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A two-queue cyclic-service system with mixed K-limited and 1-limited discipline: An application to F-net server performance

01013590 °Hisaki OOHARA *NTT Multimedia Networks Laboratories*
01304590 Yoshitaka TAKAHASHI *NTT Multimedia Networks Laboratories*
Toyofumi TAKENAKA *NTT Multimedia Networks Laboratories*

1 Introduction

We encounter a two-queue alternating (or cyclic) service operation in a traffic control server of the NTT's facsimile communication network (F-net). At the server, customers from one class cannot be delayed too much but customers from another class can be delayed to a certain extent. This situation leads to the study of a two-queue alternating service system, where one queue is served under K-limited discipline while another queue is served under 1-limited discipline. To the best of our knowledge, however, there are no literature on the two-queue system because it is very hard to apply the exact results; see [1]. We will here propose a mean-delay approximation using Bernoulli schedule for modeling the K-limited discipline together with using a decomposition method.

2 Model description

We consider a M/G/1 cyclic-service system of two infinite capacity queues which are denoted by Q_1, Q_2. *A Bernoulli service descipline*, which is parametrized by (p_1, p_2) where $0 \le p_j < 1, j = 1, 2$, is described as follows. When the server visits to a queue, the server always serves a customer if the queue is not empty. At the end of every service given to a customer in Q_j, if the queue is not empty, the server serves the next available customer in Q_j with probability p_j. With probability $1 - p_j$, the server goes to the next queue, i.e., $Q_{j+1 \text{ (mod 2)}}$. Also if Q_j is empty at the end of the service, the server goes to $Q_{j+1 \text{ (mod 2)}}$ with probability 1. Within a queue, the server discipline is FCFS. The server takes a random time, the so-called the switchover time, to go from Q_j to $Q_{j+1 \text{ (mod 2)}}$.

Customers arrive at Q_j according to independent Poisson processes with arrival rate $\lambda_j, j = 1, 2$. The service period of a customer in Q_j is independent and identically distributed(i.i.d.) random variable S_j. And the switchover time from Q_j to $Q_{j+1 \text{ (mod 2)}}$ is i.i.d. random variable D_j. The arrival processes, service times and switchover times are mutually independent.

Throughout this paper, we use the following convention. For any real and positive-valued random variable X, we use $x, x^{(2)}, \sigma_X^2$ to denote the mean, second moment and variance of X, respectively.

The offered traffic ρ_j at Q_j is defined as $\rho_j = \lambda_j s_j, j = 1, 2$. Also define $\rho = \rho_1 + \rho_2$ and set $D = D_1 + D_2$.

The system is assumed to be stable. The mean cycle time c, i.e., the mean time between two successive arrivals of the server at a queue, is given by $c = d/(1 - \rho)$. The stability condition is as follows, $\rho < 1, \lambda_j < \frac{1-\rho+\rho_j}{(1-p_j)d+s_j}$.

We are interested in analyzing mixed K-limited and 1-limited service discipline queueing system. So, we set p_1 to between 0 and 1, and p_2 to 0. We can say that Q_1 has higher priority than Q_2. (See, Fig. 1).

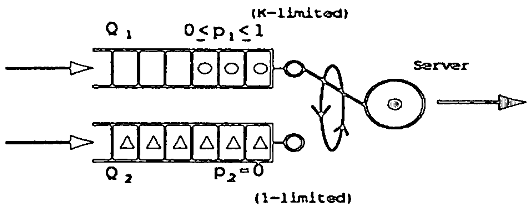

Figure 1: Two-queue cyclic-service system

3 Derivation of mean waiting time

Let W_j denote the waiting time of a customer in Q_j. From [2], *the pseudo-conservation law for Bernoulli schedule* holds.

$$\sum_{j=1}^{2} \rho_j \left(1 - \frac{\lambda_j d}{1-\rho}(1-p_j)\right) w_j = \frac{\rho}{2(1-\rho)} \sum_{j=1}^{2} \lambda_j s$$
$$+ \frac{\rho \sigma_D^2}{2d} + \frac{d}{2(1-\rho)} \sum_{j=1}^{2} (\rho_j + \rho_j^2 - 2\rho_j^2 p_j) \qquad ($$

3.1 Decomposition method

We shall derive the individual mean waiting times w_1, w_2. We use the following decomposition method. We assume that the behavior in Q_1 is mutually independent of that in Q_2. M/G/1 cyclic-service queue system is approximated as M/G/1 vacation model with Bernoulli schedule. The vacation time V is defined as a period from server's departure time from Q_1 to server's next visiting time to Q_1. When the server finishes a service of a customer in Q_1, if Q_1 is not empty, the server serves the next customer in Q_1 with probability p_1, the server starts the vacation with probability $1-p_1$. Also if Q_1 is empty, the server starts the vacation.

The vacation time V is i.i.d. random variable and conditioned by whether Q_2 is empty. The probability that server finds at least one customer when visiting to Q_2 is $\lambda_2 c$. So, we have

$$V = \begin{cases} D_1 + S_2 + D_2 & \text{with prob. } \lambda_2 c \\ D_1 + D_2 & \text{with prob. } 1 - \lambda_2 c \end{cases} \tag{2}$$

From [1, 3], we can derive the mean waiting time of a customer in Q_1, i.e.,

$$w_1 = \frac{v^{(2)}}{2v} + \frac{\lambda\{s_1^{(2)} + (1-p)(2s_1 v + v^{(2)}])\}}{2\{1-\rho-(1-p)\lambda v\}}$$

where

$$v = d + \lambda_1 c s_1,$$
$$v^{(2)} = d_0^{(2)} + d_1^{(2)} + 2d_0 d_1 + 2d\lambda_1 c s_1 + \lambda_1 c s_1^{(2)}.$$

Now, we can get w_2 by using the pseudo-conservation law(1) for the Bernoulli schedule.

4 Numerical Examples

Here are some numerical results of our approximation. We show the efficiency of our approximation by comparing with simulation results for mixed K-limited and 1-limited service discipline queueing system.

The system parameters are taken as follows. The relation between λ_1 and λ_2 is $\lambda_2 = 3\lambda_1$. The service time in Q_1 or Q_2 is supposed to be constant, $s_1 = s_2 = 1.0$. Moreover, the switchover time is assumed to be constant, $d_1 = d_2 = 0.1$. The following relation holds, $K = 1/(1-p_1)$.

When K is 10, from the stability condition, the upper bound of λ_1 is 0.22. Fig. 2 and Table 1 show the individual mean waiting times w_1, w_2 using decomposition method in Bernoulli schedule with $p_1 = 0.9, p_2 = 0.0$ and simulation results, respectively.

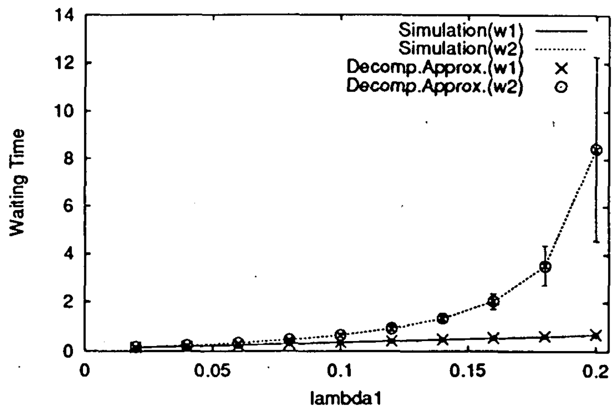

Figure 2: Analysis of 10-limited & 1-limited queueing system

Table 1: Numerical values in Fig. 2

	Analysis		Sim. (95%conf.int.)	
λ_1	w_1	w_2	w_1	w_2
0.02	0.15	0.16	0.15±0.03	0.16±0.02
0.04	0.20	0.24	0.20±0.03	0.24±0.03
0.06	0.25	0.34	0.25±0.02	0.34±0.04
0.08	0.30	0.47	0.30±0.02	0.48±0.05
0.10	0.36	0.66	0.36±0.03	0.66±0.07
0.12	0.42	0.92	0.42±0.03	0.92±0.11
0.14	0.48	1.33	0.47±0.03	1.34±0.20
0.16	0.55	2.04	0.54±0.04	2.03±0.33
0.18	0.63	3.52	0.61±0.03	3.54±0.83
0.20	0.71	8.44	0.68±0.04	8.42±3.86

5 Conclusion

From Fig. 2 and Table 1, we can see that our approach resulted in a good approximation for mixed K-limited and 1-limited discipline. We are now considering to generalize our approach to the batch arrival process.

References

[1] H. Takagi, *Queueing Analysis Vol.1: Vacation and Priority Systems, Part 1*, North-Holland, 1991.

[2] Tedijanto, "Exact Results for the Cyclic-service Queue with a Bernoulli Schedule", *Performance Evaluation*, 11, pp.107-115, 1990.

[3] L. D. Servi, "Average Delay Approximation of M/G/1 Cyclic Service Queues with Bernoulli Schedule", *IEEE. J. Sel. Areas Comm.*, Vol. 4, No. 6, pp.813-822, 1986.