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A Markov Game Model for Determining Tactical Changes in Soccer

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1. Introduction

Markov game is one of the useful tools for analyzing a tactics in sports [1]. In this paper, we formulate a soccer game as a two-player zero-sum Markov game. This is an extension of Hirotsu and Wright [2], by considering the location of the ball on the pitch as nine zones together with the change of possession of the ball, following Hirotsu et al.[3].

2. Markov game formulation

An association football match game can be seen as progressing through a set of stochastic transitions occurring due to a change of possession of the ball or the scoring of a goal. A Markov process model can be used to appropriate the progress of the match as an approximation. Hirotsu et al.[3] propose a Markov process model in which the pitch discretised into nine zones and define the states as follows:

State H_G: Home team scores a goal;
State H_I: Home team is in possession of the ball and the ball is located in the "I" zone (I=1,…, 9);
State A_I: Away team is in possession of the ball and the ball is located in the "I" zone (I=1,…, 9);
State A_G: Away team scores a goal.

Here, the "I" zone (I=1,….9) on the pitch is defined in Figure 1. There are two states for the goal scoring (states H_G and A_G) and 18 states relating to the location and team's possession of the ball.

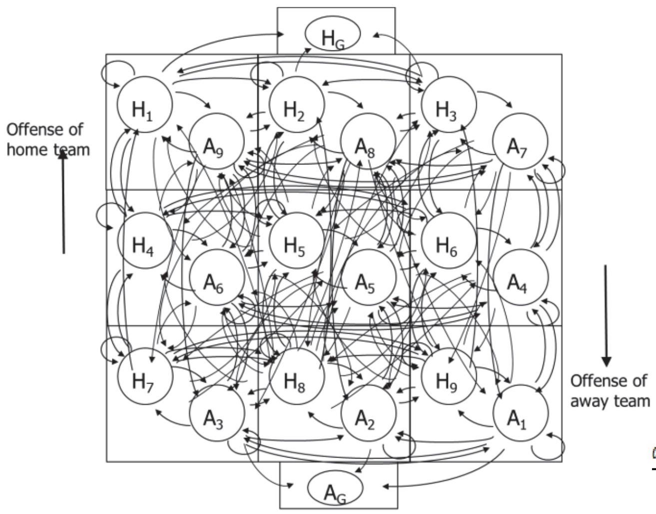

Figure 1: Image of the Markov process model

Table 1. Definition of transition probabilities from state i

Transition	Probability	Remarks
$i \to H_G$	$a_{i\,HG}\,dt$	Transition to scoring a goal for home team
$i \to j$	$a_{ij}\,dt$	Transition from state i to state j
$i \to A_G$	$a_{i\,AG}\,dt$	Transition to scoring a goal for away team

The transition probabilities between them are defined in Table 1. The 378 (=9×2×21) different states are defined in the course of a game, except H_G and A_G, are identified by the combination of the following factors:

- Location of the ball (9 possibilities);
- Possession of the ball (2 possibilities).
- Number of goals by which the home team leads (21 possibilities, assuming that the number of runs by which either team may lead will never exceed 10);

Here, we look at the winning from the perspective of the home team. Given this specification, Equation (1) determines the probability of winning from each state:

$$\frac{d\mathbf{w}(t)}{dt} = \mathbf{P} \cdot \mathbf{w}(t) \tag{1}$$

where $\mathbf{w}(t)$ is a 378×1 vector, each entry of which corresponds to the probability of home team winning from a position of leading by l goals with time t remaining, starting from state i $(i = H_1, H_2, H_3, \ldots, A_1)$. $\mathbf{P}$ is a 378×378 matrix, which represents the transition between the 378 states. We will present the detail of (1) in the Fall National Conference. We can numerically solve them with the boundary conditions at the end of the game such that $\mathbf{w}_{H1}(l|0) = \mathbf{w}_{H2}(l|0) = \ldots = \mathbf{w}_{A1}(l|0) = 1$ if $l > 0$ and 0 if $l < 0$. We set $\mathbf{w}_{H1}(l|0) = \mathbf{w}_{H2}(l|0) = \ldots = \mathbf{w}_{A1}(l|0) = 0.5$ only if $l = 0$ in the case of draw.

By solving (1), we can simultaneously obtain the probability of home team winning from a position of leading by l goals with time t remaining in each of the 378 states. This approach makes it possible to develop a Markov game formulation to identify the situations to enforce alternative transition rates between state by a tactical change. We can formulate this as follows:

$$\frac{d\mathbf{w}(t)}{dt} = \max_{k,k' \in \{0,1,\cdots,K\}} \min_{h,h' \in \{0,1,\cdots,H\}} \begin{cases} \mathbf{P}^{kh} \cdot \mathbf{w}(t) & : \textit{No tactical change} \\ \mathbf{P}^{k'h} \cdot \mathbf{w}(t) & : \textit{Tactics } k \to k' \ (\textit{Tactical change from } k \textit{ to } k' \neq k) \\ \mathbf{P}^{kh'} \cdot \mathbf{w}(t) & : \textit{Tactics } h \to h' \ (\textit{Tactical change from } h \textit{ to } h' \neq h) \\ \mathbf{P}^{k'h'} \cdot \mathbf{w}(t) & : \textit{Tactics } k \to k' \textit{ and } h \to h' \ (\textit{Tactical change from} \\ & \quad \textit{to } k' \neq k \textit{ and } h \textit{ to } h' \neq h) \end{cases} \tag{2}$$

where maximisation is taken from possible different tactics k and k' (=0,1,2, ･･･,K) for the home team and minimisation is from possible different tactics h and h' (=0,1,2, ･･･H) for the away team. We note that under the assumption that both teams do not make a tactical change in the small time dt simultaneously, we can find the Nash equilibrium and obtain the solution of (2) as a pure strategy.

3. Example

3.1 Transition rates

We now present a numerical example. The play-by-play data used in this study was provided by Data Stadium Inc. The estimates of transition rates of the J-League average which is extracted as intercept as main factors for the transition rates between states in the most suitable log-linear model presented in [3]. This means that the average number of transition from each location to the other location can be estimated by taking away of the effects of home advantage and strength of each teams. For example, the transition rate from H_2 to H_1 was 3.081 times/min.

3.2 Setting the transition rates for tactical change

Based on the estimates of the average transition rates, we set up the transition rates of the hypothetical game between the average home team and the average away team. As an advantage of using the Markov game, we can calculate the effect of the change of transition rates on the probabilities of winning the match. We set the case that the home team have home advantage addition to the average transition rates as a base. The effect of home advantage is estimated in [3] and we introduce this effect into each transition rate. For example, the effect of home advantage in the transition from H_2 to H_1 is estimated 1.08, and the transition rate for the home team is set as 3.35(=1.08×3.081) times/min.

Here, as an example concretely to see the sensitivity of the transition rate, we manipulate the transition from zone “4” to “1” for the home team and from “7” to “1” for the away team. We change the transition rates by the amount of its 1SD and 0.5SD, respectively, and see the effect of the change of transition rates. That is, we set four cases: (1) both teams take base tactics ((0,0)), (2) the home team increase the transition rate from zone “4” to “1” by 1SD ((1,0)), (3) the away team increases the transition rate from zone “7” to “1” by 0.5SD ((0,1)), (4) both teams take the tactics which increases these transition rates ((1,1)). The SD can be obtained by the annual data of J-League, and in terms of the transition from H_4 to H_1 1SD=1.418 times/min. When the home team takes Tactic 1, the unsuccessful pass will increase. Thus, the transition rate from H_4 to A_9 is also set to increase by its 2SD. (i.e. from H_4 to A_9, 2SD=0.814 (=2×0.412) times/min.) In a similar manner, Tactic 1 for the away team is assumed to increase the number of long pass from zone 7 to 1 by 0.5SD. (i.e., the transition rate from A_7 to A_1 increases by 0.5SD.) As the unsuccessful long pass will increase, transition rate from A_7 to H_9 also increase by 2SD. (i.e. from A_7 to A_1, 0.5SD=0.1781 (=0.5×0.3562) times/min.; from A_7 to H_9, 2SD=1.0626 (=2×0.5313) times/min.)

3.3 Calculation result

Now we can calculate the probability of the home team winning using Equation (2). We show the change of formations during the game under the condition that they always make their best decisions at time t remaining. Table 2 shows a whole image of the optimal tactics and timing of their tactical changes, representing the case where the number of goals by which either team leads is not to exceed 2. Both teams start off with Tactic 0 (i.e. (0,0)). If the home team falls behind by 2 goals with more than 57 min. remaining, or by 1 goal with less than 57 min. remaining, it should make a tactical change from 0 to 1. Otherwise, if the away team falls behind by 2 goals with less than 18 min. remaining, or by 1 goal with less than 7 min. remaining, it should make a tactical change from 0 to 1. This result looks reasonable because Tactic 1 is the offensive tactics for scoring goals in order to get the scores level after falling behind.

Table 2: Optimal tactical changes in the case where both teams make their best decisions

	Lead in goals for the home team				
Remaining Time	2	1	0	1	2
90–57 min.					
57–18 min.	**(1,0)**				
18– 7 min.			**(0,0)**		**(0,1)**
7– 0 min.					

4. Conclusion

In this paper, we have modelled a soccer game as a two-player zero-sum Markov game, discretising the pitch into nine zones. An example of optimal tactical changes is demonstrated in the case that two teams can make tactical changes during a game, by setting the change of transition rates.

References

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