Labor share and corporate investment

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1 Introduction

Most studies on corporate investment have focused only on capital as a production factor. In this study, we consider both labor and capital as production factors and investigate the impact of labor share on a firm's investment timing and size decisions.

2 Setup

A firm's output at time t is $Q_t = (L_t^a K_t^{1-a})^b$ where L_t and K_t denote the amount of labor and capital, respectively, and $a \in (0,1)$ and $b \in (0,1)$ denote the labor share in production and the degree of returns to scale, respectively. The product's price is $P_t = X_t Q_t^{\gamma-1}$ where $\gamma \in (0,1)$. The demand shock X_t follows

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}W_t$$

where μ and σ are positive constants and $(W_t)_{t\geq 0}$ is a standard Brownian motion. A risk-free rate is given by a positive constant r. Lumpy investment in capital K incurs costs $\delta_0 + \delta_1 K$ and labor requires wage w per unit time. The firm's profit flow after investment is

$$\pi(X_t, L_t, K_t) = P_t Q_t - wL_t = X_t (L_t^a K_t^{1-a})^{b\gamma} - wL_t.$$

3 Models and solutions

3.1 Fully adjustable labor

Suppose that the firm can adjust the amount of labor at any time without costs. The optimal labor given demand shock X and capital K is

$$L^*(X,K) = \Big(\frac{ab\gamma X K^{(1-a)b\gamma}}{w}\Big)^{\frac{1}{1-ab\gamma}},$$

and thus, the profit flow given X and K is

$$\pi(X, L^*(X,K), K) = \psi X^{\frac{1}{1-ab\gamma}} K^{\frac{(1-a)b\gamma}{1-ab\gamma}}$$

where $\psi := (1-ab\gamma)(\frac{ab\gamma}{w})^{\frac{ab\gamma}{1-ab\gamma}}$.

Proposition 1 *Given demand shock X, the firm value is*

$$V(X) = \begin{cases} A(X_I^*)\big(\frac{X}{X_I^*}\big)^{\alpha} & \text{if } X < X_I^*, \\ A(X) & \text{if } X \geq X_I^*, \end{cases}$$

where

$$A(X) = \phi K^{*\frac{(1-a)b\gamma}{1-ab\gamma}} X^{\frac{1}{1-ab\gamma}} - (\delta_0 + \delta_1 K^*)$$

with $\phi := \frac{\psi}{r - \frac{\mu}{1-ab\gamma} - \frac{ab\gamma\sigma^2}{2(1-ab\gamma)^2}}$ *and* $\alpha := \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2r}{\sigma^2}}$.

The optimal capital $K^* := K^*(X_I^*)$ *and investment threshold* $X_I^* := X_I^*(K^*)$ *are determined by*

$$K^*(X) = \Big\{\frac{(1-a)b\gamma\phi}{\delta_1(1-ab\gamma)}\Big\}^{\frac{1-ab\gamma}{1-b\gamma}} X^{\frac{1}{1-b\gamma}},$$

$$X_I^*(K) = \Big\{\frac{\alpha(\delta_0 + \delta_1 K)}{(\alpha - \frac{1}{1-ab\gamma})\phi}\Big\}^{1-ab\gamma} \Big(\frac{1}{K}\Big)^{(1-a)b\gamma},$$

and the optimal labor at the timing of investment is $L^* := L^*(X_I^*, K^*)$.

3.2 Partially adjustable labor

Now suppose that the firm can make incremental adjustment in labor but only with irreversibility. That is, once the amount of labor is increased, it cannot be reduced afterwards.

Lemma 1 *The firm's optimal choice of labor at time t given capital K is*

$$\hat{L}^*(X_t, K) = \Big(\frac{(\alpha-1) rab\gamma \bar{X}_t K^{(1-a)b\gamma}}{\alpha w(1+r)(r-\mu)}\Big)^{\frac{1}{1-ab\gamma}}$$

where $\bar{X}_t := \sup_{0\leq s\leq t} X_s$.

Proposition 2 *Given demand shock X, the firm value is*

$$\hat{V}(X) = \begin{cases} \hat{A}(\hat{X}_I^*, \hat{L}^*(\hat{X}_I^*))\Big(\frac{X}{\hat{X}_I^*}\Big)^{\alpha} & \text{if } X < \hat{X}_I^*, \\ \hat{A}(X, \hat{L}^*(X)) & \text{if } X \geq \hat{X}_I^*, \end{cases}$$

where $\hat{L}^*(X) := \hat{L}^*(X, \hat{K}^*)$ *and*

$$\hat{A}(X,L) = \frac{L^{ab\gamma}\hat{K^*}^{(1-a)b\gamma}}{r-\mu}X - \frac{wL}{r} + \chi L^{1-\alpha(1-ab\gamma)}\hat{K^*}^{\alpha(1-a)b\gamma}X^{\alpha} - (\delta_0 + \delta_1\hat{K}^*)$$

with $\chi := \left(\frac{(\alpha-1)r}{w(1+r)}\right)^{\alpha-1}\left(\frac{ab\gamma}{\alpha(r-\mu)}\right)^{\alpha}\frac{1}{\alpha(1-ab\gamma)-1}$.

The optimal capital $\hat{K}^*$ *and investment threshold* $\hat{X}_I^*$ *are determined by*

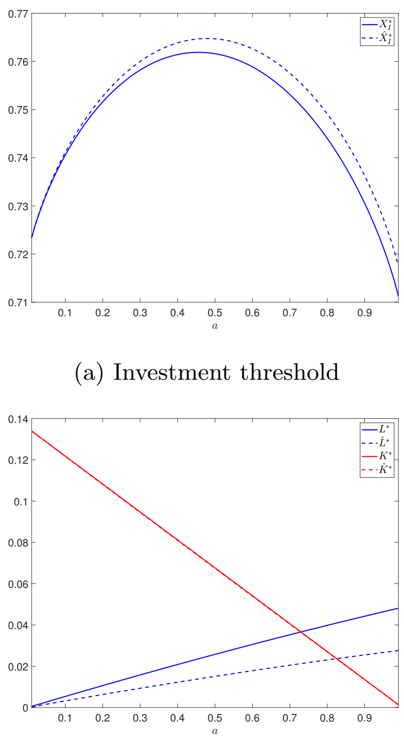
$$\hat{X}_I^* = \frac{\alpha(r-\mu)(\frac{w\hat{L}^*}{r} + \delta_0 + \delta_1\hat{K}^*)}{(\alpha-1)\hat{L^*}^{ab\gamma}\hat{K^*}^{(1-a)b\gamma}},$$

$$(1-a)b\gamma\frac{\hat{X}_I^*}{r-\mu}\hat{L^*}^{ab\gamma}\hat{K^*}^{(1-a)b\gamma-1} + \alpha(1-a)b\gamma\chi\hat{L^*}^{1-\alpha(1-ab\gamma)}\hat{K^*}^{\alpha(1-a)b\gamma-1}\hat{X_I^*}^{\alpha} = \delta_1,$$

with the optimal labor at the timing of investment $\hat{L}^* := \hat{L}^*(\hat{X}_I^*, \hat{K}^*)$.

4 Comparative statics

We adopt the following benchmark parameters for comparative statics: $r = 0.06$, $\mu = 0.02$, $\sigma = 0.2$, $b = 0.9$, $\gamma = 0.1$, $w = 1$, $\delta_0 = 5$, $\delta_1 = 10$.

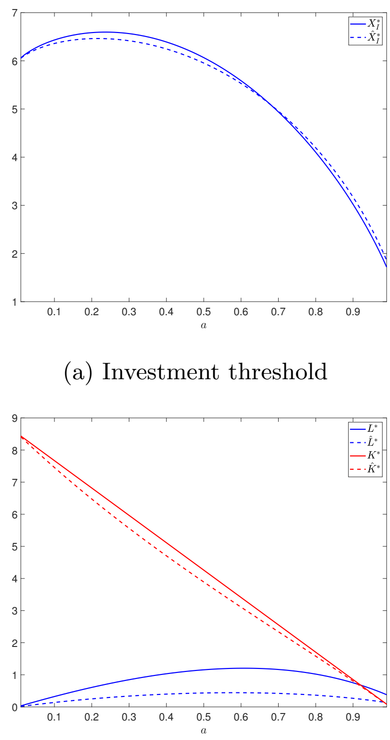


(a) Investment threshold

(b) Labor and capital

Figure 1: Labor share and investment decisions based on benchmark parameters

Figure 1a shows that the investment thresholds are concave with respect to a. This comes from the convexity of average product of capital and the concavity of average cost of capital regarding the labor share in production.



(a) Investment threshold

(b) Labor and capital

Figure 2: Labor share and investment decisions for $\sigma = 0.3$, $b = 1$, and $\gamma = 0.3$

Figure 2a shows that investment can be made earlier with partially adjustable labor than it would with fully adjustable labor (i.e., $\hat{X}_I^* < X_I^*$) when demand is volatile and price-elastic and labor share is low. This finding is associated with the sensitivity of marginal value of capital with regard to demand shocks.

A more detailed illustration on the results and related empirical evidence will be given at the presentation.

References

[1] Abel, A., 1983, *Optimal investment under uncertainty*, American Economic Review 73, 228-233.

[2] Caballero, R., 1991, *On the sign of the investment-uncertainty relationship*, American Economic Review 81, 279-288.