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Optimal default decision with an upper reflecting barrier

01208080 東京都立大学 *芝田隆志 SHIBATA Takashi
01406054 大阪大学 西原理 NISHIHARA Michi

1. Introduction

This study extends [1] to incorporate an upper reflecting barrier. In particular, this study explores how the upper reflecting barrier affects the firm's default decision.[1]

2. Model

Consider the firm under operations, which face the stochastic cash flow (earnings before interest and taxes) as

$$\frac{\mathrm{d}X(t)}{X(t)} = \mu \mathrm{d}t + \sigma \mathrm{d}z(t), \ X(0) = x > 0, \qquad (1)$$

where $\mu > 0$, $\sigma > 0$, and $z(t)$ indicates standard Brownian motion. Here, $r > 0$ indicates the interest rate, and we assume $r > \mu$ for convergence. We assume a corporate tax rate $\tau \in (0, 1)$. At time 0, firm issues perpetual debt with coupon $c \geq 0$ to maximize the firm value.

If $X(t)$ decreases and arrives at some level x_d from above, the firm defaults. We then assume a bankruptcy cost. Thus, at the default, the debt holders obtain the residual value as $(1-\alpha)$ times the firm value, where $\alpha \in (0, 1)$ indicates the proportional bankruptcy cost parameter. The notion is identical to that in [1] and [3].

In this study, following [2], we assume that there exists an upper reflecting barrier $\overline{x} > 0$. If $X(t)$ climbs to $\overline{x}$ from below, it is immediately brought back to a slightly lower level. Economically, if the cash flow is increased at some high level, a potential firm enters the market, causing a decrease in the cash flow $X(t)$.

In this study, we use the following parameters:

$$\beta = \frac{-(\mu - \frac{1}{2}\sigma^2) + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}}{\sigma^2} > 1,$$

$$\gamma = \frac{-(\mu - \frac{1}{2}\sigma^2) - \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}}{\sigma^2} < 0,$$

$$v = \frac{1-\tau}{r-\mu} > 0.$$

As the derivation of [1] and [3], the equity value, $E(x, c)$, is obtained as

$$\begin{aligned} E(x,c) &= vx - (1-\tau)\frac{c}{r} \\ &\quad + (x_d^{\beta-\gamma} - \frac{\beta}{\gamma}\overline{x}^{\beta-\gamma})^{-1} H_1(c) x^\beta \\ &\quad + (x_d^{\gamma-\beta} - \frac{\gamma}{\beta}\overline{x}^{\gamma-\beta})^{-1} H_2(c) x^\gamma, \qquad (2) \end{aligned}$$

where

$$H_1(c) = -v(x_d^{1-\gamma} - \frac{1}{\gamma}\overline{x}^{1-\gamma}) + (1-\tau)\frac{c}{r}x_d^{-\gamma}, \quad (3)$$

$$H_2(c) = -v(x_d^{1-\beta} - \frac{1}{\beta}\overline{x}^{1-\beta}) + (1-\tau)\frac{c}{r}x_d^{-\beta}. \quad (4)$$

Here, the optimal default trigger, $x_d := x_d(c)$, is obtained by solving the following equation:

$$\begin{aligned} &v + (x_d^{\beta-\gamma} - \frac{\beta}{\gamma}\overline{x}^{\beta-\gamma})^{-1} H_1(c)\beta x_d^{\beta-1} \\ &+ (x_d^{\gamma-\beta} - \frac{\gamma}{\beta}\overline{x}^{\gamma-\beta})^{-1} H_2(c)\gamma x_d^{\gamma-1} = 0. \quad (5) \end{aligned}$$

The debt value, $D(x, c)$, is given as

$$\begin{aligned} D(x,c) &= \frac{c}{r} + (x_d^{\beta-\gamma} - \frac{\beta}{\gamma}\overline{x}^{\beta-\gamma})^{-1} H_3(c) x^\beta \\ &\quad + (x_d^{\gamma-\beta} - \frac{\gamma}{\beta}\overline{x}^{\gamma-\beta})^{-1} H_4(c) x^\gamma, \quad (6) \end{aligned}$$

where

$$\begin{aligned} H_3(c) &= (1-\alpha)v(x_d^{1-\gamma} - \frac{1}{\beta}\overline{x}^{1-\beta}x_d^{\beta-\gamma}) \\ &\quad - \frac{c}{r}x_d^{-\gamma}, \qquad (7) \\ H_4(c) &= (1-\alpha)v(x_d^{1-\beta} - \frac{1}{\beta}\overline{x}^{1-\beta}) \\ &\quad - \frac{c}{r}x_d^{-\beta}. \qquad (8) \end{aligned}$$

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[1] [2] examine how the upper reflecting barrier influences the firm's investment decision.

In an extreme case of $\overline{x} \to +\infty$, we obtain

$$\begin{aligned} E(x,c) &= = vx - (1-\tau)\frac{c}{r} \\ &\quad + (\frac{x}{x_d})^{\gamma}\{(1-\tau)\frac{c}{r} - vx_d\}, \quad (9) \\ D(x,c) &= \frac{c}{r} + (\frac{x}{x_d})^{\gamma}\{(1-\alpha)vx_d - \frac{c}{r}\}, (10) \\ x_d &= \frac{\gamma}{\gamma-1}\frac{1-\tau}{v}\frac{c}{r}, \quad (11) \end{aligned}$$

which are identical to those by [1]. In addition, as $c \to 0$, we have

$$\begin{aligned} E(x,c) &= v(x - \frac{1}{\beta}\overline{x}^{1-\beta}x^{\beta}), \quad (12) \\ D(x,c) &= 0, \quad (13) \\ x_d &= 0. \quad (14) \end{aligned}$$

3. Model implication

We focus on the numerical calculation to examine the solution. The parameters are $r = 0.05$ $\sigma = 0.2$, $\mu = 0.01$, $\tau = 0.15$, and $\alpha = 0.35$. Then, we obtain $x_d = 0.2939$ for $\overline{x} \to \infty$.

Fig.1. shows the default trigger x_d with $\overline{x}$. An increase in $\overline{x}$ decreases x_d with $\lim_{\overline{x}\to\infty} x_d = 0.2939$. The larger the upper reflecting barrier, the smaller the default trigger. Economically, the more intense the competitiveness, the more likely the firm's default. This result accords with empirical study.

We will provide other interesting results at the presentation.

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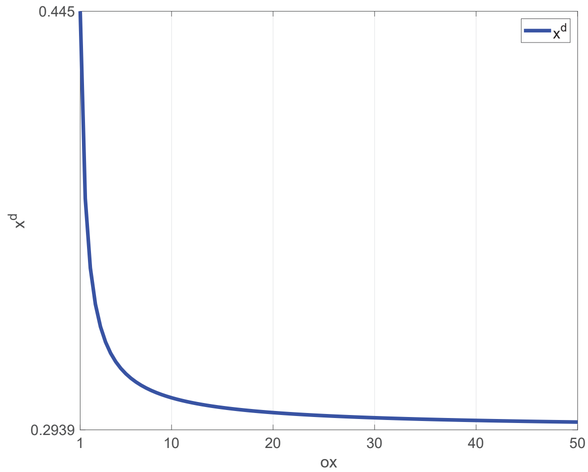

Fig.1. Default trigger x_d with $\overline{x}$

[3] Leland, H. E., 1994. Corporate debt value, bond covenants, and optimal capital structure. Journal of Finance, 49, 1213–1252.