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A (T, S) Inventory/Production System with Limited Production Capacity and Uncertain Demands

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1 Introduction

We consider a single-item, periodic-review inventory/production model with uncertain demands in which a production capacity is limited per period. Demands D_n in period n are represented by independent, identically distributed (IID) nonnegative random variables, and a (T, S) inventory control policy is considered. Under this policy, the inventory position (IP; physical stock plus lots on order minus backorders) is reviewed in every T units of time and, at a moment of review, a replenishment order is placed to the production facility in order to raise the IP to a fixed level $S > 0$. An order will be delivered at the end of each period. But, because of capacity constraint, the leadtime is generally random and the entire order may not be filled simultaneously. Demands are satisfied by on-hand inventory on a first-come-first-serve (FCFS) basis and excess demands are backordered.

Traditionally, performance measures of inventory systems are calculated in terms of cost. However, in certain industries, customer waiting times have become more important as performance measure (see, e.g., [1]). Prior treatments of this problem assume that the supply process is *exogenous*, as described in [2]; the evolution of the supply system is independent of demands and replenishment orders. However, consider a manufacturer with a few sales subsidiaries, where each subsidiary controls its own inventory system and total orders from the subsidiaries to the manufacturer are close to the production capacity per period. This scenario is typical in Japanese manufacturers. In such cases, since each order contributes to the fluctuation of the manufacturer's total workload, the supply process should be included as a part of the inventory model so that the workload determines the leadtime of a replenishment order.

2 The Associated MC

Because our concern is not optimal values of T, we henceforth fix $T = 1$ by defining the time unit appropriately. The time interval $(n-1, n]$ is called the nth period. The following notation will be used for the inventory system:

J_n = the inventory/backorder level;
R_n = the quantities ordered.

Replenishment orders are produced in the production system with a finite capacity $c > 0$:

Q_n = the remainings to be produced;
P_n = the number produced during n.

We note that $P_n = \min\{Q_n, c\}$.

The inventory/backorder level satisfies

$$J_{n+1} = J_n + \min\{c, S - J_n\} - D_{n+1} \quad (2.1)$$

Since the demands D_n are IID, $\{J_n\}$ is Markovian with the state space $\{\cdots, -1, 0, 1, \cdots, S-1, S\}$.

Next, define the *shortfall* for inventory at the end of period n: $Y_n = S - J_n$. Then $Y_n = Q_{n+1}$. This is so, because the (T, S) policy tries to keep the IP to the fixed level S. The process $\{Y_n\}$ is also Markovian with the state space $\mathcal{N}_+ = \{0, 1, 2, \cdots\}$ that satisfies

$$Y_{n+1} = \max\{0, Y_n - c\} + D_{n+1} \quad (2.2)$$

It should be noted that the recursion (2.2) does not include the order-up-to-level S. This is the advantage to consider the shortfall Y_n rather than

J_n directly. Note that the recursion (2.2) is the same as the queueing process of a bulk-arrival, batch-service M/D/1 queue when observed just after service completion. Hence, $\{Y_n\}$ is stable as long as the production capacity exceeds the mean demand, i.e. $c > E\{D\}$. (We assume that the production system is not subject to failure.)

The next result is simple but important to characterize the (T, S) inventory/production system (cf. the Kanban system).

Proposition 2.1 *We have $R_{n+1} = D_n$ for all n. That is, the ordering policy is to order so as to satisfy the previous demand.*

3 System Characteristics

In order to consider the transient behavior of system characteristics, we define $\pi(n) = \{\pi_k(n)\}$ where $\pi_k(n) = P\{Y_n = k\}$. It is important to be able to investigate the transient behavior, because we can then understand the impact of the initial inventory/backorder level on the system characteristics. The transition matrix of $\{Y_n\}$ is given by $\mathbf{P}$. It is well known that $\pi(n+1) = \pi(n)\mathbf{P}$.

Let L_n denote the leadtime of an order made at the beginning of period n. The leadtime is defined as the difference between periods in which an order is made and is fully delivered. The order made at that period will be fully delivered after $\lceil Q_n/c \rceil$ periods, where $\lceil x \rceil$ denotes the smallest integer not less than x. Hence the leadtime probability is given by

$$\ell_k(n+1) = \frac{P\{kc < Y_n \leq (k+1)c,\ D_n > 0\}}{P\{D_n > 0\}}$$

Note that the leadtime is considered only under the condition $D_n > 0$. After an algebra, we have

$$\ell_0(n+1) = \frac{\Pi_c(n) - d_0 \Pi_{2c}(n-1)}{1 - d_0},$$

where $\Pi_j(n) = \sum_{k=0}^{j} \pi_k(n)$, and $\ell_k(n+1)$ are obtained similarly. The leadtime distribution $\{\ell_k(n)\}$ does not depend on the order-up-to-level S. When $c = \infty$, it is clear that $\ell_0(n) = 1$, meaning $L_n = 0$ almost surely.

Next consider the waiting time distributions. The waiting time is defined as the difference between periods in which positive demands arrive and are fully satisfied. The waiting time for a customer who arrives during period n is denoted by W_n. It is readily seen that, given $D_n > 0$, $W_n = 0$ if and only if $J_n \geq 0$ while $W_n = k > 0$ if and only if $\lceil -J_n/c \rceil = k$. Hence the waiting time probability is given by

$$w_k(n) = P\{\lceil \max\{-J_n,\ 0\}/c \rceil = k | D_n > 0\}$$

After an algebra, we then have

$$w_0(n) = \frac{\Pi_S(n) - d_0 \Pi_{S+c}(n-1)}{1 - d_0} \qquad (3.1)$$

and $w_k(n)$ are obtained similarly. It should be noted that, in contrast to the leadtime distribution, the waiting time distribution $\{w_k(n)\}$ depends on the order-up-to-level S too.

For random variables A and B, A is said to be *stochastically greater* than B if $E[f(A)] \geq E[f(B)]$ for all nondecreasing functions f. In this case, we write $A \geq_{\mathrm{st}} B$.

In the following, we shall denote by $L_n(c)$ and $W_n(c, S)$ the leadtime and the waiting time when the production capacity is c and the order-up-to-level is S.

Proposition 3.1 *Suppose $d_0 = 0$. Then, $L_{n+1}(c) \geq_{\mathrm{st}} L_{n+1}(c+1)$ and $W_n(c, S) \geq_{\mathrm{st}} W_n(c+1, S)$ for each fixed S.*

Proposition 3.2 *Suppose $d_0 = 0$. Then, $W(c, S) \geq_{\mathrm{st}} W(c, S+1)$ for each fixed c.*

Other monotonicity results as well as a numerical example will be reported in the presentation.

References

[1] van der Heijden, M.C. and De Kok, A.G. (1992), *ZOR-Methods and Models of OR*, **36**, 315–332.

[2] Zipkin, P. (1986), *Naval Research Logistics*, **33**, 763–774.