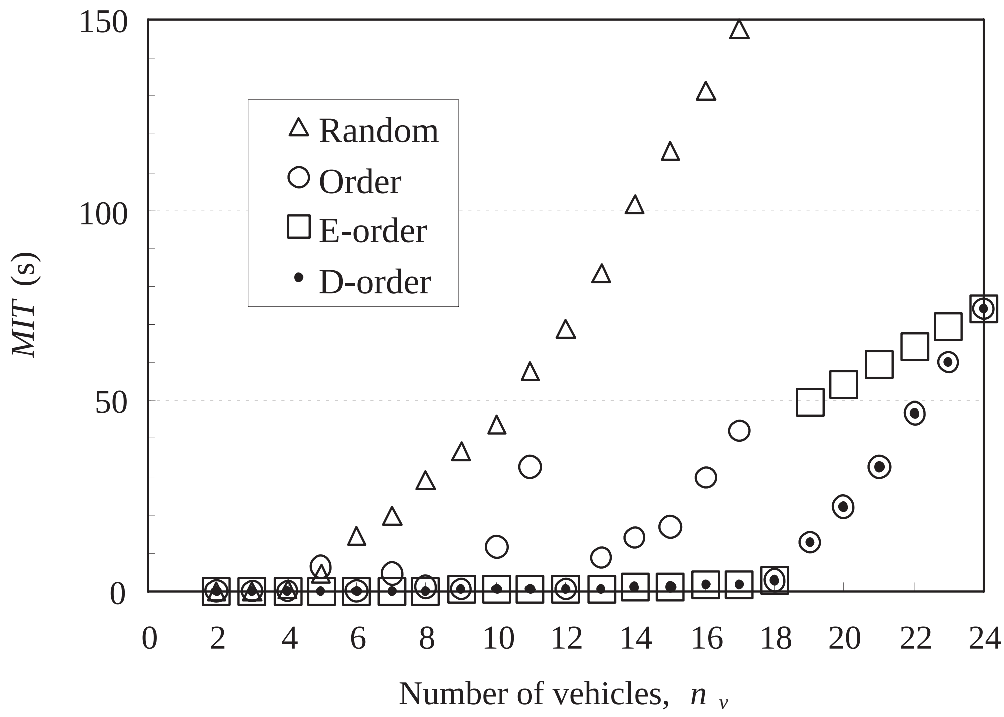

Figure 7: Effect of n_V on mean interference time MIT with the same condition as Figure 6

5.2. The D-order rule keeps the same T_P as the E-order rule till $n_V = 18$ and the same T_P as the Order rule with $n_V \geq 19$, resulting in keeping T_P close to upper bound (3.20).

Figure 7 shows the effect of n_V on the mean interference time MIT(s) (see (3.21) and (3.22)) under the same condition as the one in Figure 6. For example compare the Order rule and the E-order rule with $n_V = 10$ (and gcd(6,10)=2) in Figure 7 in which $\alpha = \infty$. the MIF distance within a sub-group is, according to (3.8).

$$d_B + p_0 v_{max} = 5 + 1 = 6(m) \tag{7.1}$$

and the one between two adjacent sub-groups is, according to (3.9).

$$d_B + p_P v_{max} = 5 + 55 = 60(m) \tag{7.2}$$

Then, in the Order rule $L_V^0(10, 2) = 270$ (by (4.7)), thus, the OLB interference is estimated to occur by (4.8). $MIT^0(10, 2) = 2.0$ by (4.10), but the real MIT is much larger due to the OLB interference. In the E-order rule $L^E(10) = 108$ (by (5.3)) and hence the steady state is estimated to be realized by (5.4). $MIT^E(10) = 0.38(s)(\text{by}(5.6))$ is close to the real value.

Figure 8 shows the MIT with the same condition as Figure 7 except $\alpha = 0.1(m/s^2)$. In this case the MIF distance within a sub-group is given by, according to (6.14).

$$d_B + (v_{max}p_0 + 2\sqrt{d_B/\alpha} - d_B) \approx 5 + (1 + 9) = 5 + 10(m) \tag{7.3}$$

because $v_{max}^2/\alpha = 10 > d_B = 5$, and hence the processing time at the I/O station is stretched from 1(s) to 10(s) due to the virtual stop. The MIF distance between adjacent sub-groups is given by, according to (6.13).

$$d_B + (v_{max}p_P + v_{max}^2/\alpha) = 5 + (55 + 10) = 5 + 65(m) \tag{7.4}$$

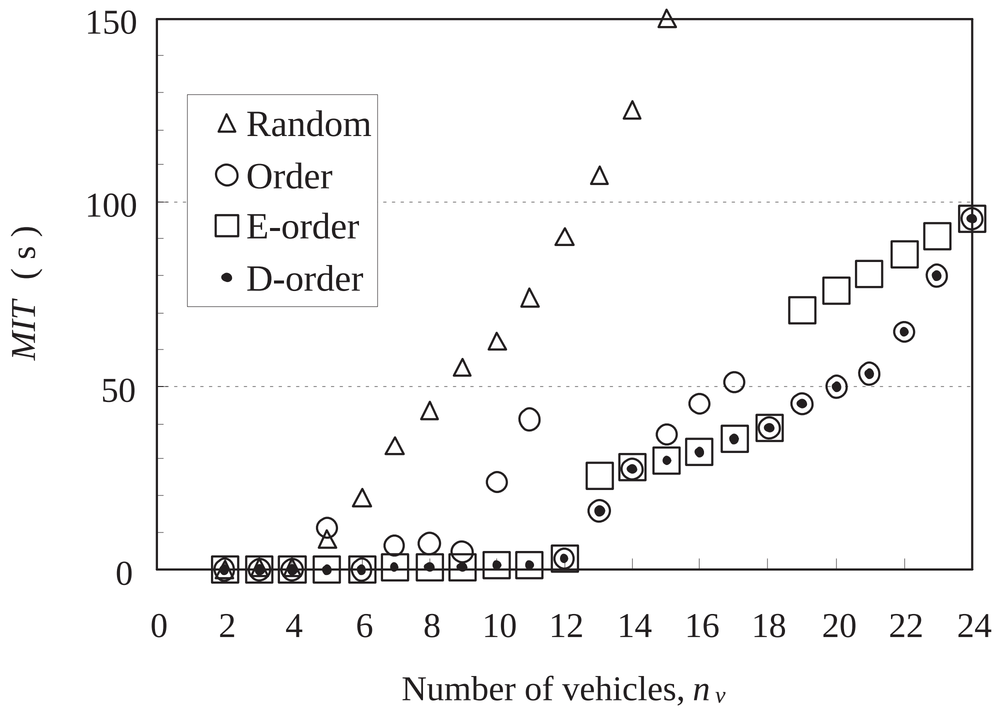

Figure 8: Mean interference time MIT with the same condition as Figure 7 except $\alpha = 0.1(m/s^2)$

because the distance between every two stations is set longer than $v_{max}^2/\alpha = 10$. The processing time is stretched from 55(s) to 65 (s). Therefore, the MFL is given by replacing $p_0 \leftarrow 10(s)$ and $p_P \leftarrow 65(s)$ in (4.7), i.e.,$L_V^0(10, 2) = 355(m)$ and hence the OLB interference occurs more frequently than the one in Figure 7. The MFL of the E-order rule becomes $L_V^E(10) = 190(m)$ and hence the OLB interference occurs even in the E-order rule but less frequently than in the Order rule.

Figure 9 shows the MIT with the same conditions as the ones in Figure 7 except different processing times. Thus, difference between both results comes from different processing times as theoretically estimated. But, the result in Figure 9 is very close to the one in Figure 8 in spite of different processing times and different α. This similarity comes from the virtual stop by (7.3) and (7.4) which equivalently changes the finite acceleration/deceleration to the infinite one as estimated theoretically.

8. A Concluding Remark

Our study was motivated by the PCVRS in a real automated warehouse where complicated interference between vehicles was observed. The warehouse told us that there is a limit of increasing the number of vehicles to improve the throughput: a fleet size larger than this limit improve no more or occasionally deteriorate the throughput. The reason had not been known and been suspected due to a vendor provided software which is of black box. We wish that our result is informative to the warehouse.

Acknowlegement

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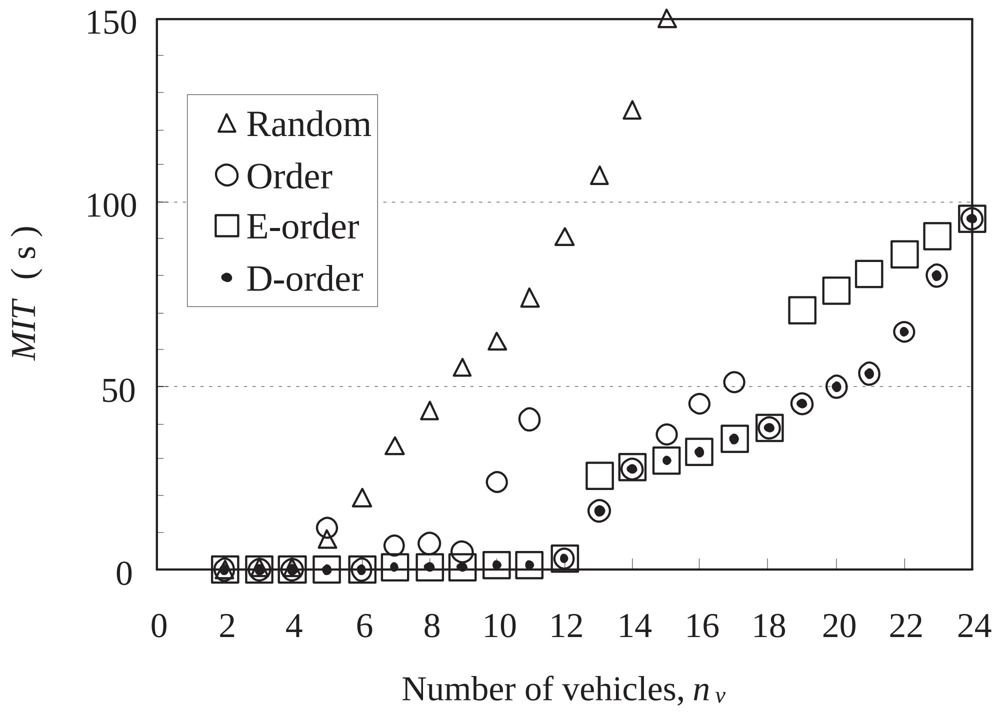

Figure 9: Mean interference time MIT with the same condition as Figure 7 except $p_P = 65$ and $p_0 = 10$

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