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THE *M/G/c* QUEUE IN LIGHT TRAFFIC

01991370 *Ronald W. Wolff Tokyo Metropolitan University
Chia-Li Wang[1] National Dong Hwa University

Abstract

We investigate the quality of a well-known approximation for first-moment performance measures for a stable *M/G/c*, and in particular, determine conditions under which the approximation is either an upper or a lower bound. We show only that these bounds hold under light traffic. However, we believe, and empirical evidence supports, that these bounds hold under moderate to heavy traffic as well.

The approximation for the average number of customers in queue, obtained by equating time-average work in system under FIFO (from the queue) with the corresponding quantity under any mode of operation that produces *insensitivity*, is

$$Q_F \approx Q_{M/M/c} \frac{E(S^2)}{2E(S)}, \tag{1}$$

where $S \sim G$ denotes a service time. We assume throughout that $E\{e^{\theta S}\} < \infty$ for some real $\theta > 0$.

Approximation (1) fails to be exact because the two time-average work quantities are in fact different. It gives a lower (upper) bound on Q_F when the insensitive system has smaller (larger) time-average work.

We show that in light traffic, (1) is a lower bound when the service distri-

1. Professor Chia-Li Wang, Institute of Applied Mathematics, National Dong Hwa University, Hualien, Taiwan, ROC

bution has the property *single crossing from above* (SCFA), and is an upper bound under the property *single crossing from below* (SCFB), as defined in the paper. Furthermore, by approximating the difference in the two quantities in (1), we obtain approximations that improve on (1).

The result is obtained by a two-step argument. In Section 2, we consider the area under the work-in-system during a busy period, and obtain an inequality on a certain portion of the expected area under either SCFA or SCFB. For this purpose, we choose an insensitive system that is a variation on preemptive LIFO.

In Section 3, we show that the inequality in Section 2 determines a corresponding inequality on time-average work under light traffic. We do this by representing time-average work as a ratio of analytic functions of arrival rate λ that may be expanded as Taylor series. While this work builds on the work of others, we take a different and simpler approach. Because the work-in-system process is regenerative, performance measures may be written as ratios of expectations taken over a single cycle. While we are dealing with a particular regenerative process here, this approach to light traffic is developed for regenerative processes in general in a separate paper by coauthor Wang [1996] that will appear in Advances in Applied Probability in 1997.