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Reliability Evaluation of Multi-state Linear Consecutive-k Systems

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1. Introduction

A linear consecutive-k-out-of-n:G system consists of n components which are arranged in a line and the system works if and only if at least k consecutive components work [1]. Zhang *et al.* [2] explained a typical application of consecutive-k-out-of-n:G systems to a railroad operation. Considerable research on the reliability and the application to the optimal assignment of the consecutive-k-out-of-n:G system have been done by a few papers [3, 4]. According to the previous studies, the system state was considered with two states, that is, working or failed. However, in actual, it is also important to discuss the state when system is near to failure. In this paper, we discuss the state of the linear consecutive-k-out-of-n:G system with multi-state, that is, working, failed and the specified state approaching to system failure. We assume that all components in this paper has independent and identically exponential lifetime distribution.

2. Multi-state of Linear Consecutive-k-out-of-n:G Systems

According to the structure features, we define a specified state when system has high possibility to failure. Then we construct components failure sequences by considering the proposed specified state.

2.1. Definition of the specified state approaching to system failure

For the consecutive-k-out-of-n:G system, since the system is working when there is at least one set of working components with number k or more, then it can be judged whether the system has reached a specified state when there is only one set of working components with number k or more in system.

As a result, the range of the number of components in such last one set is indicated. According to the structural characteristic of the consecutive-k-out-of-n:G system, we consider two extreme cases. When there is only one position in which component failure will cause system failure, then the number of working components in this last one set is equal to $2k-1$; furthermore, when the failure of component at any position will cause system failure, then the number of working components in this last one set is equal to k. Overall, we give the following result.

Definition 1. For a consecutive-k-out-of-n:G system, when exists only one set of working components with number i remaining, where i satisfies

$$k \le i \le 2k-1,$$

then system is defined as under the specified state, where system has high possibility to failure.

2.2. Path method for constructing components failure sequences

We then investigate the proposed specified state and failure sequences of the linear consecutive-k-out-of-n:G system. We first give some assumptions in this paper: a) components are either working or failed; b) components have the identical and independent exponential lifetime distribution with failure rate λ.

We consider a binary state: 1 for component working state and 0 for component failed state. At the beginning, all components are working and the system state vector can be expressed as $(1, 1, \cdots, 1)$. The component fails one by one and the component failure sequences are constructed. These sequences from the beginning state (all working component states) to the system failure state (system consists of no k consecutive working components) are called the system failure paths. Obviously, there are at most $n!$ paths to system failure for a consecutive-k-out-of-n:G system. Suppose that the number of steps, that is, the number of failed components until system failure in path j $(1 \le j \le n!)$ is denoted as

Table 1: Paths to system failure for a consecutive-2-out-of-4:G system with component failure rate λ ($*$: satisfied condition).

Path	Step i									
j	0			1			2			3
	state	α_{j0}	β_{j0}	state	α_{j1}	β_{j1}	state	α_{j2}	β_{j2}	state
1	1111	4λ	λ	0111*	3λ	λ	0011	2λ	λ	0001
2	1111	4λ	λ	0111*	3λ	λ	0011	2λ	λ	0010
3	1111	4λ	λ	0111*	3λ	λ	0101			
4	1111	4λ	λ	0111*	3λ	λ	0110	2λ	λ	0010
5	1111	4λ	λ	0111*	3λ	λ	0110	2λ	λ	0100
6	1111	4λ	λ	1011*	3λ	λ	0011	2λ	λ	0001
7	1111	4λ	λ	1011*	3λ	λ	0011	2λ	λ	0010
8	1111	4λ	λ	1011*	3λ	λ	1001			
9	1111	4λ	λ	1011*	3λ	λ	1010			
10	1111	4λ	λ	1101*	3λ	λ	0101			
11	1111	4λ	λ	1101*	3λ	λ	1001			
12	1111	4λ	λ	1101*	3λ	λ	1100	2λ	λ	0100
13	1111	4λ	λ	1101*	3λ	λ	1100	2λ	λ	1000
14	1111	4λ	λ	1110*	3λ	λ	0110	2λ	λ	0010
15	1111	4λ	λ	1110*	3λ	λ	0110	2λ	λ	0100
16	1111	4λ	λ	1110*	3λ	λ	1010			
17	1111	4λ	λ	1110*	3λ	λ	1100	2λ	λ	0100
18	1111	4λ	λ	1110*	3λ	λ	1100	2λ	λ	1000

N_j. Furthermore, in path j, the sum of failure rates of working components after the ith failure $(0 \le i \le N_j - 1)$ is denoted as α_{ji}, and the failure rate of the component which will be failed after the ith failure is denoted as β_{ji}.

As a result, the probability that system states follows path j becomes

$$\pi_j = \prod_{i=0}^{N_j-1} \frac{\beta_{ji}}{\alpha_{ji}}. \tag{1}$$

We give an example of the failure path for the consecutive-k-out-of-n:G system. We consider a consecutive-2-out-of-4:G system where all components have the same failure rate λ. The system has 18 paths to the system failure and we give the details in Table 1.

3. Expected Time to Specified State and Reliability Evaluation

Denote s_j^{sc} as the step when system reaches the specified state in path j, and P as the number of system failure paths. Then the expected time to specified condition for the consecutive-k-out-of-n: G system is derived as

$$E[t^{sc}] = \sum_{j=1}^{P} \pi_j \sum_{i=0}^{s_j^{sc}-1} \frac{1}{\alpha_{ji}}. \tag{2}$$

Using the results of path method, we can obtain the system failure probability in path j:

$$\begin{aligned} F_j(t) &= \Pr\{T_{j,N_j} \le t\}, \\ &= 1 - \sum_{i=0}^{N_j-1} A_{ji} \mathrm{e}^{-\alpha_{ji} t}, \end{aligned} \tag{3}$$

where $A_{ji} = \prod_{m=0, m\neq i}^{N_j-1} \frac{\alpha_{jm}}{\alpha_{jm} - \alpha_{ji}}$. As a result, denote that P is the number of total paths to system failure, then the system failure probability can be estimated as

$$\begin{aligned} F(t) &= \sum_{j=1}^{P} \pi_j F_j(t), \\ &= 1 - \sum_{j=1}^{P} \pi_j \sum_{i=0}^{N_j-1} A_{ji} \mathrm{e}^{-\alpha_{ji} t}, \end{aligned} \tag{4}$$

where π_j is given in Eq. (1).

4. Conclusion

In this paper, we discussed the multi-state of the linear consecutive-k-out-of-n:G system and defined the specified state when system approaching to failure. We also obtained the expected time to the specified state and the expression of system failure distribution by considering the specified state. In the future, it is interesting to apply the reliability evaluation of multi-state system to the maintenance problems.

References

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