On the Pricing of Corporate Value under Information Asymmetry

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1 Introduction

This paper examines the corporate value of a decentralized firm in the presence of principal-agency conflicts due to information asymmetries. When owners delegate the management to managers, contracts must be designed to provide incentive for managers to truthfully reveal private information. Using contingent claims approach, we demonstrate that an underlying option value of the firm can be decomposed into two components: a manager's option and an owner's option. The value of a decentralized firm is lower than that of an owner-managed firm. In particular, the implied manager's decisions in a decentralized firm differ significantly from that in an owner-managed firm.

2 Model

Throughout our analysis, we suppose that capital markets are frictionless, agents are risk neutral and can borrow and lend freely at a constant interest rate, r. The assumption of risk neutrality represents little loss of generality. The owner of a firm (principal) has an option to hire a manager (agent) to operate the company. We assume that the owner delegates the corporate operation to a manager. For simplicity, we assume that the firm finances the capital only with pure equity.

Consider a manager hired by an owner that produces a unit of output which it sells for a price, $(X_t)_{t\in\mathbb{R}_+}$. We assume that $(X_t)_{t\in\mathbb{R}_+}$ follows:

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}z_t, \quad X_0 = x \in \mathbb{R}_{++}, \qquad (1)$$

where $(z_t)_{t\in\mathbb{R}_+}$ denotes the standard Brownian motion under a risk neutral measure, $\mathbb{P}$, and where μ and σ are positive constants. For convergence, we assume that $\mu < r$. While in production, the firm incurs costs per period of $w \in \mathbb{R}_{++}$, its net earnings flow is $X_t - w$.

Here, the corporate value consists of the value of two assets, tangible asset and intangible asset. The former is observable and contractible to both the owner and the manager, while the latter is privately observed only by the manager. Let $W(x)$ represent the value of the tangible asset component with an income flow $x - w$ where $X_t = x$, and θ represent the value of the intangible asset component. Thus, the sum of values is the corporate value, $W(x) + \theta$.

The intangible asset component of corporate value, θ, may take one of two possible values: θ_1 or θ_2 with $\theta_1 > \theta_2$. We denote $\Delta\theta := \theta_1 - \theta_2$. We may regard a draw of θ_1, θ_2 as a "higher quality," "lower quality" intangible asset, respectively. The probability of drawing θ_1 equals p, an exogenous variable.

Now we assume that bankruptcy occurs when the value of the tangible asset first hits some constant γ, because the value of the tangible asset is observed by both the owner and manager. So, the value of the firm at the bankruptcy turns to be $\gamma + \theta$.

It is useful to begin our analysis by looking at the optimal contracting problem when θ is publicly observable by both the owner and the manager.

Lemma 2.1 *Let $\pi^*(x)$ denote the value of the firm in the first-best no-principal-agent setting. Then, $\pi^*(x)$ is equal to:*

$$\begin{aligned}
\pi^*(x) &= \frac{x}{r-\mu} - \frac{w}{r} \\
&\quad + p\left\{\gamma + \theta_1 - \frac{x_1^*}{r-\mu} + \frac{w}{r}\right\}\left(\frac{x}{x_1^*}\right)^{\beta} \\
&\quad + (1-p)\left\{\gamma + \theta_2 - \frac{x_2^*}{r-\mu} + \frac{w}{r}\right\}\left(\frac{x}{x_2^*}\right)^{\beta}, \qquad (2)\\
x^*(\theta) &= -\frac{\beta}{1-\beta}\left\{\frac{w}{r} + (\gamma+\theta)\right\}(r-\mu), \qquad (3)
\end{aligned}$$

where $\theta \in \{\theta_1, \theta_2\}$. *And* β *is the negative root of* $Q(y) = 0$, *where* $Q(y) = y(y-1)\frac{\sigma^2}{2} + y\mu - r$.

We then consider the principal-agent optimzation problem in a situation of information asymmetry. The owner offers the manager a contract at time zero that commits the owner to pay the manager's compensation (wage) at the time of bankruptcy. In principle, for any realized value $\tilde{x}$ of X_t obtained at the time of bankruptcy, a contracted compensation $k(\tilde{x})$ can be specified, provided that $k(\tilde{x}) \geq 0$. The contract will endogenously provide incentives to ensure that the manager declares bankruptcy in accordance with the owner's rational expectations and delivers the true scrapping value of the firm to the owner at the time of bankruptcy. Thus the contract need include two wage/bankruptcy trigger pairs (k_1, k_2, x_1, x_2).

The owner has a scrapping value of $\gamma + \theta_1 - k_1$ if $\theta = \theta_1$ at the time of bankruptcy, and $\gamma + \theta_2 - k_2$ if $\theta = \theta_2$. Thus, the value of the owner's option can be written as:

$$\frac{x}{r-\mu} - \frac{w}{r} + p\left\{(\gamma+\theta_1) - \frac{x_1}{r-\mu} + \frac{w}{r} - k_1\right\}\left(\frac{x}{x_1}\right)^{\beta} + (1-p)\left\{(\gamma+\theta_2) - \frac{x_2}{r-\mu} + \frac{w}{r} - k_2\right\}\left(\frac{x}{x_2}\right)^{\beta}.$$

The manager's option has a payoff function of k_1 if $\theta = \theta_1$ and k_2 if $\theta = \theta_2$. Thus, the value of the manager's option can be written as:

$$\pi^m(x) = p\left(\frac{x}{x_1}\right)^{\beta} k_1 + (1-p)\left(\frac{x}{x_2}\right)^{\beta} k_2.$$

In principal-agent optimal setting, the owner sets the contract pairs in order to induce the manager to do the truth-telling action at the bankruptcy trigger. For accomplishing these objectives, the owner must attempt to design the four constraints: the two incentive and two participation constraints:

$$\left(\frac{x}{x_1}\right)^{\beta} k_1 \geq \left(\frac{x}{x_2}\right)^{\beta} (k_2 + \Delta\theta), \quad (4)$$

$$\left(\frac{x}{x_1}\right)^{\beta} (k_1 - \Delta\theta) \leq \left(\frac{x}{x_2}\right)^{\beta} k_2, \quad (5)$$

$$k_1 \geq 0, \quad (6)$$

$$k_2 \geq 0. \quad (7)$$

3 Model Solution

We provide the solution to the principal-agent optimization problem described in the previous section: maximizing the owner's value function subject to the four inequality constraints (4) to (7).

Proposition 3.1 *The optimal contracts* $(x_1,\ x_2,\ k_1,\ k_2)$ *are as follows:*

$$x_1 = x_1^*, \quad x_2 = x_3^*, \quad k_1 = \left(\frac{x_1^*}{x_3^*}\right)^{\beta} \Delta\theta, \quad k_2 = 0.$$

where θ_3 *is defined by:*

$$\theta_3 = \theta_2 - \frac{p}{1-p}\Delta\theta. \quad (8)$$

Proposition 3.2 *Let* $\pi^{**}(x)$ *denote the value of the firm in the principal-agent setting. Then,* $\pi^{**}(x)$ *is equal to:*

$$\pi^{**}(x) = \frac{x}{r-\mu} - \frac{w}{r} + p\left\{\gamma + \theta_1 - \frac{x_1^*}{r-\mu} + \frac{w}{r}\right\}\left(\frac{x}{x_1^*}\right)^{\beta} + (1-p)\left\{\gamma + \theta_2 - \frac{x_3^*}{r-\mu} + \frac{w}{r}\right\}\left(\frac{x}{x_3^*}\right)^{\beta}. \quad (9)$$

4 Conclusion

The value of a decentralized firm is strictly lower than that of an owner-managed firm. The proof is obvious:

$$x_2^* = \arg\max_y \left\{\gamma + \theta_2 - \frac{y}{r-\mu} + \frac{w}{r}\right\}\left(\frac{x}{y}\right)^{\beta}. \quad (10)$$

The social loss is driven by the distance of the trigger x_3^* from x_2^*. Furthermore, we can show the several interesting implications of the model. For example, an increase in the volatility may have the possibility to give rise to the "asset substitution."

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