Systemic Risk and Short-Term Financial Networks

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1. Introduction

Since the financial crisis of 2007-2008, we have come to realize that in order to correctly assess the potential default risk present in any bilateral contractual relationship between two firms, it is essential that we have a clear picture of the network structure of each firm's bilateral contractual connections to other firms. These network connections are the channels or pathways over which default risk - and default - travel in moving through the network. Moreover, in order to correctly assess the risk of a broader network failure brought about by shocks to individual firms or groups of firms, we must know something about the strategic behavior of firms in responding to such shocks, as well as how the interplay between strategic behavior and network structure generate the dynamics which drive network formation. This "risk of a broader network failure brought about by shocks to individual firms or groups of firms" is usually referred to as *systemic risk*. While we know systemic risk when we see it, surprisingly, we have no generally agreed upon formal definition of it [1]. This research has two objectives: first, to provide a *formal definition of systemic risk* that is firmly grounded in the equilibrium dynamics of network formation; and second, to construct a discounted stochastic game (DSG) model of the *emergence of these equilibrium network dynamics* that fully takes into account the feedback between network structure, strategic behavior, and risk. We also provide a strategic approach to making global assessments of systemic risk in networks based on three key facts about equilibrium network dynamics with concepts of basins of attraction, sphere of influence and recurrent set.

2. Endogenous Systemic Risk

A useful visualization devise for understanding of our approach to systemic risk is to think of the equilibrium (state) dynamics as being represented by a *supernetwork* where the nodes represent state-network pairs and the directed arcs pointing from one state-network pair to another are labeled by the equilibrium transition probabilities of moving from one state-network pair to another. These equilibrium transition probabilities are a function of the profile of players' network formation strategies. If we then think of this supernetwork as representing the transportation network over which the network will travel in moving from one state to another, we can then compute the probabilities that the current network, departing from its current state, arrives at any other state or set of states at or before a particular time. With our transportation analogy in mind, we are led *to define the endogenous systemic risk of the current network as the equilibrium first passage probability from the current network to some future network or set of networks identified as having a particular subset of defaulted players.* We then have for each possible current network in a particular state a schedule of systemic risk measures indexed by times and states. Under our definition of endogenous systemic risk, systemic risk is inextricably linked to the equilibrium network dynamics determined by the interplay between strategic behavior, network structure, and risk. Moreover, by its very definition, our notion of endogenous systemic risk takes into account the timing and severity of the risks being measured. The presence of finitely many basins of attraction, each containing a recurrent set of states having a particular subset of defaulted players, together with the fact that the current network - no matter what its current state - will arrive at one of these recurrent sets belonging to a particular basin (i.e., will arrive at some state-network pair contained in one of these recurrent sets) in finite time with probability one, has major implications for our understanding of how best to measure and control systemic risk. Moreover, the presence of a unique set of spheres of influence, serving as an early warning system for impending defaults, enhances our ability to guide the network formation process toward less systemically risky states by providing a set of navigation beacons.

3. Short-Term Financial Networks

In developing our definition of endogenous systemic risk, we focus on short-term financing and investing networks (i.e., short term financial networks) - saving for future work an analysis of the interconnections of systemic risk and the maturity structure of financing and investing. In the discounted stochastic game model of short-term financial network formation constructed here, each of n players forms two networks: (i) one consisting of short-term borrowing-lending-repayment (BLR) connections with the other players, and (ii) one consisting of investment connections with some subset of m possible (perfectly divisible) risky investment projects. Players borrow or lend short term in order to adjust their levels of investable funds available for the risky projects. The formation of the borrowing and lending network takes place in two steps. First, each player, i, *proposes* a profile of borrowing or lending contracts to the other players. Each such proposed contract, for example one from player i to player j, is specified by a proposed amount l^0_{ij} to be borrowed (< 0) or lent (> 0) at the beginning of the period and a proposed amount l^1_{ij} to be repaid at the end of the period. Player $i's$ contract proposal, (l^0_{ij}, l^1_{ij}), to j becomes a real connection between i and j in the borrowing and lending network if - given $i's$ proposal, (l^0_{ij}, l^1_{ij}) - $j's$ counterproposal, (l^0_{ji}, l^1_{ji}), to i is matching - that is, if $l^t_{ij} + l^t_{ji} = 0$ for $t = 0$ and 1. If player pair ij's contract proposals fail to match, then a matching is reached through bargaining between players i and j. Here, rather than model this bargaining process explicitly, we instead assume that there is a matching (or bargaining) transition kernel which incentivizes players to reach a matching in borrowing or lending proposals. Our representation of the bargaining process captures the risk inherit in bargaining. Once players have reached their borrowing and lending matches - as given by the state and proposal dependent probability measure over matching outcomes - their borrowing-lending network is determined - thereby determining their levels of investable funds. With financing in place, players choose an allocation of their investable funds across the feasible subset of m risky investment projects available to each player. If as a result of prior investment, borrowing and lending activity, a player begins the period with negative cash, then the player becomes a permanent member of the set of defaulted players - and remains inactive in perpetuity. In order to take into account the unintended network-wide, negative cash flow consequences of a player's (or players') default, we adjust non-defaulted players' debt repayments to reflect the actual debt repayments players are able to make after a default. Here, using the Eisenberg-Noe approach [2], we obtain a stationary default adjustment function which allows us to compute the equilibrium default adjusted repayments.

References

[1] P. Glasserman and H. P. Young (2015), How likely is contagion in financial networks?, *Journal of Banking & Finance*, vol. 50, pp. 383-399.

[2] L. Eisenberg and T. H. Noe (2001). Systemic risk in financial systems. *Management Science*, vol. 47(2), pp. 236–249.