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Asymptotic Properties of LSE for the Regression Model with Stochastic Regressor

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Model: $y_t = \beta_0 + \beta_1 x_t + u_t, \quad u_t = \phi u_{t-1} + \varepsilon_t, \quad t = 1,2,\cdots,T, \quad |\phi| < 1 \quad (\Sigma = E(uu'))$

Estimators: OLS: $\hat{\theta}_O = (\hat{\beta}_{0O}, \hat{\beta}_{1O})' = (X'X)^{-1}X'y$

GLS: $\hat{\theta}_G = (\hat{\beta}_{0G}, \hat{\beta}_{1G})' = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}y$

[Case 1] Assumption 1 $x_t = \alpha x_{t-1} + v_t, \quad |\alpha| < 1$

Assumption 2 $(v_t, \varepsilon_t)' \sim IID\left(0, Diag\left(\sigma_v^2, \sigma_\varepsilon^2\right)\right)$

$$\sqrt{T}(\hat{\theta}_O - \theta) \longrightarrow N\left(0, Diag\left(\frac{\sigma_\varepsilon^2}{(1-\phi)^2}, \left(\frac{1-\alpha^2}{\sigma_v^2}\right)^2 2\pi \int_{-\pi}^{\pi} fx(\lambda) fu(\lambda) d\lambda\right)\right)$$

$$\sqrt{T}(\hat{\theta}_G - \theta) \longrightarrow N\left(0, Diag\left(\frac{\sigma_\varepsilon^2}{(1-\phi)^2}, \left(\frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{fx(\lambda)}{fu(\lambda)} d\lambda\right)^{-1}\right)\right)$$

$$f_x(\lambda) = \frac{\sigma_u^2}{2\pi}\frac{1}{\left|1-\alpha e^{i\lambda}\right|^2}, \quad f_u(\lambda) = \frac{\sigma_\varepsilon^2}{2\pi}\frac{1}{\left|1-\phi e^{i\lambda}\right|^2} \quad (-\pi \le \lambda \le \pi)$$

$asy.\ eff\ (\hat{\beta}_{1O}) \fallingdotseq 0.078 \quad (\alpha = 0.6, \phi = 0.9)$

[Case 2] Assumption 1' $x_t = (1 - \frac{c}{T})x_{t-1} + e_t \quad c \ge 0$

$$e_t = \psi(L)v_t, \quad \psi(L) = \sum_{i=0}^{\infty}\psi_i L^i, \quad \psi_0 = 1, \quad \sum_{i=0}^{\infty} i|\psi_i| < \infty$$

All roots of $\psi(z) = 0$ are outside the unit circle.

$$Diag\left(\sqrt{T}, T\right)(\hat{\theta}_O - \theta),\ Diag\left(\sqrt{T}, T\right)(\hat{\theta}_G - \theta)$$

$$\longrightarrow \frac{\sigma_\varepsilon}{\phi(1)}\begin{pmatrix} 1 & \sigma_v\psi(1)\int_0^1 X_c(r)dr \\ * & \sigma_v^2\psi^2(1)\int_0^1 X_c^2(r)dr \end{pmatrix}^{-1}\begin{pmatrix} W_\varepsilon(1) \\ \sigma_v\psi(1)\int_0^1 X_c(r)dW_\varepsilon(r) \end{pmatrix}$$

$$\equiv \frac{\sigma_\varepsilon}{\phi(1)}H^{-1}K \sim MN\left(0, \frac{\sigma_\varepsilon^2}{\phi^2(1)}H^{-1}\right)$$

$X_c : O-U$ Process

[Case 3] Assumption 1 " $(1-L)^d x_t = e_t$, $d > \frac{1}{2}$, $(1-L)^d \equiv \sum_{k=0}^{t} \frac{(-d)_k}{k!} L^k$, $(d)_k = \frac{\Gamma(d+k)}{\Gamma(d)}$

$$e_t = \psi(L)v_t,\ \psi(L) = \sum_{i=0}^{\infty} \psi_i L^i,\ \psi_0 = 1,\ \sum_{i=0}^{\infty} i|\psi_i| < \infty$$

All roots of $\psi(z) = 0$ are outside the unit circle.

$$Diag\left(\sqrt{T}, T^d\right)(\hat{\theta}_O - \theta), Diag\left(\sqrt{T}, T^d\right)(\hat{\theta}_G - \theta)$$

$$\longrightarrow \frac{\sigma_\varepsilon}{\phi(1)} \begin{pmatrix} 1 & \int_0^1 F_{d-1}(r)dr \\ * & \int_0^1 F_{d-1}^2(r)dr \end{pmatrix}^{-1} \begin{pmatrix} W_\varepsilon(1) \\ \int_0^1 F_{d-1}(r)dW_\varepsilon(r) \end{pmatrix}$$

$$\equiv \frac{\sigma_\varepsilon}{\phi(1)} H^{-1}K \sim MN\left(0, \frac{\sigma_\varepsilon^2}{\phi^2(1)} H^{-1}\right)$$

$$F_{d-1}(r) \equiv \frac{\sigma_v \psi(1)}{\Gamma(d)} \int_0^r (r-s)^{d-1} dW_v(s)$$

References

[1] Billingsley, P.(1968), *Convergence of Probability Measures*, John Wiley, New York.

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[3] Phillips, P.C.B. and Park, J.Y. (1988), "Asymptotic Equivalence of Ordinary Least Squares and Generalized Least Squares in Regression with Integrated Regressors." *Journal of the American Statistical Association*, 83, 111-115.