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Corporate investment, financing, and exit decisions with an earnings-based borrowing constraint

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1. Introduction

Economics literature has traditionally investigated the effects of liquidation value-based borrowing constraint (LBC) on corporate investment and financing decisions. Under LBC, a firm faces a debt issuance capacity based on the liquidation value of the firm's specific assets. However, recent empirical studies, such as [1], have shown the prevalence of earnings-based borrowing constraint (EBC) over LBC. Under EBC, debt capacity is based on operating earnings rather than asset values. This paper shows how differently from LBC, EBC affects corporate investment, financing, and exit decisions.

2. Model Setup

The model builds on the standard setup of investment with optimal capital structure based on tradeoff theory. Consider a firm that has an option to invest in a new project by incurring capital expenditure $I(>0)$. At the investment time, the firm can issue consol debt with coupon C. The project generates continuous streams of earnings before interest and taxes (EBIT) $X(t)$, which follows

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}B(t)\ (t>0), \quad X(0)=x,$$

where $B(t)$ denotes the standard Brownian motion, and $\mu, \sigma(>0)$ and $x(>0)$ are constants. For convergence, we assume that $r>\mu$, where a positive constant r denotes the risk-free interest rate. For $X(t)-C$, the corporate tax rate $\tau \in (0,1)$ is applied.

The firm optimizes both investment time T^i and coupon C to maximize the investment option value. At investment time T^i, EBC

$$D^d(X(T^i), C) \le \phi_E X(T^i) \qquad (1)$$

is enforced, where $D^d(X(T^i), C)$ and ϕ_E denote the risky debt value at time T^i and the tightness parameter of EBC. EBC (1) means that the cap of debt is based on EBIT. This type of EBC is the most prevalent among various types.

We assume partial investment reversibility, where a fraction $k \in (0,1)$ of capital expenditure I remains as liquidation value. The firm choose an exit type among sellout, liquidation bankruptcy, and reorganization bankruptcy when EBIT $X(t)$ deteriorates. On sellout, debt holders are repaid the principal of debt, and shareholders receive positive residual value. On liquidation bankruptcy, debt holders receive liquidation value, while shareholders receive nothing. On reorganization bankruptcy, debt holder receive going-concern value, while shareholders receive nothing.

3. Model Solutions

The model is solved backward. First, we derive the going-concern value of the firm for debt holders. At bankruptcy threshold $x^d(C)$, the going-concern value is derived as

$$G(x^d(C)) = (1-\tau)(1-\alpha)\left(\frac{x^d(C)}{r-\mu} + \left(\frac{x^d(C)}{x^l}\right)^{\gamma}\left(kI - \frac{x^l}{r-\mu}\right)\right) \quad (x^d(C) \ge x^l),$$

where $x^l = \gamma(r-\mu)kI/(\gamma-1)$ is the liquidation threshold (optimized by former debt holders), and $\alpha \in (0,1)$ denotes bankruptcy cost. Notation $\gamma = 0.5 - \mu/\sigma^2 - \sqrt{(\mu/\sigma^2 - 0.5)^2 + 2r/\sigma^2}$ is the negative characteristic root. Debt holders choose reorganization bankruptcy if $G(x^d(C))$ is higher than liquidation bankruptcy value $(1-\alpha)kI$. Otherwise, they choose liquidation bankruptcy. With lower k and higher C and $x^d(C)$, the firm is more likely to go into reorganization bankruptcy, which is consistent with empirical evidence.

Next, we consider shareholders' exit choice between sellout and default. Shareholders choose sellout for positive residual value, i.e., $C \le C^s = r(1-\tau)kI$.

In this region, at investment threshold x^i, the equity value is derived as

$$E^s(x^i, C) = (1-\tau)\left(\frac{x^i}{r-\mu} - \frac{C}{r} + \left(\frac{x^i}{x^s(C)}\right)^\gamma \left(kI - \frac{\tau C}{(1-\tau)r} - \frac{x^s(C)}{r-\mu}\right)\right) \quad (x^i \geq x^s(C)),$$

where $x^s(C) = \gamma(r-\mu)(kI - \tau C/(1-\tau)r)/(\gamma-1)$ is the sellout threshold (optimized by shareholders). The debt value is the riskless value $D^d(x^i, C) = C/r$.

Shareholders choose default for negative residual value, i.e., $C > C^s$. In this region, the equity value is derived as

$$E^d(x^i, C) = (1-\tau)\left(\frac{x^i}{r-\mu} - \frac{C}{r} + \left(\frac{x^i}{x^d(C)}\right)^\gamma \left(\frac{C}{r} - \frac{x^d(C)}{r-\mu}\right)\right) \quad (x^i \geq x^d(C)),$$

where $x^d(C) = \gamma(r-\mu)C/(\gamma-1)r$ is the default threshold (optimized by shareholders). The debt value is

$$D^d(x^i, C) = \frac{C}{r} + \left(\frac{x^i}{x^d(C)}\right)^\gamma \left(\max\{(1-\alpha)kI, G(x^d(C))\} - \frac{C}{r}\right).$$

Finally, we consider the firm's investment and financing decisions. For riskless debt financing, i.e., $C \in [0, C^s]$, no financial constraints is imposed. The firm optimally chooses C^s that maximizes the tax benefits of debt. At time 0, the riskless firm value becomes

$$V^s(x) = \sup_{x^i \geq x} \left(\frac{x}{x^i}\right)^\beta \left(\frac{(1-\tau)x}{r-\mu} + \frac{\tau C^s}{r} - I + \left(\frac{x^i}{x^s(C^s)}\right)^\gamma \left((1-\tau)kI - \frac{(1-\tau)x^s(C^s)}{r-\mu} - \frac{\tau C^s}{r}\right)\right),$$

where x^i is the investment threshold (optimized by ex-ante shareholders). Notation $\beta = 0.5 - \mu/\sigma^2 + \sqrt{(\mu/\sigma^2 - 0.5)^2 + 2r/\sigma^2}$ is the positive characteristic root.

For risky debt financing, i.e., $C > C^s$, EBC (1) is imposed. The risky firm value becomes

$$V^d(x) = \sup_{x^i \geq x, C > C^s} \left(\frac{x}{x^i}\right)^\beta (E^d(x^i, C) + D^d(x^i, C) - I)$$
$$\text{subject to } D^d(x^i, C) \leq \phi_E x^i.$$

The firm optimally chooses between riskless and risky debt financing, and hence, the initial firm value is $V(x) = \max\{V^s(x), V^d(x)\}$.

We also examine a model with LBC to highlight the differences between EBC and LBC. In the LBC model, we constrain the risky firm by $D^d(x^i, C) \leq \phi_L(1-\alpha)kI$, where ϕ_L denotes the tightness parameter of LBC. This LBC model is essentially the same as in [2].

We numerically show the following results. Although very tight EBC (i.e., very low ϕ_E) induces the firm to resort to riskless debt, plausible levels of EBC lead the firm to use risky debt financing with realistic levels of leverage. The firm can increase the cap of debt under EBC by delaying investment, although investment timing is not related to the cap of debt under LBC. Then, unlike with LBC, the firm with EBC delays investment to utilize more debt financing. Investment reversibility k does not largely affect the firm with EBC, although it greatly affects the firm with LBC. The difference implies that the firm with low investment reversibility, which leads to reorganization bankruptcy rather than liquidation bankruptcy in case of financial distress, prefers EBC to LBC. This also leads to a positive relation between prevalence of EBC and reorganization bankruptcy through low liquidation value. Higher volatility σ increases the cap of debt under EBC by delaying investment, although the cap of debt under LBC does not depend on investment timing. Then, contrary to the results in unconstrained and LBC models, under EBC, the firm with higher volatility increases leverage and choose reorganization bankruptcy in case of financial distress.

References

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[2] T. Shibata and M. Nishihara. Investment timing, reversibility, and financing constraints. *Journal of Corporate Finance*, 48:771–796, 2018.