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DEA with Controllable Category Levels

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1. Introduction

In this paper, DEA models under categorical environments will be discussed where the category is under the control of decision makers. For example, suppose that the DMUs are a set of shops with three levels of service, i.e., *poor, average* and *good.* A shop owner in the *poor* service category has the option to remain *poor* or upgrade *average* or *good.* An *average* shop can move within the *average* level or further up to the *good* level.

Suppose that each shop (DMU) can be rated as having one of L different categorical classes, where the categories range from category 1 (representing the lowest service orientation) to category L (the highest).

The problem here is to find, for each DMU, the DEA projected point in the same or higher category levels.

2. Algorithm

In the algorithm below, we consider the case for DMU_o, which is currently at level l $(1 \leq l \leq L)$ and try to find the reference set and the DEA projected points on the frontier with levels in the same category or higher. As for the DEA model employed, we can choose any model, e.g. CCR, BCC, Additive.

[Algorithm]

For $h = l, l+1, \ldots, L$, repeat the following steps:

- **Step 1.**
  Organize a set of DMUs composed of ones with level h or *higher* and DMU_o. Evaluate the efficiency of DMU_o with respect to this group via the DEA model chosen.

- **Step2.**
  (i) If DMU_o is found to be efficient, go to Step 3.
  (ii) If DMU_o is inefficient, then record its reference set and reference (projected) point on the frontier.
  If $h = L$, go to Step 3. Otherwise, replace h by $h+1$ and go back to Step 1.

- **Step 3**
  Look into the reference set, reference point, and category level, obtained from Step 2, and choose the most appropriate point and category level for DMU_o.

One of the characteristics of this algorithm is that it allows DMUs at different levels to mix when forming a reference point. For example, suppose a *poor* DMU_1 has its reference set composed of *average* DMU_2 and *good* DMU_3 with weights 0.75 and 0.25. A categorical service level of $0.75 \times average\ +\ 0.25 \times good$ is assumed for the projected point, i.e. it has a quality level close to *average* and slightly upgraded to *good.*

3. Example

Table 1 exhibits nine single input and single output DMUs, each having either a *poor, average* or *good* category level. We applied this algorithm using the BCC model and obtained the results shown in the far right column of Table 1, where the number in parenthesis designates the λ-value to the referent DMU. DMU A (*poor*) is judged to be efficient, even compared with DMUs in the same or higher category levels and it is reasonable for A to remain in this category. DMU B (*poor*) is enveloped by D (*average*) and E (*average*) and is suitable for upgrading to *average*. DMU C (*poor*) has two sets of reference, i.e. one composed of D (*average*) and E (*average*) and the other G (*good*). So, it has two possibilities to upgrade its level. Similarly, DMU F has two possibilities.

4. Notes

The categorical inputs and outputs models were introduced by Banker and Morey (1986). They formulated the controllable categorical variable problem within the framework of the mixed-integer LP model under the BCC model. Kamakura (1988) pointed out the shortcomings of their formulation and presented another mixed-integer model. Rousseau and Semple (1993) presented a new algorithm for this case, as an extension of Kamakura's work (1988). This method eliminated the difficulties of computation that had accompanied earlier mixed integer models. However, these methods are associated with the BCC model and make use of its characteristics in formulation. It should be noted that the algorithm developed in this paper can be coupled with any DEA model, in contrast to earlier methods. Cook, Kress and Seiford (1993) proposed a method for using ordinal data in DEA. This method is erroneous in that it assumes substitutionability between numerical and categorical values.

References

[1] Banker, R.D. and R.D.Morey (1986), "Data Envelopment Analysis with Categorical Inputs and Outputs," *Management Science*, 32, 1613-1627.

[2] Cook, W.D., M. Kress and L.M. Seiford (1993), "On the Use of Ordinal Data in Data Envelopment Analysis," *J. Opl Res. Soc.*, 44, 133-140.

[3] Kamakura, W.A. (1988), "A Note on the Use of Categorical Variables in Data Envelopment Analysis," *Management Science*, 34, 1273-1276.

[4] Rousseau, J.J. and J. Semple (1993), "Categorical Outputs in Data Envelopment Analysis," *Management Science*, 39, 384-386.

Table 1: Nine DMUs with Three Category Levels

DMU	In	Out	Category	Reference set
A	3	1	*poor*	$A(1)$
B	7	7	*poor*	$D(.6)$, $E(.4)$
C	12	6	*poor*	$D(.8)$, $E(.2)$ $G(1)$
D	4	5	*average*	$D(1)$
E	6	10	*average*	$E(1)$
F	11	11	*average*	$E(.667)$, $H(.333)$ $G(1)$
G	8	11	*good*	$G(1)$
H	9	13	*good*	$H(1)$
I	13	15	*good*	$I(1)$