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A Note on Component Criticality Importance Measures of Reliability Models under Epistemic Uncertainty

05001387 Hiroshima University ZHANG Jiahao
05000255 Ritsumeikan University ZHENG Junjun
01013754 Hiroshima University OKAMURA Hiroyuki
01307065 Hiroshima University DOHI Tadashi

1. Introduction

Fault tree (FT) as one of the model-based evaluation methods plays an important role in reliability and represents the relationship between component failures and system failure. FT provides some quantitative measures for the system reliability from component reliabilities, which brings convenience when maintenance the system. However, actual systems often consist of multiple components, which brings insights into system maintenance, e.g., which component should be maintened first. However, actual systems often consist of multiple components, which causes high cost of maintenance. Importance analysis is the well-known analysis method for estimating the importance of system components that contributes to the whole system, aiming at figuring out the important or critical components by the order of their importance. As far as concerned, criticality importance (CI) measure plays as one of the center roles in estimating components importance. Since the computation of importance measure requires the model parameters estimated from data samples or expert's knowledge. Another problem should be taken into account is that since CI measure is the degree of components failure to system failure, the maintenance of components is from low importance of CI measure. However, the limitation of numbers of data may bring the unpredictable impact on the estimation accuracy of model parameters, which is called epistemic uncertainty. Due to the uncertainty in the model parameters, the estimated importance measure could be different from what it should be. In general, the uncertainty can be solved by applying the Bayes estimation, but the credibility of the results is hard to measure. So in this paper, we want to present an importance analysis method through the stationary analysis of continues-time Markov chain (CTMC) based on CI measure in FT, aiming at ensuring whether the component importance when model parameters contain uncertainty is reliable or not.

2. Fault Tree

Consider a FT to represent the relationship between system and component failures. The FT can be represented by a structure function. Let $F(x_1, \ldots, x_n)$ be a structure function where $x_1, \ldots, x_n$ are the states of basic events. Then $F(x_1, \ldots, x_n)$ means the state of top event. In the structure function, AND and OR gates are expressed by $x_1 x_2$ and $1-(1-x_1)(1-x_2)$, respectively. Note that this is slightly different from a simple boolean function in the Boolean algebra. By using such expression, the system reliability R_s can be obtained by $R_s = F(R_1, \ldots, R_n)$ where $R_1, \ldots, R_n$ are component reliabilities.

3. Importance under Epistemic Uncertainty in FT

Consider the CI measure in FT, for i-th component, the definition of CI measure can be obtained by;

$$I_i^{CR} = \frac{\partial F(R_1(t), \ldots, R_n(t))}{\partial R_i(t)} \frac{R_i(t)}{F(R_1(t), \ldots, R_n(t))}. \quad (1)$$

Since the uncertainty in model parameters can affect the final results, the uncertainty analysis should be focused on. In the Bayes theory, let the model parameters involve uncertainty as the random variable, assume that the prior probability density function $f_\Lambda(\lambda)$ of the failure rate Λ, and given the independent and identically distributed (i.i.d.) samples $\mathcal{D} = (t_1, \cdots, t_n)$ as observed failure times of a component in the system. The effects from data samples can be obtained from the posterior density function of the failure rate $f_\Lambda(\lambda|\mathcal{D})$. Assume the reliability of the components follows the formula $R(\lambda; t)$, the mean of

$R(\lambda;t)$ is defined as;

$$E[R(\lambda;t)] = \int_0^{\infty} R(\lambda;t) f_{\Lambda}(\lambda|\mathcal{D}) d\lambda. \quad (2)$$

On the other hand, although Bayes estimation can help to estimate importance measure when model parameters involve uncertainty, the estimated results should be verified. We know that CI measure indicates how much components failure devote to the system failure, which means that the larger the CI measure, the more critical the corresponding component is. However, due to the uncertainty, the value of CI measure can be changed, which means there exists a probability that the CI measure of component i can be larger than component i (without uncertainty component i is smaller than component j). The dynamic of this relationship can be captured by CTMC, where the change probability can be regarded as a transition rate. Consider a FT composed by only AND gates, which means $F(R_1(t), \ldots, R_n(t)) = 1 - \prod_{i=1}^{n}(1 - R_i(t))$. Then the probability when component i is critical than j can be computed by;

$$\begin{aligned} p_{i,j} &= P(I_i^{CR} > I_j^{CR}) = P(R_i(t) > R_j(t)) = P(\Lambda_i > \Lambda_j) \\ &= \int_0^{\infty} \bar{F}_{\Lambda_i}(\lambda|\mathcal{D}) dF_{\Lambda_j}(\lambda|\mathcal{D}), \quad (3) \end{aligned}$$

where $F_{\Lambda}(\lambda|\mathcal{D})$ is the posterior cumulative distribution function (c.d.f.) of Λ, and $\bar{F}_{\Lambda}(\lambda|\mathcal{D}) = 1 - F_{\Lambda}(\lambda|\mathcal{D})$. Obviously, $p_{j,i} = 1 - p_{i,j}$. Assume that there exist n components. Defining the transition rate matrix (i.e., infinitesimal generator) of the underlying CTMC in terms of $p_{i,j}$ as follows,

$$\boldsymbol{Q} = \begin{pmatrix} p_{i,i} & \cdots & p_{i,n} \\ \vdots & \ddots & \vdots \\ p_{n,i} & \cdots & p_{n,n} \end{pmatrix} \quad (4)$$

where $p_{i,i} = -\sum_{k \neq i}^{n} p_{i,k}$. Since the dynamics of the transition process can be captured by the above CTMC, the probability that a component is most critical among the current component's set can be computed by solving the linear equation $\boldsymbol{\pi Q} = \boldsymbol{0}, \boldsymbol{\pi 1} = 1$.

4. Numerical Illustration

In this section we consider CI measure when model parameters contain uncertainty in FT. In particular, the results are verified through the stationary analysis of CTMC. Assume a FT composed of four components with a AND gate, the system reliability can be computed as $R_S = 1 - \prod_{i \in \mathrm{H}}(1 - R_i)$ where H = {A, B, C, D}, and the component reliabilities can be computed by the well-known formular $R(\lambda;t) = e^{-\lambda t}$. the failure rate λ of component i $\in$ H are set as $\lambda_A = 1/2,50,000$, $\lambda_B = 1/480,000$, $\lambda_C = 1/670,000$, $\lambda_D = 1/120,000$. Also, the uncertainty of model parameters is injected by set the number of samples as 5.

Table 1 shows the CI measure for epistemic uncertainty of system reliability. It's obvious that the results when model parameters contain uncertainty are different from the ones without uncertainty, even the ranks of importance are different. But whether the result is credible or not, we apply the stationary solution of CTMC to verify. By using the Eq. (3) to (4), the transition rate matrix can be computed and the probability of importance rank can be obtained from Table 2. In the table, 'rank1' means the most critical component that devotes to the system failure, the probability of 'rank1' means the probability of components being the most critical component. From the Table 2, we can see that component A has the highest probability to become the most critical component, and by the comparison of the results between Table 1 and 2, the rank of components become the 'rank1' or 'rank2' has the same rank of the CI measure rank with uncertainty. Although there is no 'Rank4', it can be obtained component D is the last critical component in the system.

Table 1: CI measure with uncertainty.

Component	Without uncertainty		Sample size = 5	
	Importance	rank	Importance	rank
A	3.24e-12	1	3.73e-12	1
B	6.22e-13	3	1.45e-12	2
C	8.68e-13	2	1.00e-12	3
D	1.55e-13	4	2.64e-13	4

Table 2: Probability of importance rank.

Component	Rank1	Rank2	Rank3
A	9.02e-01		
B	7.15e-02	9.03e-01	
C	2.56e-02	7.15e-02	9.24e-01
D	6.03e-04	2.56e-02	7.56e-02

References

[1] R.M. Fricks, and K.S. Trivedi, "Importance analysis with Markov chains," In: Proc. Annual Reliability and Maintainability Symposium, IEEE, pp. 89-95, 2003.