

# Corporate sustainability, investment, and capital structure

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## 1. Introduction

Although an increasing number of finance papers, such as [3], have recently studied ESG investment, few real options papers have focused on the effects of sustainability on project valuation and investment timing. By developing a real options model including project sustainability, this paper clarifies the conditions under which a firm prefers sustainable project investment and the interactions between sustainability and capital structure.

## 2. Model Setup

Consider a firm that has an option to invest in either sustainable or unsustainable project. Each project generates continuous streams of earnings before interest and taxes (EBIT)  $X(t)$ , which follows

$$dX(t) = \mu X(t)dt + \sigma X(t)dB(t) \quad (t > 0), \quad X(0) = x,$$

where  $B(t)$  denotes the standard Brownian motion, and  $\mu, \sigma (> 0)$  and  $x (> 0)$  are constants. For convergence, we assume that  $r > \mu$ , where  $r$  denotes the risk-free interest rate. Corporate tax rate  $\tau (> 0)$  is assumed. The unsustainable project requires a low investment cost  $I_1 (> 0)$  and yields  $X(t)$  till a random maturity, which follows an exponential distribution with rate  $\lambda$ , whereas the sustainable project requires a high investment cost  $I_2 (> I_1)$  and yields  $X(t)$  perpetually. Note that the unsustainable firm will exit at the maturity.

In the unlevered case, the firm chooses the investment time and project choice to maximize its financial value. This means that the firm is not ESG-motivated but ESG-aware (for the difference, see [3]). In the levered case, the firm also optimizes capital structure, where as in the standard literature, consol debt is issued at the investment time. On bankruptcy, a fraction  $\alpha$  of the unlevered firm value is lost as the bankruptcy cost.

## 3. Model Solutions

The unlevered problem is expressed as follows:

$$V^U(x) = \sup_T \mathbb{E}[e^{-rT} \max_{i=1,2} \{a_i^U X(T) - I_i\}], \quad (1)$$

where  $a_1^U = (1-\tau)/(r-\mu+\lambda)$  and  $a_2^U = (1-\tau)/(r-\mu)$  are coefficients for the unsustainable and sustainable projects, respectively. The superscript  $U$  stands for the unlevered case. As in [1] and [2], problem (1) is solved as follows, where  $\beta = 0.5 - \mu/\sigma^2 + \sqrt{(\mu/\sigma^2 - 0.5)^2 + 2r/\sigma^2} (> 1)$ .

**Proposition 1** *If the sustainability condition*

$$\left(\frac{r-\mu+\lambda}{r-\mu}\right)^{\frac{\beta}{\beta-1}} \geq \frac{I_2}{I_1},$$

*holds, the firm chooses the sustainable project.*

$$V^U(x) = \begin{cases} (a_2^U x_2^U - I_2) \left(\frac{x}{x_2^U}\right)^\beta & (x < x_2^U) \\ a_2^U x - I_2 & (x \geq x_2^U), \end{cases}$$

$$x_2^U = \frac{\beta I_2}{(\beta-1)a_2^U}.$$

*Otherwise, the firm chooses either sustainable or unsustainable project. The firm chooses the unsustainable project when  $X(t)$  starts from a sufficiently low  $X(0) = x$ .*

$$V^U(x) = \begin{cases} (a_1^U x_{11}^U - I_1) \left(\frac{x}{x_{11}^U}\right)^\beta & (x < x_{11}^U) \\ a_1^U x - I_1 & (x \in [x_{11}^U, x_{12}^U]) \\ (a_1^U x_{12}^U - I_1)\Sigma^U(x) + (a_2^U x_{21}^U - I_2)\Delta^U(x) & (x \in (x_{12}^U, x_{21}^U)) \\ a_2^U x - I_2 & (x \geq x_{21}^U), \end{cases}$$

$$x_{11}^U = \frac{\beta I_1}{(\beta-1)a_1^U},$$

*where  $\Sigma^U(x)$  and  $\Delta^U(x)$  denote the state prices, and  $x_{12}^U$  and  $x_{21}^U$  are determined by the smooth pasting conditions.*

This proposition implies that an economic downturn can trigger the unsustainable project investment. The next proposition clarifies the situations where the firm prefers the sustainable project.

**Proposition 2** *The sustainability condition is more likely to hold for higher  $\lambda, \sigma$ , and  $\mu$ , as well as for lower  $r$ .*

The levered problem is solved backward. First, derive the equity, debt, and firm values for debt in place. Second, derive the default threshold to maximize the equity value. Third, derive the initial capital structure (coupon of debt) to maximize the firm value, based on the tradeoff between the tax benefits of debt and bankruptcy cost.

By doing these steps for the sustainable and unsustainable firms, I have the levered problem:

$$V^L(x) = \sup_T \mathbb{E}[e^{-rT} \max_{i=1,2} \{a_i^L X(T) - I_i\}], \quad (2)$$

where  $a_1^L = \phi_1 a_1^U$  and  $a_2^L = \phi_2 a_2^U$ . The superscript  $L$  stands for the levered case. The unsustainable and sustainable project leverage multipliers,  $\phi_1$  and  $\phi_2$ , are defined by

$$\phi_i = 1 + \frac{\tau}{(1-\tau)h_i} (> 1),$$

$$h_i = \left[ 1 - \gamma_i \left( 1 - \alpha + \frac{\alpha}{\tau} \right) \right]^{-\frac{1}{\gamma_i}} (> 1),$$

$$\gamma_1 = 0.5 - \mu/\sigma^2 - \sqrt{(\mu/\sigma^2 - 0.5)^2 + 2(r + \lambda)/\sigma^2} (< 0),$$

where  $\gamma_2$  is defined by  $\gamma_1$  with  $\lambda = 0$ . Problem (2) is solved as follows.

**Proposition 3** *If the sustainability condition*

$$\left( \frac{(r - \mu + \lambda)\phi_2}{(r - \mu)\phi_1} \right)^{\frac{\beta}{\beta-1}} \geq \frac{I_2}{I_1},$$

*holds, the firm chooses the sustainable project.*

$$V^L(x) = \begin{cases} (a_2^L x_2^L - I_2) \left( \frac{x}{x_2^L} \right)^\beta & (x < x_2^L) \\ a_2^L x - I_2 & (x \geq x_2^L), \end{cases}$$

$$x_2^L = \frac{\beta I_2}{(\beta - 1)a_2^L} = x_2^U / \phi_2.$$

*Otherwise, the firm chooses either sustainable or unsustainable project. The firm chooses the unsustainable project when  $X(t)$  starts from a sufficiently*

*low  $X(0) = x$ .*

$$V^L(x) = \begin{cases} (a_1^L x_{11}^L - I_1) \left( \frac{x}{x_{11}^L} \right)^\beta & (x < x_{11}^L) \\ a_1^L x - I_1 & (x \in [x_{11}^L, x_{12}^L]) \\ (a_1^L x_{12}^L - I_1) \Sigma^L(x) + (a_2^L x_{21}^L - I_2) \Delta^L(x) & (x \in (x_{12}^L, x_{21}^L)) \\ a_2^L x - I_2 & (x \geq x_{21}^L), \end{cases}$$

$$x_{11}^L = \frac{\beta I_1}{(\beta - 1)a_1^L} = x_{11}^U / \phi_1,$$

*where  $\Sigma^L(x)$  and  $\Delta^L(x)$  denote the state prices, and  $x_{12}^L$  and  $x_{21}^L$  are determined by the smooth pasting conditions.*

**Proposition 4**  *$\partial \phi_1 / \partial \lambda > 0$  holds. In particular,  $\phi_1 > \phi_2$  holds, and hence, the sustainability condition is less likely to hold in the levered case than in the unlevered case.*

This proposition implies that the leverage effect is stronger for the less sustainable firm and that access to debt financing can lead the firm to choose the unsustainable project.

The following results are also shown numerically. The less sustainable firm takes the higher leverage and credit spreads. In contrast to the well-known results, lower discount rate and higher growth rate can delay investment through switch from the unsustainable project to the sustainable project.

## References

- [1] J. Décamps, T. Mariotti, and S. Villeneuve. Irreversible investment in alternative projects. *Economic Theory*, 28:425–448, 2006.
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- [3] L. Pedersen, S. Fitzgibbons, and L. Pomorski. Responsible investing: The ESG-efficient frontier. *Journal of Financial Economics*, 142:572–597, 2021.