Repeated Game of Criminal vs Police, I & II.

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Part I : Complete Information Case.

Abstract. Section 1. Formulation of the Game —— Criminal vs. Police 2 ~ 5. Games $\Gamma_{n,n}(n)$, $\Gamma_{1,1}(n)$, $\Gamma_{1,n}(n)$, $\Gamma_{n,1}(n)$, resp. 6. Remarks and a Numerical Example.

Five theorems with proofs.

Part II : Incomplete Information Case

Abstract. Section 1. The Game of Criminal vs. Police under Incomplete Information. 2. One-Period Games under Incomplete Information 3. Two-Period Game under Symmetric Information.

Four theorems with proofs, and a numerical example.

The game is played as a repeated game over n periods between a potenti al criminal offender (hereafter called a criminal, or player I) and a law-enforcement authorities (hereafter called police, or player II) Being a repeated game implies that the fundamentals of the game are the same in each period. There are two pure strategies available in each period to player I: to commit a crime (C) and to act honestly (H). Similarly, player II has two pure strategies: to enforce the law (E) or to do nothing (N). If player I chooses H he earns his leagal income $r > 0$ (dollars). If he chooses C, illegal income in amount of $\pi > 0$, in addition to his legal income r, may be earned. However if I's crime is detected and arrested by II, I is punished by having to pay a fine in amount of $f > 0$, and Inprisoned until the end of the game. When caught in prison, I earns no income at all, of course.

If player II chooses E, with a cost of $c > 0$ (dollars), he can (cannot), catch I's crime with probability $p(\bar{p} = 1-p)$. In case that I commits crime that goes unpunished, a loss of $l > 0$ is inflicted upon society.

So a single stage of this game has the game tree as shown by Figure 1, and is represented by a bimatrix game with payoff bimatrix (1).

(1) (I) / (II)

	E	N
C	$-pf+\bar{p}(r+\pi),\ -(c+\bar{p}l)$	$r+\pi,\ -l$
H	$r,\ -c$	$r,\ 0$

We assume that $c < pl$ i.e. the strategy E for player II has a positive merit of choosing. This condition is very important as is seen in the proofs of the subsequent theorems.

We shall disacuss the n-stage game, where player I wants to commit crime at most k of n periods, and player II attempts to prevent I's illegal act by taking enforcement action at most m times during n periods. After each period is over, the outcome in that period becomes known to both players. The total payoff during n periods is the sum of the payoffs on each period. We assume that all of the above information is known to both players.

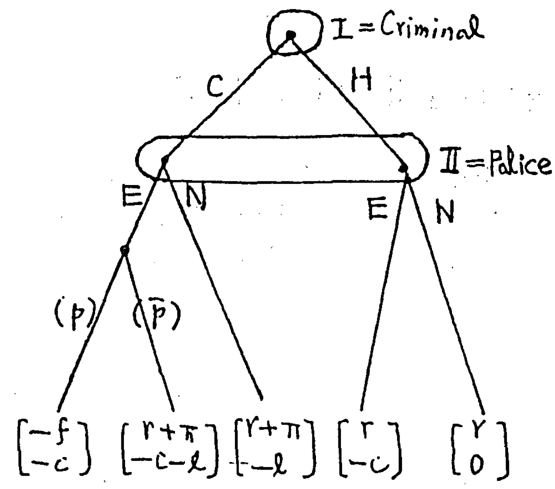

Figure 1. Game tree of a single stage

C: commit a crime
H: be honest
E: law-enforcement
N: do nothing
p: prob. of being punished
f: amount of fine
r(π): leagal (illeagal) income to I (if unpunished)
c: II's cost of law-enforcement
ℓ: social loss to II for an unpunished crime

Let $\Gamma_{k,m}(n)$ denote the game described above. (n, k, m) denotes the state of the system in which players I and II possess k and m times to take actions, respectively, and they have n periods to go as their "mission time." Let $(u_{k,m}(n), v_{k,m}(n))$ represent the equilibrium values of this non-zero-sum n-stage game $\Gamma_{k,m}(n)$. Then the Optimality Equation of dynamic programming gives a system of equations

$(u_{k,m}(n), v_{k,m}(n)) =$ Eq.Val.

(2)

	E	N
C	$-pf+\bar{p}\{r+\pi+u_{k-1,m-1}(n-1)\}$, $-(c+\bar{p}\ell)+\bar{p}v_{k-1,m-1}(n-1)$	$r+\pi+u_{k-1,m}(n-1)$, $-\ell+v_{k-1,m}(n-1)$
H	$r+u_{k,m-1}(n-1)$, $-c+v_{k,m-1}(n-1)$	$r+u_{k,m}(n-1)$, $v_{k,m}(n-1)$

(if the equilibrium values exist uniquely), with the boundary conditions : (以下略)

In the present paper we shall investigate the incomplete-information version of the above game. Each player may not know his opponent's and/or his own allowed number of actions, and is able to estimate only by some probability dis tribution Suppose that (k, m) is a bivariate random variable with independent Bernoulli marginal distributions with parameters α and β (See Table 1) This distribution is assumed to be a common knowledge for each player.

Table 1 Bivariate type distribution

(3)

(I) \ (II)	$m=m'$	$m=m''$	
$k=k'$	$\bar{\alpha}\bar{\beta}$	$\bar{\alpha}\beta$	$\bar{\alpha}$
$k=k''$	$\alpha\bar{\beta}$	$\alpha\beta$	α
	$\bar{\beta}$	β	

Among the possible $2^4=16$ ISs we shall focus our attention to the following four ISs.

(1°) $I^{11:11}$ i.e. complete information : Both players know both of k and m.

(2°) $I^{10:01}$ i.e. symmetric (or private) information : Each player knows his owntype, but not the opponents.

(3°) $I^{11:01}$ and $I^{10:11}$ i.e. asymmetric information ; One player knows both players' types whereas the other can know his own type only. (以下略)

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Part II. —— Game Th. & Appl., IV, Nova Sci. Publishers, New York, 1998.