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Optimal Software Rejuvenation Policies with Discounting

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1. Introduction

Software rejuvenation is a preventive maintenance technique that has been extensively studied in the recent literature [1, 2]. In this paper, we consider a generalized problem to estimate the optimal software rejuvenation schedule. More precisely, the software rejuvenation models are formulated via the semi-Markov processes, and the optimal software rejuvenation schedule which minimizes the expected total discounted cost over an infinite time horizon are derived analytically.

2. Model Description

Following Huang *et. al* [1] and Dohi *et al.* [2], we consider the two-step failure model to describe the aging phenomenon in telecommunications billing applications. Define the following four states:

State 0: highly robust state (normal operation state)

State 1: failure probable state

State 2: failure state

State 3: software rejuvenation state

Suppose that all the states mentioned above are regeneration points. More specifically, let Z be the random time interval when the highly robust state changes to the failure probable state, having the common distribution function $\Pr\{Z \leq t\} = F_0(t)$ with density $f_0(t)$ and finite mean μ_0 (> 0). Just after the state becomes the failure probable state, a system failure may occur with a positive probability. Without any loss of generality, it is assumed that the random variable Z is observable during the system operation [1, 2].

Define the failure time X (from State 1) and the repair time Y, having the distribution functions $\Pr\{X \leq t\} = F_f(t)$ and $\Pr\{Y \leq t\} = F_a(t)$, respectively, where $f_f(t)$ and $f_a(t)$ are the associated probability density functions and λ_f (> 0) and μ_a (> 0) are their mean values, respectively. If the system failure occurs before triggering a software rejuvenation, then the repair is started immediately at that time and is completed after the random time Y elapses. Otherwise, the software rejuvenation is started as a preventive maintenance of the software system. Denote the distribution functions of the time to software rejuvenation and its system overhead by $F(t)$ and $F_r(t)$ (with density $f_r(t)$ and mean μ_r (> 0)), respectively. After completing the repair or the rejuvenation, the software system becomes as good as new, and the software age is initiated at the begining of the next highly robust state. Consequently, we define the time interval from the begining of the system operation to the next one as one cycle, and the same cycle is repeated again and again over an infinite time horizon.

Note that the software rejuvenation cycle is measured from the time instant just after the system enters State 1 from State 0. If we consider the time to software rejuvenation as a constant t_0, then it follows that

$$F(t) = U(t - t_0) = \begin{cases} 1 & \text{if } t \geq t_0 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

We call t_0 (≥ 0) as *the software rejuvenation schedule* in this paper and $U(\cdot)$ is the unit step function. Hence, the underlying stochastic process is a semi-Markov rprocess with four regenerative states.

3. Analysis

Define the following cost components:

c_s (> 0): repair cost per unit time

c_p (> 0): rejuvenation cost per unit time

β (> 0): discount factor.

For convenience, we define

$$\mathcal{L}\{f(\alpha)\} = \int_0^\infty \exp(-\alpha s) f(s) ds \tag{2}$$

for an arbitrary continuous function $f(\cdot)$ and a complex number α. That is, the function $\mathcal{L}\{f(\alpha)\}$ is the Laplace transform of the function $f(\cdot)$.

The discounted unit cost for one cycle, *i.e.* the net present value of one dollar after one cycle, is formulated as

$$\delta(t_0) = \int_0^\infty \int_0^\infty \int_0^{t_0} e^{-\beta(z+x+y)} dF_f(x) dF_0(z) dF_a(y) + \int_0^\infty \int_0^\infty \int_{t_0}^\infty e^{-\beta(z+t_0+s)} dF_f(x) dF_0(z) dF_r(s).$$

(3)

The expected total discounted cost for one cycle is

$$\begin{aligned} V(t_0) &= \int_0^\infty \int_0^\infty \int_0^{t_0} \int_0^y c_s e^{-\beta(t+x+y)} \\ &\times dt dF_f(x) dF_0(z) dF_a(y) \\ &+ \int_0^\infty \int_0^\infty \int_{t_0}^\infty \int_0^s c_p e^{-\beta(t+t_0+y)} \\ &\times dt dF_f(x) dF_0(z) dF_r(s). \end{aligned} \quad (4)$$

The expected total discounted cost over an infinite time horizon is given by

$$TC(t_0) = \sum_{k=0}^{\infty} V(t_0)\delta(t_0)^k = V(t_0)/\overline{\delta}(t_0). \quad (5)$$

Then the problem is to seek the optimal software rejuvenation schedule t_0^* which minimizes $TC(t_0)$.

Define the numerator of the derivative of $TC(t_0)$ with respect to t_0, divided by the factor $\overline{F}_r(t_0)e^{-\beta t_0}$, as $q(t_0)$, *i.e.*

$$\begin{aligned} q(t_0) &= \frac{\mathcal{L}\{f_0(\beta)\}}{\beta}\Big[c_s\overline{\mathcal{L}}\{f_a(\beta)\}r_f(t_0) - c_p\overline{\mathcal{L}}\{f_r(\beta)\} \\ &\times \big(\beta + r_f(t_0)\big)\Big]\overline{\delta}(t_0) + \mathcal{L}\{f_0(\beta)\} \\ &\times \Big[\big(\mathcal{L}\{f_a(\beta)\} - \mathcal{L}\{f_r(\beta)\}\big)r_f(t_0) \\ &-\beta\mathcal{L}\{f_r(\beta)\}\Big]V(t_0), \end{aligned} \quad (6)$$

where $r_f(t) = f_f(t)/\overline{F}_f(t)$ is the failure rate and in general $\overline{\psi}(\cdot) = 1 - \psi(\cdot)$. It is assumed that $r_f(t)$ is continuous and differentiable with respect to t.

We make the following assumption:

(A-1) $\frac{c_s\overline{\mathcal{L}}\{f_a(\beta)\} - c_p\overline{\mathcal{L}}\{f_r(\beta)\}}{\beta[\mathcal{L}\{f_r(\beta)\} - \mathcal{L}\{f_a(\beta)\}]} > TC(t_0).$

In [2], it is proved through the reduction argument that $(c_s\mu_a - c_p\mu_r)/(\mu_a - \mu_r) > \lim_{\beta\to 0}\beta \cdot TC(t_0)$ for all t_0 if two parametric assumptions $\mu_a > \mu_r$ and $c_s > c_p$ hold. In the assumption (A-1), taking $\beta \to 0$ yields

$$\lim_{\beta\to 0} \frac{c_s\overline{\mathcal{L}}\{f_a(\beta)\} - c_p\overline{\mathcal{L}}\{f_r(\beta)\}}{\beta[\mathcal{L}\{f_r(\beta)\} - \mathcal{L}\{f_a(\beta)\}]} = \frac{c_s\mu_a - c_p\mu_r}{\mu_a - \mu_r} \quad (7)$$

from the l'Hospital's theorem. In other words, the assumption (A-1) may hold in the most cases with sufficiently small discount factor.

The following result gives the optimal software rejuvenation policy with discounting.

Theorem: (1) Suppose that the failure time distribution is strictly IFR (increasing failure rate) under the assumption (A-1).

(i) If $q(0) < 0$ and $q(\infty) > 0$, then there exists a finite and unique optimal software rejuvenation schedule t_0^* $(0 < t_0^* < \infty)$ satisfying $q(t_0^*) = 0$, and the minimum expected total discounted cost is

$$TC(t_0^*) = \frac{c_s\overline{\mathcal{L}}\{f_a(\beta)\}r_f(t_0^*) - c_p\overline{\mathcal{L}}\{f_r(\beta)\}(\beta + r_f(t_0^*))}{\beta\Big\{\beta\mathcal{L}\{f_r(\beta)\} + \big[\mathcal{L}\{f_r(\beta)\} - \mathcal{L}\{f_a(\beta)\}\big]r_f(t_0^*)\Big\}}. \quad (8)$$

(ii) If $q(0) \geq 0$, then the optimal software rejuvenation schedule is $t_0^* = 0$, *i.e.* it is optimal to start the software rejuvenation just after entering the failure probable state, and the minimum expected total discounted cost is given by $TC(0)$.

(iii) If $q(\infty) \leq 0$, then the optimal software rejuvenation schedule is $t_0^* \to \infty$, *i.e.* it is optimal not to carry out the software rejuvenation, and the minimum expected total discounted cost is given by $TC(\infty)$.

(2) Suppose that the failure time distribution is DFR (decreasing failure rate) under the assumption (A-1). Then, the expected total discounted cost function $TC(t_0)$ is a concave function of t_0, and the optimal software rejuvenation schedule is $t_0^* = 0$ or $t_0^* \to \infty$.

From the theorem above, it is seen that the optimal software rejuvenation policy t_0^* can be calculated if the underlying failure time distribution $F_f(t)$ and other model parameters are given. On the other hand, if the knowledge of $F_f(t)$ is not available, a nonparametric method based on the modified total time on test statistics [3] can be applied to estimate t_0^* from the complete sample of failure time data. These detailed results will be reported in the conference.

References

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