Seemingly Unrelated Regression Model with I(d) Regressors (d>1/2) and Its Estimation

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Ⅰ. Introduction

We derive the asymptotic properties of GLSE of Seemingly Unrelated Regression Model with nonstationary I(d) regressors (d>1/2).

Ⅱ. The Model and Assumptions

Let us consider the following seemingly unrelated regression model with nonstationary I(d) regressors x_{1t} and x_{2t}, where L is a lag operator.

$$y_{it} = \alpha_i + \beta_i x_{it} + u_{it},$$
$$(1-L)^{d_i} x_{it} = w_{it}, \qquad w_{it} = \psi_i(L)\varepsilon_{it}, \qquad i = 1,2, \quad t = 1,2,\ldots,T.$$

We make the following Assumptions 1 and 2.

Assumption 1 $d_i > \frac{1}{2}$, $\psi_i(L) = \sum_{j=0}^{\infty} \psi_{ij} L^j$ $(\psi_{10} = 1)$, $\sum_{j=0}^{\infty} j|\psi_{ij}| < \infty$, and all roots of $\psi_i(z) = 0$ are outside the unit circle. $x_{it}=0$ $(t\leqq 0)$. $(i=1, 2)$

Assumption 2 $\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim IID\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{\varepsilon_{11}} & 0 \\ 0 & \sigma_{\varepsilon_{22}} \end{pmatrix}\right)$, $\begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} \sim IID\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \Sigma\right)$, $\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$.

ε_{it} and u_{js} $(i, j=1,2, \quad t,s=1,2,\ldots,T)$ are independent.

The model can be written in conventional matrix form as

$$y = X\theta + u,$$

where y, X, u and θ are given as follows :

$$y = (y_1', y_2')', \quad y_1 = (y_{11}, y_{12}, \cdots, y_{1T})', y_2 = (y_{21}, y_{22}, \cdots, y_{2T})',$$

$$X = \begin{pmatrix} e & x_1 & 0 & 0 \\ 0 & 0 & e & x_2 \end{pmatrix}, \quad e = (1,1,\cdots,1)', x_1 = (x_{11}, x_{12}, \cdots, x_{1T})', x_2 = (x_{21}, x_{22}, \cdots, x_{2T})',$$

$$u = (u_1', u_2')', \quad u_1 = (u_{11}, u_{12}, \cdots, u_{1T})', u_2 = (u_{21}, u_{22}, \cdots, u_{2T})'$$

$$\theta = (\theta_1', \theta_2')', \quad \theta_1 = (\alpha_1, \beta_1)', \quad \theta_2 = (\alpha_2, \beta_2)'$$

Ⅲ. Estimation of θ and Its Properties

Let $\hat{\Sigma}$ be the estimator of Σ given below.

$$\hat{\Sigma} = \begin{pmatrix} \hat{\sigma}_{11} & \hat{\sigma}_{12} \\ \hat{\sigma}_{21} & \hat{\sigma}_{22} \end{pmatrix}$$

where $\hat{\sigma}_{ij} = \frac{1}{T-2}\tilde{u}_i'\tilde{u}_j$ and $\tilde{u}_i$ is the OLS residual vector.

Then the (feasible) GLSE of θ is defined as follows,

$$\hat{\theta} = (X'(\hat{\Sigma}\otimes I_T)^{-1}X)^{-1}X'(\hat{\Sigma}\otimes I_T)^{-1}y$$

Using the normalizer given by

$$D = Diag(T^{\frac{1}{2}}, T^{d_1}, T^{\frac{1}{2}}, T^{d_2}),$$

$D(\hat{\theta}-\theta)$ can be written as

$$D(\hat{\theta}-\theta) = (D^{-1}X'(\hat{\Sigma}\otimes I_T)^{-1}XD^{-1})^{-1}D^{-1}X'(\hat{\Sigma}\otimes I_T)^{-1}u.$$

We can show that $D^{-1}X'(\hat{\Sigma}\otimes I_T)^{-1}XD^{-1}$ and $D^{-1}X'(\hat{\Sigma}\otimes I_T)^{-1}u$ converge weakly to the following.

$$D^{-1}X'(\hat{\Sigma}\otimes I_T)^{-1}XD^{-1} \Rightarrow \begin{pmatrix} \sigma_{22}H_{11} & -\sigma_{12}H_{12} \\ -\sigma_{12}H_{21} & \sigma_{11}H_{22} \end{pmatrix} \equiv H$$

where $H_{11} = \begin{pmatrix} 1 & \int_0^1 F_{d_1-1}(r)dr \\ * & \int_0^1 F_{d_1-1}^2(r)dr \end{pmatrix}$, $H_{12} = \begin{pmatrix} 1 & \int_0^1 F_{d_2-1}(r)dr \\ \int_0^1 F_{d_1-1}(r)dr & \int_0^1 F_{d_1-1}(r)F_{d_2-1}(r)dr \end{pmatrix}$, $H_{21} = H_{12}'$, $H_{22} = \begin{pmatrix} 1 & \int_0^1 F_{d_2-1}(r)dr \\ * & \int_0^1 F_{d_2-1}^2(r)dr \end{pmatrix}$,

$F_{d_i-1}(r) \equiv (\psi_i(1)\sigma_{\varepsilon_{ii}}^{\frac{1}{2}}/\Gamma(d_i))\int_0^r (r-s)^{d_i-1}dW_{\varepsilon_i}(s)$ and $W_{\varepsilon_i}(s)$ is standard Brownian motion.

$$D^{-1}X'(\hat{\Sigma}\otimes I_T)^{-1}u \Rightarrow \begin{pmatrix} \sigma_{22}W_{u_1}(1) - \sigma_{12}W_{u_2}(1) \\ \sigma_{22}\int_0^1 F_{d_1-1}(r)dW_{u_1}(r) - \sigma_{12}\int_0^1 F_{d_1-1}(r)dW_{u_2}(r) \\ -\sigma_{12}W_{u_1}(1) + \sigma_{11}W_{u_2}(1) \\ -\sigma_{12}\int_0^1 F_{d_2-1}(r)dW_{u_1}(r) + \sigma_{11}\int_0^1 F_{d_2-1}(r)dW_{u_2}(r) \end{pmatrix} \equiv K.$$

Therefore we get the following Theorem.

Theorem

$$D(\hat{\theta}-\theta) \Rightarrow H^{-1}K.$$

References

Billingsley. P. (1968), *Convergence of Probability Measure*, John Wiley, New York.

Maekawa, K. and H. Hisamatsu. (1996), SUR Models with I(1) Regressors.