Optimal Opportunity-based Age Replacement Policies with NPV Approach

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1. Introduction

In order to run the system steadily and effectively, various simple but realistic replacement policies were studied. In recent years, as an extension of the preventive replacement modeling, the standard age replacement was mixed with random age replacement. These new replacement disciplines were called the replacement first (RF) and replacement last (RL) [1, 2]. In RF, the preventive replacement is performed at a pre-scheduled time or a random time, whichever occurs first. Conversely, in RL, the preventive replacement is made at a pre-scheduled time or a random time, whichever occurs last.

Most optimal policies of opportunity-based replacement models [1, 2] are derived by minimizing the long-run average cost in the steady state, i.e., the time average of the undiscounted total cost, assuming that the system will continue operating over an infinite planning horizon. However, the average cost approach does not reflect the net present value (NPV) of money. In economy, money is endowed with time and its value ought to reduce as time passes if a greater economic change or revolution does not take place. Since the cost of capital tied up in maintenance planning is included as a part of the replacement cost, in theory, a more accurate approach would be to determine the optimal preventive replacement policy by minimizing the net present value of the expected total cost over all future time [3,4].

Our objective in the present article is to study opportunity-based replacement models with the NPV approach for better understanding the economic impact in preventive maintenance. We reformulate the opportunity-based replacement models in the sense of Zhao and Nakagawa [1] under the framework of the NPV.

2. Models Description

Let us consider a single-unit system with a non-repairable item. It is assumed that the time interval between opportunity arrivals for replacement obey a common distribution $G(t)$ with the probability density function $dG(t)/dt = g(t)$, the hazard rate $h(t) = g(t)/\overline{G}(t)$ and reversed hazard failure rate $h(t) = g(t)/G(t)$. Let $F(t)$, $f(t)$ and $\overline{F}(t)$ denote the cumulative distribution function *c.d.f.*, the probability density function *p.d.f.* and the survivor function of the failure time of a unit, respectively. It is convenient to introduce the failure rate as $r(t) = f(t)/\overline{F}(t)$.

In addition, we denote that c_1 is a cost for corrective (failure) replacement per failure, c_2 is a cost for each preventive replacement, c_3 is a cost for each opportunistic replacement. Without any loss of generality, it is essential to assume that $c_1 > c_2 \geq c_3$, so that the preventive replacement at an opportunity arrival is more economical than the preventive replacement at a scheduled time.

3. RF with NPV Approach

First, we define the discount factor $\alpha\left(>0\right)$ to represent the net present value (NPV) of the total expected cost over an infinite time horizon. Let $C_\alpha\left(T\right)$ and $C\left(T\right)$ denote the expected total discounted cost over an infinite time horizon and the long-run average cost in [1], where T is the preventive age replacement time. Then we have

$$C_\alpha\left(T\right) = \frac{B_\alpha\left(T\right)}{1 - A_\alpha\left(T\right)}, \tag{1}$$

where $A_\alpha\left(T\right)$ and $B_\alpha\left(T\right)$ are the expected cycle length and the expected cost per cycle, respectively. It is straightforward to derive $A_\alpha\left(T\right)$ and $B_\alpha\left(T\right)$ by

$$\begin{aligned} A_\alpha(T) &= \int_0^T e^{-\alpha t}\overline{G}(t)dF(t) \\ &+ \int_0^T e^{-\alpha t}\overline{F}(t)dG(t) \\ &+ e^{-\alpha T}\overline{G}(T)\overline{F}(T) \qquad (2) \\ &= e^{-\alpha T}\overline{G}(T) + \int_0^T e^{-\alpha t}g(t)dt \\ &+ \alpha\int_0^T e^{-\alpha t}F(t)\overline{G}(t)dt, \end{aligned}$$

$$B_\alpha(T) = c_1\int_0^T e^{-\alpha t}\overline{G}(t)dF(t) + c_3\int_0^T e^{-\alpha t}\overline{F}(t)dG(t) + c_2 e^{-\alpha T}\overline{G}(T)\overline{F}(T). \quad (3)$$

Theorem 1 $\lim_{\alpha\to 0} C_\alpha(T) = C(T)$. (4)

Taking the differentiation of $C_\alpha(T)$ with respect to T and setting it equal to zero yield

$$\left[(c_1-c_2)r(T)-(c_2-c_3)h(T)-\alpha c_2\right] \times(1-B_\alpha(T))+\alpha A_\alpha(T)=0. \quad (5)$$

Let $Q_1(T)$ be the left-hand-side of Eq.(5).

Theorem 2 *Suppose that r(T) is IFR and h(T) is DFR*

(1) *If* $Q_1(0)<0$ *and* $Q_1(\infty)>0$, *then there exists a finite and unique optimal preventive replacement time* $T^*(0<T^*<\infty)$, *and its resulting NPV given by*

$$C_\alpha(T^*) = \left[(c_1-c_2)r(T^*) -(c_2-c_3)h(T^*)\right]/\alpha - c_2. \quad (6)$$

(2) *If* $Q_1(\infty)\le 0$, *then the optimal preventive replacement time is given by* $T^*\to\infty$.

(3) *If* $Q_1(0)\ge 0$, *then the optimal preventive replacement time is given by* $T^*=0$.

4. RL with NPV Approach

Next, we calculate the optimal preventive replacement policy under RF discipline. Let $C_\alpha(T)$ denote the expected total discounted cost over an infinite time horizon;

$$C_\alpha(T) = \frac{B_\alpha(T)}{1-A_\alpha(T)}, \quad (7)$$

where

$$\begin{aligned} A_\alpha(T) &= \int_0^T e^{-\alpha t}dF(t) + \int_T^\infty e^{-\alpha t}\overline{G}(t)dF(t) + \int_T^\infty e^{-\alpha t}\overline{F}(t)dG(t) + e^{-\alpha T}G(T)\overline{F}(T) \\ &= \alpha\int_0^T F(t)e^{-\alpha t}dt + \alpha\int_0^T e^{-\alpha t}F(t)G(t)dt + \alpha\int_0^T e^{-\alpha t}F(t)\overline{G}(t)dt, \end{aligned} \quad (8)$$

$$B_\alpha(T) = c_1\left[\int_0^T e^{-\alpha t}dF(t) + \int_T^\infty e^{-\alpha t}\overline{G}(t)dF(t)\right] + c_3\int_T^\infty e^{-\alpha t}\overline{F}(T)dG(t) + c_2 e^{-\alpha T}G(T)\overline{F}(T). \quad (9)$$

Theorem 3 $\lim_{\alpha\to 0} C_\alpha(T) = C(T)$. (10)

Unfortunately, it seems different to show analytically the uniqueness of the optimal opportunity-based age replacement last policy under middle conditions. But it is quite easy to show the existence of a unique and finite solution numerically.

References

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