The effects of labor choice on investment and output dynamics

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1 Introduction

Most studies on corporate investment have focused on capital as an input, leaving the effects of labor on investment decisions mostly unanswered. In this study, we incorporate both labor and capital as inputs and investigate the impact of labor choice on a firm's investment timing and size decisions. Furthermore, we analyze the dynamics of labor employment, output, and price after the investment.

2 Setup

A firm's output at time t is $Q_t = L_t^a K_t^{1-a}$ where L_t and K_t denote the amount of labor and capital, respectively, and $a \in (0,1)$ denotes labor share. Its price is $P_t = X_t Q_t^{\gamma-1}$ where $\gamma \in (0,1)$. The demand shock X_t follows $\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}W_t$ where μ and σ are positive constants and $(W_t)_{t\geq 0}$ is a standard Brownian motion. A risk-free rate is given by a positive constant r. Lumpy investment in capital K incurs costs $\delta_0+\delta_1 K$ and labor requires wage w per unit time. The firm's profit flow after investment is

$$\pi(X_t, L_t, K_t) = P_t Q_t - wL_t = X_t (L_t^a K_t^{1-a})^\gamma - wL_t.$$

3 Models and solutions

Suppose that the firm can adjust the amount of labor at any time without costs. The optimal labor with full flexibility at time t is

$$L^*(X_t, K) = \Big(\frac{a\gamma X_t K^{(1-a)\gamma}}{w}\Big)^{\frac{1}{1-a\gamma}},$$

and thus, the instant profit flow is

$$\pi(X_t, L^*(X_t,K), K) = \psi X_t^{\frac{1}{1-a\gamma}} K^{\frac{(1-a)\gamma}{1-a\gamma}},$$

where $\psi := (1-a\gamma)(\frac{a\gamma}{w})^{\frac{a\gamma}{1-a\gamma}}$.

Proposition 1 *Given initial demand shock X, the firm value with fully flexible labor is*

$$V(X) = \begin{cases} A(X_I^*, K^*)\big(\frac{X}{X_I^*}\big)^\alpha & \text{if } X < X_I^*, \\ A(X, K^*(X)) & \text{if } X \geq X_I^*, \end{cases}$$

where

$$A(X,K) := \phi K^{\frac{(1-a)\gamma}{1-a\gamma}} X^{\frac{1}{1-a\gamma}} - (\delta_0 + \delta_1 K),$$

with $\phi := \psi/\big(r - \frac{\mu}{1-a\gamma} - \frac{a\gamma\sigma^2}{2(1-a\gamma)^2}\big)$ *and* $\alpha := 1/2 - \mu/\sigma^2 + \sqrt{(1/2-\mu/\sigma^2)^2 + 2r/\sigma^2}$.

The optimal capital $K^* := K^*(X_I^*)$ and investment threshold $X_I^* := X_I^*(K^*)$ are determined by

$$K^*(X) = \Big\{\frac{(1-a)\gamma\phi}{\delta_1(1-a\gamma)}\Big\}^{\frac{1-a\gamma}{1-\gamma}} X^{\frac{1}{1-\gamma}},$$

$$X_I^*(K) = \Big\{\frac{\alpha(\delta_0+\delta_1 K)}{(\alpha - \frac{1}{1-a\gamma})\phi}\Big\}^{1-a\gamma} \Big(\frac{1}{K}\Big)^{(1-a)\gamma},$$

and the optimal labor at the investment timing is $L^* := L^*(X_I^*, K^*)$.

Now suppose that if the amount of labor is increased, it cannot be reduced afterwards (e.g., employment protection).

Lemma 1 *The optimal labor with upward-only adjustability at time $t(\geq \tau)$ given capital K is*

$$\hat{L}^*(M_t, K) = \Big(\frac{(\alpha-1)ra\gamma M_t K^{(1-a)\gamma}}{\alpha w(r-\mu)}\Big)^{\frac{1}{1-a\gamma}},$$

where $M_t := \max_{s\in[\tau,t]} X_s$.

Proposition 2 *Given initial demand shock X, the firm value with upward-adjustable labor is*

$$\hat{V}(X) = \begin{cases} \hat{A}(\hat{X}_I^*, \hat{L}^*, \hat{K}^*)\big(\frac{X}{\hat{X}_I^*}\big)^\alpha & \text{if } X < \hat{X}_I^*, \\ \hat{A}(X, \hat{L}^*(X, \hat{K}^*(X)), \hat{K}^*(X, \hat{L}^*(X))) & \text{if } X \geq \hat{X}_I^*, \end{cases}$$

where

$$\hat{A}(X,L,K) := \frac{XL^{a\gamma}K^{(1-a)\gamma}}{r-\mu} - \frac{wL}{r} + \chi L^{1-\alpha(1-a\gamma)} K^{\alpha(1-a)\gamma} X^\alpha - (\delta_0+\delta_1 K),$$

with $\chi := \big(\frac{(\alpha-1)r}{w}\big)^{\alpha-1}\big(\frac{a\gamma}{\alpha(r-\mu)}\big)^\alpha \frac{1}{\alpha(1-a\gamma)-1}$.

The optimal capital $\hat{K}^*$ and investment threshold $\hat{X}_I^*$ are determined by

$$\hat{X}_I^* = \frac{\alpha(r-\mu)(\frac{wL}{r}+\delta_0+\delta_1\hat{K}^*)}{(\alpha-1)(\hat{L}^*)^{a\gamma}(\hat{K}^*)^{(1-a)\gamma}},$$

$$\frac{(1-a)\gamma\hat{X}_I^*(\hat{L}^*)^{a\gamma}(\hat{K}^*)^{(1-a)\gamma-1}}{r-\mu} + \alpha(1-a)\gamma\chi(\hat{L}^*)^{1-\alpha(1-a\gamma)}(\hat{K}^*)^{\alpha(1-a)\gamma-1}(\hat{X}^*)^{\alpha} = \delta_1,$$

with the optimal labor at the investment timing $\hat{L}^* := \hat{L}^*(\hat{X}_I^*, \hat{K}^*)$.

Now let us suppose that the firm can dismiss workers but cannot recruit employees after the investment (e.g., tight labor market).

Lemma 2 *The optimal labor with downward-only adjustability at time $t(\geq \tau)$ given capital K is*

$$\check{L}^*(m_t, K) = \Big(\frac{(\beta-1)ra\gamma m_t K^{(1-a)\gamma}}{\beta w(r-\mu)}\Big)^{\frac{1}{1-a\gamma}},$$

where $m_t := \min_{s\in[\tau,t]} X_s$ and $\beta := 1/2 - \mu/\sigma^2 - \sqrt{(1/2-\mu/\sigma^2)^2 + 2r/\sigma^2}$.

Proposition 3 *Given initial demand shock X, the firm value with downward-adjustable labor is*

$$\check{V}(X) = \begin{cases} \check{A}(\check{X}_I^*, \check{L}^*, \check{K}^*)\big(\frac{X}{\check{X}_I^*}\big)^{\alpha} & \text{if } X < \check{X}_I^*, \\ \check{A}(X, \check{L}^*(X, \check{K}^*(X)), \check{K}^*(X, \check{L}^*(X))) & \text{if } X \geq \check{X}_I^*, \end{cases}$$

where

$$\check{A}(X, L, K) := \frac{XL^{a\gamma}K^{(1-a)\gamma}}{r-\mu} - \frac{wL}{r} + \zeta L^{1-\beta(1-a\gamma)}K^{\beta(1-a)\gamma}X^{\beta} - (\delta_0+\delta_1 K),$$

with $\zeta := \big(\frac{(\beta-1)r}{w}\big)^{\beta-1}\big(\frac{a\gamma}{\beta(r-\mu)}\big)^{\beta}\frac{1}{\beta(1-a\gamma)-1}$. The optimal capital $\check{K}^*$ and investment threshold $\check{X}_I^*$ are determined by

$$\check{X}_I^* = \frac{r-\mu}{(\alpha-1)(\check{L}^*)^{a\gamma}(\check{K}^*)^{(1-a)\gamma}} \times \Big\{\alpha\Big(\frac{w\check{L}^*}{r}+\delta_0+\delta_1\check{K}^*\Big) - \frac{(\alpha-\beta)w\check{L}^*}{(1-\beta)(1-\beta(1-a\gamma))r}\Big\},$$

$$\frac{(1-a)\gamma\check{X}_I^*(\check{L}^*)^{a\gamma}(\check{K}^*)^{(1-a)\gamma-1}}{r-\mu} + \beta(1-a)\gamma\zeta(\check{L}^*)^{1-\beta(1-a\gamma)}(\check{K}^*)^{\beta(1-a)\gamma-1}(\check{X}_I^*)^{\beta} = \delta_1,$$

with the optimal labor at the investment timing $\check{L}^* := \check{L}^*(\check{X}_I^*, \check{K}^*)$.

4 Dynamics analysis

Given the optimal investment strategies, we can evaluate how labor employment and output are expected to grow in the long-run.

Proposition 4 *Expected labor with full flexibility, upward-only and downward-only adjustability after time T elapses from investment timing τ are*

$$\mathbb{E}[L_{\tau+T}^*] = \Big(\frac{a\gamma(K^*)^{(1-a)\gamma}X_I^* e^{\lambda_L T}}{w}\Big)^{\frac{1}{1-a\gamma}},$$

$$\mathbb{E}[\hat{L}_{\tau+T}^*] = \Big(\frac{(\alpha-1)ra\gamma(\hat{K}^*)^{(1-a)\gamma}\hat{X}_I^*}{\alpha w(r-\mu)}\Big)^{\frac{1}{1-a\gamma}}\Big[e^{\frac{\lambda_L T}{1-a\gamma}}\Big(1+\frac{\kappa_L}{\lambda_L}\Big) \times \Phi\Big(\frac{(\lambda_L+\kappa_L)\sqrt{T}}{\sigma}\Big) + \Big(1-\frac{\kappa_L}{\lambda_L}\Big)\Phi\Big(\frac{-(\lambda_L-\kappa_L)\sqrt{T}}{\sigma}\Big)\Big],$$

$$\mathbb{E}[\check{L}_{\tau+T}^*] = \Big(\frac{(\beta-1)ra\gamma(\check{K}^*)^{(1-a)\gamma}\check{X}_I^*}{\beta w(r-\mu)}\Big)^{\frac{1}{1-a\gamma}}\Big[e^{\frac{\lambda_L T}{1-a\gamma}}\Big(1+\frac{\kappa_L}{\lambda_L}\Big) \times \Phi\Big(\frac{-(\lambda_L+\kappa_L)\sqrt{T}}{\sigma}\Big) + \Big(1-\frac{\kappa_L}{\lambda_L}\Big)\Phi\Big(\frac{(\lambda_L-\kappa_L)\sqrt{T}}{\sigma}\Big)\Big],$$

respectively, where $\lambda(n) := \mu + (n-1)\sigma^2/2$, $\kappa(n) := n\sigma^2/2$, $\lambda_L := \lambda(1/(1-a\gamma))$, $\kappa_L := \kappa(1/(1-a\gamma))$, and $\Phi(\cdot)$ denotes the cumulative distribution function of normal distribution.

Proposition 5 *Expected output with fully flexible, upward-adjustable, and downward-adjustable labor after time T elapses from investment timing τ are*

$$\mathbb{E}[Q_{\tau+T}^*] = \Big(\frac{a\gamma X_I^* e^{\lambda_Q T}}{w}\Big)^{\frac{a}{1-a\gamma}}(K^*)^{\frac{1-a}{1-a\gamma}},$$

$$\mathbb{E}[\hat{Q}_{\tau+T}^*] = \Big(\frac{(\alpha-1)ra\gamma\hat{X}_I^*}{\alpha w(r-\mu)}\Big)^{\frac{a}{1-a\gamma}}(\hat{K}^*)^{\frac{1-a}{1-a\gamma}}\Big[e^{\frac{a\lambda_Q T}{1-a\gamma}}\Big(1+\frac{\kappa_Q}{\lambda_Q}\Big) \times \Phi\Big(\frac{(\lambda_Q+\kappa_Q)\sqrt{T}}{\sigma}\Big) + \Big(1-\frac{\kappa_Q}{\lambda_Q}\Big)\Phi\Big(\frac{-(\lambda_Q-\kappa_Q)\sqrt{T}}{\sigma}\Big)\Big],$$

$$\mathbb{E}[\check{Q}_{\tau+T}^*] = \Big(\frac{(\beta-1)ra\gamma\check{X}_I^*}{\beta w(r-\mu)}\Big)^{\frac{a}{1-a\gamma}}(\check{K}^*)^{\frac{1-a}{1-a\gamma}}\Big[e^{\frac{a\lambda_Q T}{1-a\gamma}}\Big(1+\frac{\kappa_Q}{\lambda_Q}\Big) \times \Phi\Big(-\frac{(\lambda_Q+\kappa_Q)\sqrt{T}}{\sigma}\Big) + \Big(1-\frac{\kappa_Q}{\lambda_Q}\Big)\Phi\Big(\frac{(\lambda_Q-\kappa_Q)\sqrt{T}}{\sigma}\Big)\Big],$$

respectively, where $\lambda_Q := \lambda(a/(1-a\gamma))$ and $\kappa_Q := \kappa(a/(1-a\gamma))$.

A detailed illustration on the results from comparative statics and relevant empirical evidence will be given at the presentation.

References

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