DEA Sensitivity Analysis by Changing a Reference Set: Regional Contribution to Japanese Industrial Development

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We report the paper which is submitted for publication in *OMEGA*[2]

1 Introduction

The purpose of this article is to propose a new type of DEA sensitivity analysis, which is designed to examine the stability of DEA efficiency by changing a reference set of DMUs. Using the proposed technique, this article empirically examines an industrial policy issue as to how to develop Japanese industries from the perspective of regional contributions and local governments.

2 DEA Efficiency Analysis: Slack-Adjusted BCC Model[3]

$$\begin{aligned}
\min \quad & \eta_a = \theta - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{R_i^-} - \frac{1}{k}\sum_{i=1}^{k}\frac{s_r^+}{R_r^+} \\
\text{s.t.} \quad & -\sum_{j\in J} x_{ij}\lambda_j + \theta x_{ia} - s_i^- = 0, \ (i=1,\ldots,m), \\
& \sum_{j\in J} y_{rj}\lambda_j - s_r^+ = y_{ra}, \ (r=1,\ldots,k), \qquad (1) \\
& \sum_{j\in J}\lambda_j = 1, \\
& \lambda_j \geq 0 \ (j\in J), s_i^- \geq 0, \text{ and } s_r^+ \geq 0.
\end{aligned}$$

where $R_i^- = \max_j x_{ij}$ and $R_i^+ = \max_j y_{rj}$.

3 DEA Efficiency Evaluation by Changing a Reference Set

3.1 DEA Cross-Reference (DCR)

Using the SA-BCC model, this section proposes a new type of efficiency measure in which our approach changes the reference set for (1). Hence, we can measure an efficiency score of SA-BCC under the condition that its reference set is variable. This article refers to it as "DEA Cross-Reference (DCR) efficiency measure", which can be used as a means for evaluating sensitivity analysis between two DMUs.

To describe mathematical features of the DCR, this article formulates the following input-oriented model:

$$\begin{aligned}
\min \quad & \delta_{a,b} = \theta - \frac{1}{m}\sum_{i=1}^{m}\frac{t_i^-}{R_i^-} - \frac{1}{k}\sum_{i=1}^{k}\frac{t_r^+}{R_r^+} \\
\text{s.t.} \quad & -\sum_{j\in J-\{b\}} x_{ij}\lambda_j + \theta x_{ia} - t_i^- = 0, \ (i=1,\ldots,m), \\
& \sum_{j\in J-\{b\}} y_{rj}\lambda_j - t_r^+ = y_{ra}, \ (r=1,\ldots,k), \qquad (2) \\
& \sum_{j\in J-\{b\}}\lambda_j = 1, \\
& \lambda_j \geq 0 \ (j\in J-\{b\}), t_i^- \geq 0, \text{and } t_r^+ \geq 0.
\end{aligned}$$

This article summarizes the following perspectives:

(a) BCC: If $\{b\} = \phi$, then $\delta^*_{a,b}$ equals the (input-oriented) SA-BCC efficiency score.

(b) Andersen and Petersen[1]: If $\{b\} = \{a\}$ in (2), then $\delta^*_{a,b}$ equals the extended DEA measure(EDM) proposed by Andersen and Petersen. (2) may be considered as a general model of [1].

(c) Frontier Shift: If the ath DMU is evaluated as efficient by the conventional BCC model, $\delta^*_{a,b}$ may become more than unity due to a location shift of its efficiency frontier.

3.2 Properties of DCR

Property 1. $\delta^*_{a,b} \geq \eta^*_a$.

Property 2. If the ath DMU is the same as the bth DMU in (2), then $\delta^*_{a,a}$ may take either

(i) $\delta^*_{a,a} \geq 1$ is observed on optimality, or

(ii) (2) becomes infeasible if and only if the ath DMU does not belong to a production possibility set of the ath DMU, whose efficiency is evaluated without itself.

Property 3. If the ath DMU is not the same as the bth DMU, then (2) may produce a feasible solution.

Property 4. If the bth DMU is not inefficient in SA-BCC, then $\delta^*_{a,b} = \eta^*_a$ is obtained on optimality.

Property 5. If the ath DMU is efficient in SA-BCC and different from the bth DMU, then $\delta^*_{a,b} = 1$.

3.3 Cross-Sensitivity Analysis

It can be easily thought that a difference between a DCR score ($\delta^*_{a,b}$) and a SA-BCC score (η^*_a) may serve to make a new type of DEA measure. This article refers to it as "DEA Cross-Sensitivity (DCS) measure", hereafter. The degree of DCS, denoted by $D^*_{a,b}$, is formally defined as the following measure:

$$D^*_{a,b} = \delta^*_{a,b} - \eta^*_a.$$

4 Regional Contribution to Japanese Industrial Development and Policy Implications

4.1 A Data Set

This article is interested in the measurement of each regional contribution to the whole Japanese industrial development, and considers the Prefecture as a DMU for our DEA application. The production process of these Prefectures is characterized by three outputs and eight inputs in this study.

Three outputs:

(a) the total amount of production in the primary industry(e.g., agriculture and fishery industries)

(b) the total amount of production in the secondary industry(e.g., manufacturing industries)

(c) the total amount of production in the tertiary industry(e.g., service industries)

Eight inputs:

(a) the number of individuals in the primary industry

(b) the number of individuals in the secondary industry

(c) the number of individuals in the tertiary industry

(d) the total amount of public spending(budget allocation) by the Japanese government

(e) the total amount of loans from commercial banks

(f) the number of JA offices

(g) the number of offices and manufacturing sites related to the secondary industries

(h) the number of whole sellers and retailers

4.2 SA-BCC/AR Results

Finding 1. 17 DMUs(Prefectures) are efficient while the remaining 31 DMUs are inefficient in terms of SA-BCC/AR. Since all the multipliers related to these efficiency scores are not zero, we can consider that all inputs and outputs are fully utilized for our DEA efficiency evaluation. Based upon the fact, we extend the resulting efficiencies and multipliers further into the sensitivity study proposed in this article.

Finding 2. The status of efficiency can be found in highly-populated regions(Tokyo, *etc.*), all of which have well-known large manufacturing and/or service industries. Conversely, the status of efficiency can be identified in other DMUs (Hokkaido, *etc.*). These Prefectures attain the efficiency due to their large primary industries.

Finding 3. An interesting result may be found in Osaka, which does not attain the status of efficiency in the SA-BCC/AR measurement. Furthermore, Miyagi does not attain the efficiency, as well. Why?

This article explores the above empirical questions using the proposed DCR approach, so that we can clearly demonstrate the use and importance of DCR in DEA efficiency evaluation.

4.3 Resulting DCR and DCS Estimates

Finding 4. Osaka and Miyagi become efficient after Tokyo is omitted from their reference sets. This result indicates that the two Prefectures have industrial structures similar to Tokyo, consequently becoming inefficient due to the existence of Tokyo.

Finding 5. There are two types of SA-BCC/AR efficient DMUs. A group of efficient DMUs (Hokkaido, Tokyo, *etc.*) considerably influence the magnitudes of DCR estimates of other inefficient DMUs at the level that some inefficient DMU(s) may become efficient if the DMU belonging to the group is omitted. The other group of efficient DMUs (i.e., Ibaraki, Tochigi, *etc.*) do not have such a major influence on the inefficient DMUs.

Finding 6. The EDM of Tokyo becomes an infeasible solution because its production achievement does not belong to the production possibility set of all the other DMUs (without Tokyo). As found in Tokyo that dominates the other DMUs in the magnitude of inputs and outputs, an EDM may usually face to an infeasible solution under variable RTS.

Finding 7. It is easily identified from the magnitude of DCS that Tokyo has a major impact upon many other DMUs, particularly on Osaka.

References

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