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Strategic Liquidity Provision in High Frequency Trading

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1. Introduction

We construct a Kyle (1985)-type market model in which fast and slow traders are present. We will show with numerical calculations that a fast trader who has an advan- tage in trade frequency plays a role as a liquidity provider in the sense that he takes the opposite position against a slow trader if the difference in frequency is significant.

2. Model Setup

There is one risky and one risk-free assets in the market. The risk-free rate is set to be zero to simplify the analysis. The final payoff of the risky asset is assumed to follow a normal with mean p_0 and Σ_0. There is information asymmetry among market participants in that only some can observe the realization of the payoff v.

There are four types of traders in the market. The first type is a fast trader (hereafter FT) who has more advanced technology to process the orders. The second one is a slow trader (hereafter ST) who is not able to trade the asset as frequently as the high frequency trader. The third one is noise traders whose order amount is random. Finally, the fourth one is dealers who set the pricing schedule at each trading time. The dealers are risk-neutral and perfectly competitive, implying that the market price satisfies the efficiency condition as we will mention later.

In our discrete-time market model, there are totally $N \times T$ auctions. The auctions consist of N session, each of which has T sub-sessions. We denote the trading time at t-th sub-session in n-th session by $t_{n,t}$, and thus have $0 < t_{1,1} < t_{1,2} < \cdots < t_{1,T} < t_{2,1} < \cdots < t_{N,T-1} < t_{N,T} = 1$. After time 1, the payoff of the risky asset is realized and all the traders' positions are settled. In session n, FT trades the risky asset at all T sub-sessions $t_{n,1}$ to $t_{n,T}$. On the other hand, ST can submit the order at sub-session $t_{n,1}$, but the order is settled at $t_{n,T}$ due to the inferiority in the technology in information and/or order submission processing.

Let $x^F_{n,t}$ denote the order amount of FT at sub-session (n,t), x^S_n the order amount of ST at session n, $u_{n,t}$ the total order amount of noise traders at sub-session (n,t). As in Kyle (1985) and other extensions, HF and ST observe the realization of v and the price just before the auction. The total order from noise traders at sub-session (n,t) follows a normal distribution with mean 0 and variance σ_u^2 and is independent of all random variables. The market maker sets the price $p_{n,t}$ at sub-session (n,t) observing the total order flow $y_{1,1} \ldots, y_{n,t}$, where

$$y_{n,t} = \begin{cases} x^F_{n,t} + u_{n,t}, & t < T, \\ x^F_{n,T} + x^S_n + u_{n,T}, & t = T \end{cases}$$

for $n = 1, \ldots, N$.

Definition 1 *An equilibrium is defined as the triplet (x^{F*}, x^{S*}, p) such that*

1. *FT maximizes the expected profit*

$$\mathbb{E}\left[\sum_{(k,\tau)\geq(n,t)} (v - p_{k,\tau}) x^F_{k,\tau} \,\middle|\, v, p_{1,1}, \ldots p_{n,t-1} \right]$$

over x^F at sub-session (n,t),

2. *ST maximizes the expected profit*

$$\mathbb{E}\left[\sum_{k\geq n} (v - p_{k,T}) x^S_k \,\middle|\, v, p_{1,1}, \ldots p_{n-1,T} \right]$$

over x^S at session n,

3. *the price satisfies the market efficiency condition*

$$p_{n,t} = \mathbb{E}[v | y_{1,1} \ldots, y_{n,t}]$$

at sub-session (n,t).

3. Equilibrium Solution

Proposition 1 *In a linear equilibrium, there are constants $\alpha_{n,t}$, $\tilde{\beta}^F_{n,t}$, β^S_n, $\delta^F_{n,t}$, δ^S_n, κ_n, $\lambda_{n,t}$, and $\Sigma_{n,t}$, where*

$$\begin{aligned} x^{F*}_{n,t} &= \tilde{\beta}^F_{n,t}(v - p_{n,t-1}), \\ x^{S*}_n &= \beta^S_n (v - p_{n-1,T}), \\ p_{n,t} &= \begin{cases} p_{n,t-1} + \lambda_{n,t}(x^F_{n,t} + u_{n,t}), & t < T, \\ p_{n,T-1} + \lambda_{n,T}(x^F_{n,T} + x^S_{n,T} + u_{n,T}) \\ \qquad - \kappa_n (p_{n,T-1} - p_{n,0}), & t = T, \end{cases} \\ \Sigma_{n,t} &= \mathbb{V}^U_{n,t}[v], \\ v^F_{n,t-1} &= \alpha^F_{n,t-1}(v - p_{n,t-1})^2 + \delta^F_{n,t-1}, \end{aligned}$$

$$v_{n-1}^S = \alpha_{n-1}^S (v - p_{n-1,T})^2 + \delta_{n-1}^S$$

for $t = 1, \ldots, T$ *and* $n = 1, \ldots, N$ *with* $p_{n-1,T} = p_{n,0}$, *where* $\mathbb{V}_{n,t}^U$ *is the conditional variance operator given* $\mathscr{F}_{n,t}^U$. *The constants are the solution to the simultaneous difference equation system, which will be shown in the presentation.*

The second-order conditions are given by

$$\lambda_{n,t}(1 - \alpha_{n,t}^F \lambda_{n,t}) > 0$$

for all $n = 1, \ldots, N$ *and* $t = 1, \ldots, T$, *and*

$$\lambda_{n,T}(1 - \alpha_n^S \lambda_{n,T}) > 0$$

for all $n = 1, \ldots, N$.

4. Numerical Calculations

Figures 1 to 4 are the results of numerical calculations.

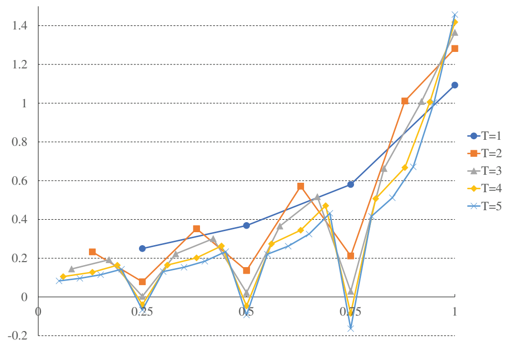

Figure 1: Graph of $\tilde{\beta}_{n,t}^F$

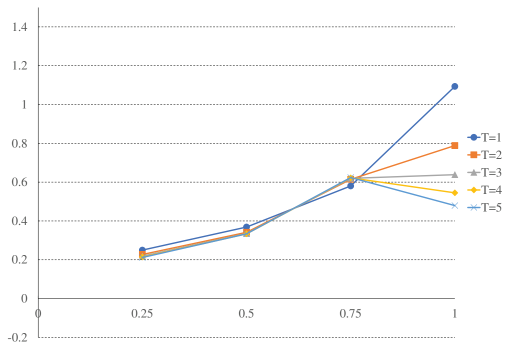

Figure 2: Graph of β_n^S

The intuitions and implications are described in the presentation.

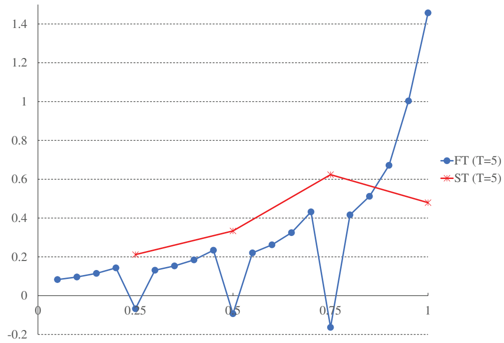

Figure 3: Graph of $\beta_{n,t}^F$ and β_n^S

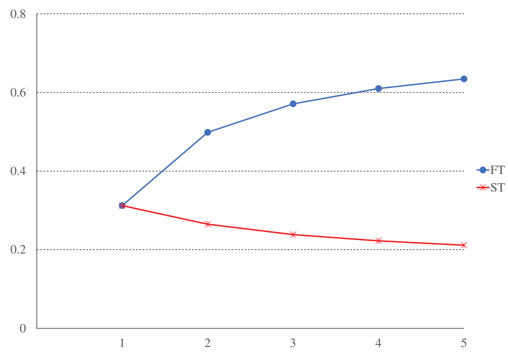

Figure 4: Graph of the expected profits of HF and ST

5. Conclusion

In this paper, we construct a Kyle (1985)-type market model where fast and slow traders with private information are present. The major conclusion is that *the trade frequency matters for the market quality*. In particular, if the asymmetry in trade frequency is substantial, FT can trade in the opposite direction to accommodate a short-term loss at the auctions with ST orders' arrivals. This trading behavior provides the liquidity to ST and other market participants.

References

Kyle, A. S. (1985), "Continuous Auctions and Insider Trading," *Econometrica*, **53**(6), 1315–1335.