SINGLE-LEVEL STRATEGIES FOR FULL-INFORMATION BEST-CHOICE PROBLEMS, I

名古屋商科大　坂口 実* (SAKAGUCHI, Minoru)
ブロツワフ工科大　クルジストフ シャヨウスキ (Szajowski, Krzysztof)

§1 Introduction & Summary.

§2 Best-choice Problems for Choosing One of the k Bests.

Let $X_1, \cdots, X_n$ be iid with $U_{[0,1]}$ distrib. By observing $\{X_i\}$ one by one seq. we want to select one of the k bests (without recall)

Strategy: Choose $z \in [0,1]$ and reject r.v.s as long as $< z$ and accept the earliest r.v. that is $\geqq z$.

Objective: $p^{(k)}(z) \equiv \Pr\{\text{accept one of } k\text{-bests} \mid \text{level } z\} \longrightarrow \max_{z\in[0,1]}$

We find that

$$p^{(k)}(z) = \sum_{m=1}^{k}\left[\sum_{j=1}^{n-k} z^{j-1}\int_z^1 \binom{n-j}{m-1}(1-x)^{m-1}x^{n-j-m+1}\right] + z^{n-k}(1-z^k),$$

where $[\cdots]$ is $\Pr\{$accepted r.v. on or before $(n-k)$-th is the m-th best

Let $$H_n(z) \equiv \sum_{i=1}^{n} i^{-1}(z^{-i}-1), \qquad 0<z\leqq 1.$$

Theorem 1 (A) For $k=1$, optimal z_0 is determined by

$$H_n(z) = n^{-1}\sum_{i=1}^{n} z^{-i}$$

and the prob. of winning is $P^{(1)}(z_0) = \frac{1}{n}\sum_{i=0}^{n-1} z_0^i$.

For $k=2$, optimal z_0 is determined by

$$H_n(z) = n^{-1}\sum_{i=1}^{n} z^{-i} + \frac{n-1}{2}z^{-1} - \frac{n}{2},$$

and prob. of winning is $$P^{(2)}(z_0) = \frac{2}{n}\sum_{i=0}^{n-1} z_0^i - z_0^{n-1}.$$

For $k=3$, optimal z_0 is determined by

$$H_n(z) = n^{-1}\sum_{i=1}^{n} z^{-i} + \frac{(n-1)(n-2)}{6}z^{-2} - \frac{(n-1)(n-5)}{6}z^{-1} - \frac{n+1}{3},$$

and $$P^{(3)}(z_0) = \frac{3}{n}\sum_{i=0}^{n-1} z_0^i + \frac{(n-1)(n-4)}{4}(z_0^{n-2} - z_0^n) + \frac{n-5}{2}z_0^{n-1}.$$

(B) Let $z = e^{-a/n}$ and $n\to\infty$, then the a.d.l. for k is given by $e^{-a_k/n}$, where a_k is a unique root in $(1,\infty)$ of the equation

$$G_k(a) \equiv \sum_{j=k}^{\infty}\frac{a^j}{j(j+1)!} = \frac{1}{k}.$$

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$a_k \doteqdot 1.5029,\ 2.0177,\ 2.4934$; for $k=1, 2, 3$, resp.
a.p.w. $(1-e^{-a_1})/a_1 \doteqdot 0.5174$, $2(1-e^{-a_2})/a_2 - e^{-a_2} \doteqdot 0.7265$, $3(1-e^{-a_3})/a_3 - (2+\frac{a_3}{2})e^{-a_3} \doteqdot 0.8355$; for $k=1, 2, 3$, resp.

§ 3. A ZERO-SUM BEST CHOICE GAME WHERE PLAYERS' PRIORITY IS GIVEN

Selecting-best / Players'-priority / Common / ZS, with FI.

Players observe a common seq. $\{X_t\}$. Player I(II) chooses d.l. $z(w) \in [0,1]$. I[II] accepts the earliest r.v. that is in $(z,\infty)\cap(0, z\vee w)$ $[(w,\infty)\cap(0,z\vee w)]$. If the earliest r.v. that is $\geq z\vee w$ appears, it is accepted by I, and the game is left as II's one-person game thereafter. A player stopping at the best is the winner, and gets 1 from the opponent. I(II) wants to maxi(mini)mize I's exp. payoff.

We find that, for $z > w$,

$$M(z,w) = w^n \sum_{i=1}^{n} i^{-1} w^{-i}(1 - 2z^i + w^i)$$
$$+ z^n\left(\frac{z}{w} - 1\right)\sum_{j=1}^{n}\left\{\sum_{i=1}^{n-j}\left(\frac{w}{z}\right)^i\right\}\frac{z^{-j}-1}{j}$$
$$+ w^n \sum_{j=1}^{n}(n-j)\left(\frac{z}{w}\right)^{j+1}\left\{\frac{z^{-j}-1}{j} - \frac{z^{-j-1}-1}{j+1}\right\}.$$

Let $z = e^{-a/n}$, $w = e^{-b/n}$ with $a, b > 0$, and $\Phi(a) \equiv \int_0^1 t^{-1}(e^{at}-1)dt$, then, for $0<a<b$,

$$M(a,b) = e^{-b}\Big[(b-a)\big(\Phi(b)-\Phi(b-a)\big) - \Phi(b) - a(b^{-1}-1)\Big] + e^{-a}\big(\Phi(a)+1\big) + a/b - 1.$$

This is a continuous game on $[0,\infty)^2$ with $M(a,a) = ae^{-a}$.

Theorem 2. A saddle point $(a_0, b_0) \doteqdot (1.5721,\ 2.9963)$ exists and the saddle value is $\doteqdot 0.3233$.

I's decision level $z_0 = e^{-a_0/n}$ is higher than his rival's $w_0 = e^{-b_0/n}$ and the saddle value is >0, reflecting I's advantage.

§ 4. A NON-ZERO SUM BEST CHOICE GAME WHERE WINNING REQUIRES EARLIER STOP

Selecting-best / Earlier-stop / Each / NZS, with FI.

I(II) observes his own $\{X_i\}$ $(\{Y_i\})$. A player who is the first to stop at the best in his set of r.v.s is the winner. Each wants to maximize Pr.(become a single winner). (以下略)

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