A Round-Trip Airline Booking With Dependent Demand

Membership Number: 02991570 Chang Jung University P.S. You*

Abstract This paper deals with a round-trip airline booking problem in which rejected customers may diverse to higher fare classes. The round-trip refers to a journey entirely by an outbound OW trip and an inbound OW trip. To develop a dynamic-nested reservation system, this paper proposes a dynamic model that enables the airline reservations system to devise a set of dynamic decision rules at any actual booking status. The booking process is controlled by some set of booking thresholds.

1 Introduction

It is a business practice to divided a pool of identical seats at different prices in accordance with different classes of customers to improve revenues. Under this practice, A central problem in airline reservation system is determining decision rules for sequentially accepting or denying reservation requests.

A common approach to this problem is to set the booking-limit for distinct fare class. That is, whether or not to accept customers' request depends on the booking-limit. Many authors have taken very considerations on this problem. For details, the reader is referred to references Belobaba (1987), Weatherford and Bodily (1992), and Jeffrey, McGill and Ryzin (1999).

In general, previous approaches to the booking-limit can be categorized into three categories: (1) non-nested, (2) static-nested, and (3) dynamic-nested. The airlines expect to have a functions in the Computer-Reservation-System which is the function of revising the booking decision based on the actual booking status. Many authors have devoted to developing dynamic-nested booking policy. However, the number of the data storage from dynamic-nested booking policy is many times than that from the static-dynamic booking policy. Thus, research on how to eliminate unnecessary data can not be ignored when developing dynamic-nested booking policy.

If the booking policy is a piecewise-constant functions of the time to flight departure, then the data storage can be reduced by only storing the switching points. Such a policy is called a booking-limit policy.

A large portion of travel requests comprise an outbound OW trip and an inbound OW trip simultaneously, this paper tries to develop booking-limit policy for an airline network, which is composed of an outbound OW trip, inbound OW trip, and a round-trip. The round-trip refers to a journey entirely by the outbound OW trip and the inbound OW trip. In addition, we allow that rejected customers may diverse to higher fare classes.

2 The Model

For convenience, we divide the total planning horizon into T decision periods which are small enough so that no more than one customer arrives during each period. In addition, we will count our periods in reverse time sequence. Moreover, this paper assume that there are no-shows, go-shows and cancellations of booking. Although customers do not necessarily know what fare class they are making inquiry or purchase. However, according to their requests, an airline operators can know what fare class they request. Thus, we further make a realistic assumption that the airline company can help identity the fare class requested by the customer. Denote by $j = 1$, $j = 2$ and $j = 3$ the outbound OW trip, inbound OW trip, and a round-trip, respectively. Let i_1, i_2 and $i_2 = \min\{i_1, i_2\}$ be the seats available for outbound OW trip, inbound OW trip and round-trip, respectively (initially, i_j = the total booking capacity of the flight for trip j).

In this paper, fare classes are classified into ordered types ℓ^j, $\ell^j \in \{1, 2, \cdots L^j\}$, where fare class 1 is the most expensive class and L^j is the least expensive class in trip j. Let x^j_ℓ be the revenues generated by selling a ticket to a customer who requests fare class ℓ in trip j; let $\lambda^j_{t\ell}$ be the probability that a request for fare class ℓ in trip j will arrive during a decision period t with $\sum_{\ell=1}^{3}\sum_{\ell=1}^{L^j}\lambda^j_{t\ell} \leq 1$; and let $r^j_{\ell,n}$ be the probability that the customer being denied a requested fare class ℓ in trip j will request an upgrade to fare class n.

The problem is formulated as follows: flights #A and #B will be departing at t_1 and $t_2 = 0$, respectively. The airline company tries to set a strategy for selling seats within T decision periods so as to maximize their total expected revenue. During each decision period, if a customer requests a seat, then the airline must decide on whether to reject the customer's request for a possible upgrade to a higher fare class or accept the request. Denote by $v_t(\mathbf{i})$ $\mathbf{i} = (i_1, i_2)$ the maximum total expected revenue that can be generated within t periods and i seats remaining; and let $u^j_{t\ell}(\mathbf{i})$ be the same as the meaning of $v_t(\mathbf{i})$ under the condition that a request for class ℓ in trip j is denied. Then, we have the following recursive function:

$$v_t(\mathbf{i}) = \begin{cases} (1 - \sum_{j=1}^{3}\sum_{\ell=1}^{L^j}\lambda^j_{t\ell} + \sum_{j=1}^{3}\sum_{\ell=1}^{L^j}\lambda^j_{t\ell} I(i_j = 0))v_{t-1}(\mathbf{i}) & , \\ + \sum_{j=1}^{3}\sum_{\ell=1}^{L^j}\lambda^j_{t\ell}\max\{u^j_{t\ell}(\mathbf{i}), x^j_\ell + v_{t-1}(\mathbf{i} - I_j)\} I(i_j \geq 1) & , t \geq t_1 \\ (1 - \sum_{\ell=1}^{L^2}\lambda^2_{t\ell} + \sum_{\ell=1}^{L^2}\lambda^2_{t\ell} I(i_2 = 0))v_{t-1}(\mathbf{i}) & \\ + \sum_{\ell=1}^{L^2}\lambda^2_{t\ell}\max\{u^2_{t\ell}(\mathbf{i}), x^2_\ell + v_{t-1}(\mathbf{i} - I_j)\} I(i_2 \geq 1) & , t_1 < t \geq t_2 \end{cases} \quad (1)$$

$$u^j_{t\ell}(i) = \max_{\boldsymbol{C} \subset \boldsymbol{S}^j_\ell}\{\sum_{n \in \boldsymbol{C}} r^j_{\ell,n}(x^j_n + v_{t-1}(\mathbf{i} - I_j)) + (1 - \sum_{n \in \boldsymbol{C}} r^j_{\ell,n})v_{t-1}(\mathbf{i} - I_j)\}, \quad (2)$$

where I_j is a vector with $I_1 = (1, 0)$, $I_2 = (0, 1)$ and $I_3 = (1, 1)$; the set $\boldsymbol{s}^j_\ell = \{1, 2, \cdots, \ell - 1\}$; and the set $\boldsymbol{c}$ is a decision variable, i.e., the offering decision.

Theorem 2.1. *For given t, i_1, and ℓ, there is a booking-limit policy such that requests should be accepted if and only if i_j is larger then the booking-limit $i_j(t, \ell)$.*

Theorem 2.2. *Booking-limit values is nondecreasing in the time to flight.*

Refenence

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[3] L.R. Weatherford and S.E. Bodily (1993) "A Taxonomy and Research Overview of Perishable-Asset Revenue Management: Yield Management, Overbooking, and Pricing," Operations Research, 40, 831-844.